DETERMINANTS OF PROBLEM-SOLVING PERFORMANCE: BASIS FOR MATHEMATICAL MODEL DEVELOPMENT

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Abstract

This study aimed to look into the determinants of problem solving performance among the pre-service mathematics teachers with the end view of developing a mathematical model. Specifically, it looked for the description of the pre-service mathematics teachers in terms of the following variables: epistemological belief, motivation, curiosity, cognition, and metacognition; level of problem solving performance, relationship of the predictor variables under study and the mathematics problem solving performance, best predictor of mathematics problem solving if taken singly or in combination. Mathematical model was developed as output of the study. A researcher-made questionnaire and test were the instruments used in gathering the data needed in the correlational research. Purposive sampling method was used to obtain the 118 pre-service mathematics teachers who served as respondents. The statistical tools used were weighted mean , mean and standard deviation, Pearson's product moment correlation coefficient and stepwise multiple regression analysis. The study revealed that the pre-service mathematics teachers had strong epistemological belief, strong motivational belief, belief in curiosity, belief in cognitive style and metacognitive learning style as reflected in strong agreement with those variables. It also revealed that the pre-service mathematics teachers had average performance in mathematics problem solving as shown in the result in problem solving performance test. Findings also revealed that the problem solving performance was significantly related to epistemological belief, curiosity, cognitive and metacognitive learning style. When taken singly, the best predictor among those variables was the epistemological belief and when in combination, their epistemological belief, cognitive and the metacognitive learning style gave the best results. As a result of this study, a mathematical PSP model was produced to enhance the mathematics problem solving performance of pre-service mathematics teachers.

Keywords: Epistemological belief; Motivation; Curiosity ; cognitive style; metacognitive style, Problem solving in Mathematics

INTRODUCTION

Problem solving is knowing what to do when one does not immediately know what to do. Not a problem solving if answer can immediately seen or solve. It is the ability to apply mathematics (skill and concept) in different situations. It is important because it requires combination of skills and concepts in order to deal a with specific mathematical situation. It implies therefore when skills and concepts cannot be used or put together well, the individual cannot do as well in mathematics. Mathematical literacy is totality of skill, concept and problem solving. Mathematics teachers believe that mathematical problem solving involves much more than the routine use of algorithms. Instead, problem solving should engage students in rigorous and complex tasks that require them to think, reason, communicate, and apply their understanding of number concepts and operations. Problem solving is fundamental to the acquisition of deeper mathematical understanding but that frequently students find such activities to be difficult and de-motivating. Solving problem is not just a means for finding correct answers. Rather, it is a vehicle for developing logical thinking; it provides a context for mathematics and an opportunity for the transfer of newly acquired concepts and ideas, to the degree that problems simulate real life; solving them endows mathematics with meaning. Unfortunately, solving word problems is one of the least popular and least addressed aspects of the mathematics curriculum among students and teachers.

One important factor related to problem- solving success is the intrinsic motivation, that is students' willingness to persist in the solving. It has long been acknowledged that motivation is important: for example if the students try to participate in the problem situation and interact with peers positively, that behavior is regarded as a highly motivated activity. By referring to their behaviors, problem solvers' problem solving behaviors in the situation may be influenced by motivation.

Belief also plays a great role in teaching and learning for producing mathematically literate individuals. Epistemology is an area of philosophy concerned with the nature and justification of human knowledge. A growing area of interest for psychologists and educators is that of personal epistemological development and epistemological beliefs: how individuals come to know, the theories and beliefs they hold about knowing, and the manner in which such epistemological premises are a part of and an influence on the cognitive processes of thinking and reasoning. As defined by Schommer (2004) , epistemological beliefs refer to beliefs about the nature of knowledge (including its structure and certainty) and knowledge acquisition (including sources and justification of math knowledge) Students' epistemological beliefs have become one of the critical components of understanding student learning, deeply influencing and mediating the learning process and the learning outcome. These beliefs are likely to influence how students learn, how teachers knowingly or unknowingly modify students' epistemological beliefs. Evidence is accumulating to support the notion that the students' epistemological beliefs play an important role in their learning. For example, various studies indicate that the more students believe in certain knowledge, the more likely they are to draw absolute conclusion from tentative text. The more students believe in fixed ability, simple knowledge and quick learning, the more likely they are to comprehend text poorly or earn lower grade point. The more students believe in fixed ability, the less likely they are to value academic task. If the teacher can ascertain students' epistemological beliefs, they can perhaps adapt instruction to guide lower achieving students into higher level thinking, and conversely, they can adapt instruction for higher achieving students to help them grow

Many teachers agree that solving problems mathematically involves important cognitive dispositions and skills. Cognition refers to knowing and thinking (Ashman, 2000). It involves taking in, storing, retrieving, transforming, and manipulating information that is obtained through the senses. It also involves perception, awareness, judgment, the understanding of emotions and, memory and learning. Almost everything people do during their waking lives involves thinking and perhaps while asleep as well!. Indeed, it is fairly difficult to identify aspects of daily lives that involve no thinking..

Mathematics is a living subject which seeks to understand patterns that permeate both the world and the mind. Although the language of mathematics is based on rules that must be learned, it is important for motivation that students move beyond rules to be able to express things in t It is almost a universal thinking that mathematics is a difficult subject for many students. Educators and education researchers have identified difficulties in learning a broad concept of mathematics. Mathematics education teachers and researchers have to look at variables which are both in cognitive and noncognitive variables affecting the mathematics performance, for example looking at students' attitude, expectation, epistemologies. For instance, students may think that mathematics is just simply memorizing formulas and can be best learned by absorbing information from the authority or building up their own ideas. This discipline-specific epistemology research builds on extensive research on more generalized epistemology.

The biggest challenge of teachers in the classroom is listening to the students, responding to their difficulties, and facilitating the use of productive cognitive resources they possess. In diagnosing student learning, mathematics teachers must consider the strengths and difficulties of an epistemological nature. Specifically, teachers must learn to identify the epistemological resources that students possess and to understand which resources they are using during the learning process, so that they help students to choose the more productive approaches to learning.

It has been observed and noticed in any level of education that students have negative attitude in mathematics. If students cannot avoid taking it, they will do it but it is impossible since mathematics is part of any curriculum. It is not only the attitude but also the mathematics performance, which is a great issue not only in Philippine education but universally. As an educator there is a need to address those issues to trace the root cause of those weaknesses. The goal of mathematics for the pre- service education teachers focuses on ensuring that they understand the basic mathematics concepts they will teach and have access to developmentally appropriate pedagogy and practices.

It is also observed when the episode in the classroom is on problem solving, sudden change in the degree of motivation, attitude occur, as such teacher finds difficulty dealing with these situations. Problem solving cannot just be put aside in teaching mathematics because this is the ultimate application of teaching mathematics. So in this part of learning where the belief that students are able to solve the problem contributes to the success of doing it. Beliefs can be a gate to developing and learning problem solving to the maximum, as educators, therefore they need to find out more about these beliefs.

There is continuous deterioration in mathematics performance in spite of many interventions and studies on the strategies, and performance to address those difficulties. Results in studies were found to be affective but it seems the problems in the mathematics education are in the affective domain of learning, the attitude, beliefs of the learner in the subject. Specifically, the pre service mathematics teachers who later on will be mathematics teacher should develop a strong belief so that they can also be agents of developing strong belief of their students in the future.

Many teachers are concerned that for reasons of poor cognitive and meta-cognitive dispositions, or because of poor motivation, some students approach mathematics word problems aimlessly, randomly, and unsystematically. If these teachers are right, then students who perform poorly need to learn how to process mathematics word problems. They need instruction that targets the processes of problem solving they fail to do efficiently. Most teachers are aware of some of the obvious, but not all the important factors that determine the difficulty of solving word problems. Teachers must consider the critical role of pre- requisite mathematics knowledge (concepts and operations), and often must consider the language requirements and the level of complexity of the problems (number of steps or amount of information to be considered). With word problems, however, teachers often fail to consider the more subtle cognitive and meta-cognitive functions that discriminate between successful and failing attempts at solutions.

Students' epistemological beliefs about mathematical problem solving is an important component of their learning experience since they affect the students' involvement in mathematics learning activities as well as their mathematics performance. Being aware of how the students view mathematical problem solving, the teachers can provide more appropriate and effective instruction for the students in mathematic learning. Teachers may be able to help the students with poor performance become more interested in problem solving by cultivating their mathematical beliefs (Muis, 2007).

Epistemology is the study of the nature and scope of knowledge and justified belief. It analyzes the nature of knowledge and how it relates to similar notions such as truth, belief and justification. It also deals with the means of production of knowledge, as well as skepticism about different knowledge claims (Kuhn, D., Cheney, R., & Weinstock, M., 2000).

In comprehensive review of epistemological belief, Hofer and Pintrich (2002) offered that epistemological belief undoubtedly plays a role in academic motivation. Specifically, a particular course task may elicit consistent knowledge approaches to the materials. Depending on epistemological belief development, different courses may produce different approaches or the same approaches used habitually to the determinant of the students' academic achievement. Students with well developed epistemological belief may have more cognitive choice available for the selection of adaptive responses to instructor decision on classroom goal structure. Students with less developed epistemological belief may be potentially disadvantaged when faced with a course demand to adjust to an instructor course design embedded in a goal structure.

Objectives

This study aimed to look into determinants of problem solving performance among pre-service mathematics teachers with the end view of developing a mathematical model. Specifically the study aimed to described the pre-service mathematics teachers with regards to epistemological belief, motivation, curiosity , cognition and metacognition; to determine the level of problem solving performance; to relate epistemological belief, motivation, curiosity, cognition and metacognition to problem solving performance; to determine the best predictor of the problem solving performance.

Methodology

The study used the correlational design to study the relationship between the predictor variables and the criterion variables, Cohen (2003) suggests the focus of this type of design is to determine if relationship exists between two or more variables, thus enabling the researcher to predict, with quantified accuracy, the effect one variable (predictor) has on another variable. The purpose of correlational study is to determine the relationship between variables or to use relationship to make prediction. Frequently this relationship is described by an inferential statistics termed the Pearson r or simply by the correlational coefficient. This was used in this study to look for relationship of the defined variables in the study and to predict which variable described the problem solving performance. The subjects of the study were 118 pre-service mathematics teachers of Batangas State University system and Teacher Education colleges in Batangas City. Purposive sampling was used to determine the respondents of the present study. The sample of this study comprised of 118 pre-service mathematics teachers who met the minimum members in a sample for multiple regression analysis. As mentioned by Tabachnick & Fidell (2007) there are a number of recommendations for a suitable sample size for multiple regression analysis As a simple rule, the researcher calculated the following two values: 104 + m or 50 + 8 m, where m is the number of independent variables. This study used five independent variables so the needed sample size ranged from 90 to 109, thus 118 was acceptable as a sample size for multiple regression analysis.

The researcher used a questionnaire in gathering pertinent data as main instrument. It covered questions on epistemological belief inventory (EB), motivation (M), curiosity (C), cognitive learning (CL) and metacognitive learning (ML). The other instrument was the problem solving test prepared by the researcher. The items of the questionnaire were prepared based on readings and existing questionnaire on the variables under study. For the questionnaire was based on the instrument developed by Walker Wheeler (2007) ;the cognitive-style inventory based on Martin Lorna P(1998),Metacognitive Awareness Inventory (MAI) by Schraw, G. and Dennison, R.S. (1994), Curiosity Inventory by Kashdan et al. (2009) and Litman (2003) and the Motivation Learning Inventory by Wigfield A (2010).

The internal consistency reliability coefficients of the instrument were 0.880, 0.856. 0.870, 0.767, and 0.846 of EB, M, C, CL and ML, respectively indicating good reliability. A commonly accepted rule of thumb is that an alpha of 0.7 indicates acceptability and 0.8 or higher indicates good reliability (Peer, Willie van, Hakemulder, Frank, and Zyngier, Sonia.2012). If there are multiple factors in a study such as in the present research , there is a need to split the questionnaire into n factors and calculate the Cronbach's alpha for each n factor. The present research had five independent factors so the internal consistency was established for each factor.

RESULT AND DISCUSSION

Description of the Pre- service Mathematics Teacher

Epistemological belief. This is a belief about the nature of knowledge including its structure and certainty; and knowledge acquisition which includes sources and justification of math knowledge. The present study describes the pre-service mathematics in this variable. Table 1 presents the description of the pre-service mathematics teachers in terms of their epistemological belief. It can be noted that the pre-service mathematics teachers strongly agreed that best learning mathematics problem solving is facilitated by working practice problem which was in the first rank with a weighted mean of 3.71. This means that the pre-service mathematics teachers believe that better learning of problem solving is obtained through exposure to more exercises or practice problem.

This is in accordance with law of exercises of Thorndike that practice helps in increasing efficiency and durability of learning. To obtain mastery in the mathematics problem solving needs exercises or practice problem many times. Moreover, the pre-service mathematics teachers strongly agreed that the source of knowledge is the teacher who provides more practice problem which means knowledge in mathematics problem solving comes from an authority who is the mathematics teacher. As noted by BonJour (2002), knowledge is justified on the basis of observational evidence collected by experts yet the best way to learn that knowledge is by having a teacher explain it quickly

Table 1

Description of Pre-Service Mathematics Teacher in Terms of Epistemological Belief

Epistemological Belief	WM	VI
1. Best learning of mathematics problem solving is facilitated by working practice problems.	3.71	SA
2. You are confident that mathematical problem solving be learned if enough effort is exerted.	3.69	SA
3. Mathematic problem solving can be learned by being patient.	3.69	SA
4. Better study habits are key to success for students who have difficulties in mathematics.	3.69	SA
5. Mathematics teachers must show different ways to solve the same problem.	3.67	SA
6. In mathematics problem solving situation, you can be creative and discover things on your own.	3.66	SA
7. It is interesting to find different ways to work mathematics problem.	3.64	SA
8. When learning mathematics problem solving you can understand the material better if I relate it to real world.	3.63	SA
9. Learning good study skills can improve a person's mathematical ability.	3.61	SA
10. Mathematics is like a game that uses number, symbol and formulas.	3.57	SA

The epistemological beliefs of the pre-service mathematics teacher in learning mathematics problem solving were: confidence that mathematical problem solving could be learned if enough effort is exerted; and can be learned by being patient. Also better study habits are key to success for students who have difficulties in mathematics. These had same weighted mean of 3.69 indicating pre-service mathematics teachers strongly agreed on these beliefs. The above beliefs of learning mathematics problem solving are described as the control of knowledge acquisition which means ability to learn is genetically predetermined, ability to learn is acquired through experience. Pre-service mathematics teacher believed that efforts will result in improved mathematical ability and thus long-term success in mathematics. They also believe that it is the attitude that has something to do in their performance in mathematics problem solving. This is supported by Abedalaziz's and Akmar's belief that the efforts variable was the best predictor of dependent variable that had the most significant effect in predicting mathematics achievement. Some students believed that they lacked the ability to learn mathematics, whereas others believed that they can learn mathematics and improve their mathematical ability with sufficient effort.

Another epistemological beliefs that pre-service mathematics teachers strongly agreed on was the belief that mathematics teachers must show different ways to solve the same problem, as shown in weighted mean of 3.67. This belief refers to the nature of knowledge that mathematics is learned by showing different solutions of the experienced mathematics teacher. Respondents were aware that in problem solving there is no single problem that can be solved with a single strategy. This means, there are more than one solution in a single problem such as a mathematics problem that can be solved by working backward strategy and at the same time can be solved by using equation.

Furthermore belief that mathematics problem solving, students can be creative and discover things on own where the pre-service mathematics teachers strongly agreed on as shown in weighted mean of 3.66. It implies that pre-service mathematics teachers also believe that they themselves can also be sources of knowledge because by being creative, they are able to discover something new related to their present knowledge. In this sense, source of knowledge of pre-service mathematics teachers is from knowledge handed down by an omniscient authority *to* knowledge reasoned out through objective and subjective means. Trying to use different strategies in mathematical problem solving would help in developing the creativity of an individual which should be an important personal quality of the pre-service mathematics teachers

The pre-service mathematics teachers also strongly agreed that to learn problem solving one should have interest of finding different ways to work . Finding different ways to work in a problem to arrive at the answer will boost the students' confidence in learning mathematics problem solving. From the structured interview conducted, pre-service mathematics teachers were interested in different ways to solve a problem and believed that no single problem that could be solved with single a strategy. This implies that there are more than one strategies that can be used to solve for a single problem.

The mathematical belief where the pre-service mathematics teachers strongly agreed on was the belief that better learning of mathematics problem solving could be understood by students if the materials were related to real world, shown in weighted mean of 3.63. This implies that better learning is obtained if the knowledge presented is related to real life situation of the students. In this situation, the students are able to see the immediate application of mathematics in daily life. As cited by Nabeel Abedalaziz and Sharifah Norul Akmar (2012), students' sophisticated beliefs about problem solving may be due to the mathematics curriculum and teaching and evaluation processes. For instance, mathematics textbooks pay more attention to relating mathematical problem solving to real life situations with which the students are familiar. The contents are introduced through the description of real life situations and a number of open-ended questions are included in the textbooks. This enables students to understand more clearly that mathematical problem solving skills are dynamic and closely related to real life.

Belief that learning good study skills can improve a person's mathematical ability was in the ninth rank and with weighted mean of 3.61 with the pre-service mathematics teachers strongly agreeing on this epistemological belief. This is the belief of the respondents in terms of nature of learning, that learning acquisition of learning is through having good study habits. Study habits may serve as vehicle of learning mathematics problems and in other disciplines and there are times the success and failure of each student depends upon study habits.

Motivation. It refers to the forces encouraging a person to engage on a task or to pursue a goal; in the school setting, it is the reason for which a student works persistently to reach a desirable result .The study looked at the motivational description of the pre-service mathematics teachers. Table 2 presents the description of the pre-service mathematics teachers with regards to motivation.

Table 2

Description of Pre-Service Mathematics Teachers in Terms of Motivation

Motivational Belief	Weighted Mean	Verbal Interpretation
1. As you are taking mathematics class, you believe that you can succeed if you try hard enough.	3.75	SA
2. Solving challenging questions is helpful in developing higher thinking skill.	3.70	SA
3. Mathematical problem solving is useful and important to learn.	3.69	SA
4. You like mathematics course materials because they really challenge you to learn new things.	3.64	SA
5. You are confident you will to learn the basic concepts taught in math course.	3.64	SA
6. You prefer mathematics course materials because it they arouse your curiosity even it is difficult to learn.	3.53	SA
7. The most satisfying thing although they are in mathematics course is trying to understand the context as thoroughly as possible.	3.53	SA
8. Content area in mathematics course is very interesting.	3.53	SA
9. In the mathematics class you try to set and achieve high standard of excellence.	3.47	SA
10. You feel satisfied with what you are getting from mathematics course.	3.47	SA
11. You are confident you will understand the most complex materials presented by instructor in a mathematics class.	3.47	SA

It can be noted from the table that pre-service mathematics teachers strongly agreed that in taking mathematics, they could succeed if they try hard enough. This had highest weighted mean of 3.75 and was ranked first. This shows that pre- service mathematics teachers appraised their competence to succeed in problem solving through the effort they exerted. For example if students view themselves as capable to solve mathematical problems they will choose to perform that task. This implies that pre-service mathematics teachers believe that abilities are developed through effort and this could be beneficial to them and therefore should be also developed among students.

This motivational belief refers to self efficacy which is students' beliefs concerning the degree to which they are confident in accomplishing an academic task. This is in consonance with the study of Liem, Lau , and Nie (2007) who cited self-efficacy had direct influences on the deep learning and the achievement outcome. Moreover, in elementary math class it was reported that there was a positive correlation between effort and self- competence in mathematics. It is also proven in the study of Seo Daeryong (2000) that effort as a mediator of mathematical ability is defined as the extent to which students feel that they can increase their mathematical ability by studying math.

Another motivational beliefs among the pre-service mathematics teachers was the belief that solving challenging questions is helpful in developing higher thinking skills (task value) reflected weighted mean of 3.70 with pre-service mathematics teachers strongly agreeing on this. Pre-service mathematics teachers are motivated because they believed that through problem solving they will develop HOTS. They are motivated because of the task value in engaging in problem solving and also see the usefulness of learning problem solving. They perceived the importance and usefulness of the mathematical problem solving and value. This view was supported by the study of Badiee (2013) that if math tasks are challenging for students, they become more involved in learning and choose some goals that will help them to succeed. Moreover, according to Johnson, learners are motivated to learn and perform when they deem that learning or performance important.

The belief that mathematical problem solving is useful and important to learn which had a weighted mean of 3.69 with pre-service mathematics teachers strongly agreeing on this. Pre-service mathematics teachers perceive the importance of learning the problem solving and the usefulness of knowledge in problem solving to their future works. These motivate them to be knowledgeable about the content they will teach in the future, aware that they also need to know how to teach it, to become effective teachers.

Pre-service mathematics teachers are motivated to learn mathematics problem solving if the materials course presented challenge them to learn new things and are also motivated to learn the basic concepts taught because of the confidence that they are capable (self Efficacy). These two had a weighted mean of 3.64. These are the reasons of the respondents of engaging in the mathematical problem solving. Moreover, pre-service mathematics teachers anticipate their inborn capacity in learning and strongly agreed that this is a strong contributor into learning mathematics. This finding is parallel to the research of Pajares (2001) who tested the joint contribution of self-efficacy and intelligent to the prediction of achievement which found out that students' self-efficacy belief made a powerful and independent contribution to the prediction of the academic performance. This finding is also in consonance with the study of Marcou and George Philippou (2005) that elementary school students had high efficacy beliefs with respect to problem solving were confidence in their skills were more likely to achieve higher performance in mathematics problem solving.

The other motivational beliefs of the pre-service mathematics teachers such as preferring mathematics course because it aroused their curiosity even difficult to learn (goal orientation); most satisfying in mathematics course is trying to understand the context as thoroughly as possible (goal orientation); and content area in mathematics course is very interesting (task value) had equal weighted mean of 3.53 indicating the pre-service mathematics teachers' strong agreement in all of these which were ranked seventh. The first two were the reasons why students engage in mathematics problem solving, while the last one was the perception of interest, usefulness and importance of the task. Pre-service mathematics teachers perceived the mathematics course is very interesting. Research suggests that students who attach a high value to the task will use deeper cognitive and metacognitive strategies (McWhaw and Abrami, 2001).

The motivational beliefs among the pre-service mathematics teachers which were ranked tenth were: in mathematics class trying to set and achieving high standard of excellence (goal orientation); feeling satisfied with what is gotten from the mathematics course(task value) ; confidence to understand the most complex material presented by instructor in a mathematics class(self-efficacy). These had same mean of 3.47 indicating strong agreement among pre-service mathematics teachers on these motivational factors. Beliefs on setting and achieving high standard of excellence motivate them to learn mathematics problem solving, which according to mathematics authorities are the motivating powers in learning . This agrees with the research of Badiee (2013) that if students perceived that they were evaluated in the process of math problem solving based on mastery level, students become mastery- oriented and aware of the learning process.

Pre-service mathematics teachers are motivated to learn the mathematics problem solving because of their confidence that they could understand the most complex materials and were confident of their capability in performing the task. Awareness of the benefits gained from mathematics problem solving is also considered as a motivating power , if students know the usefulness of the concepts they will try to cope with them. Researchers have found that students with high self-efficacy were more likely to make use of deep cognitive strategies and engage in self-regulation than students with low self-efficacy (Silver, Smith, and Greene, 2001) and that motivational variables and students' learning engagement were linked in various ways as cited by Neuville,Frenay and Ebourgeois.

1.3 Curiosity. This is a desire to acquire new knowledge and new sensory experiences that motivate exploratory behavior. When people feel curious, they devote more attention to an activity, process information more deeply, remember information better, and are more likely to persist on tasks until goals are met. The immediate function of curiosity is to learn, explore, and immerse oneself in the activity. Table 3 gives the description of pre-service mathematics teachers in term of curiosity.

It can be seen from the table that curiosity brings pleasure when new solution to difficult mathematics is found. This had weighted mean of 3.61 indicating the pre-service mathematics teachers' agreement on this epistemic curiosity. This implies the desire of the pre-service mathematics teachers to obtain new knowledge. This is true, because the education students are well informed that education is a continuous process of acquiring knowledge.

Another curiosity description of the pre-service mathematics was the curiosity when given an incomplete puzzle and the tendency to find the final answer. This had a weighted mean of 3.60 giving strong agreement by the pre-service mathematics teachers. Curiosity is aroused because by nature, puzzles are teasers to an inquisitive mind and the inquisitive mind asks question and asking questions leads to new knowledge.

Moreover the pre-service mathematics teachers strongly agreed on their curiosity on how complicated problem is solved as shown in its weighted mean of 3.48. This is epistemic in nature, and because of this curiosity, they try to understand what the problem is. This shows the pre-service mathematics teachers' urge and their the intellectual interest in dealing with difficult mathematics

Table 3

Description of Pre-Service Mathematics Teachers in Terms of Curiosity

	Weighted Mean	Verbal Interpretation
1. It brings pleasure when a new solution to a difficult mathematics problem is found.	3.61	SA
2. If you are given an incomplete puzzle, you like to try to find final answer.	3.60	SA
3. Tend or try to understand how a complicated mathematics problem is solved.	3.48	SA
4. You are interested in discovering how the solution works in mathematics.	3.47	SA
5. When you learn something new, you try to find out more about this.	3.46	SA
6. Something that puzzles keeps you reading until you understand it.	3.42	SA
7. When someone asks you to solve a puzzle or riddle, you are interested in solving it.	3.42	SA
8. Keep thinking deeply about the solution of difficult mathematics problem	3.41	SA
Composite Mean	3.38	SA

Legend: **SA - Strongly Agree , A-Agree**

problem solving cases. This curiosity in dealing with mathematics problem solving contributes to enhance their skills in mathematics problem solving. At the same time, pre-service mathematics teachers don't get frustrated and do not easily give up in solving difficult mathematics problems as one of the personal characteristics of becoming a professional teacher is being patient.

Cognition. This is the mental action or process of acquiring knowledge and understanding through thought, experience, and the senses. Need for cognition' refers to the tendency of an individual to engage in effortful cognitive activities and to enjoy thinking. Table 4 presents the description of the pre-service mathematics teachers in term of their cognition. It can be seen from the table that the cognitive learning style of the pre-service mathematics teacher which was not being good enough in simply getting the answer but knowing how and why it is the answer. This had weighted mean of 3.53 with pre-service mathematics teachers strongly agreeing on this. As pre-service mathematics teachers, it is not enough to only give the answer but they should know how and why the answer was obtained. This would lessen abstractness on the solution process and enhance the mathematical development. This is the cognitive learning of the pre-service mathematics teachers.

The cognitive learning of the satisfaction after completing mathematical problem solving that required a lot of mental effort had a weighted mean of 3.47 which was strongly agreed on by pre-service mathematics teachers. This is the common feeling of students when a difficult mathematical task is correctly done; this also boosts their morale. Pre-service mathematics teachers believe that exposure to difficult mathematical problems sharpen and prepare them in their field of work as a good mathematics teacher.

Table 4

Description of Pre-Service Mathematics Teachers in Terms of Cognition

	Weighted Mean	Verbal Interpretation
1. It is not good enough for me just to simply get the answer but know how and why it is the answer.	3.53	SA
2. I feel satisfaction after completing a mathematical problem solving that requires a lot of mental effort.	3.47	SA
3. I really enjoy mathematical problem solving that involves coming up with new solution to the problems.	3.33	SA
4. I usually don’t quit trying to figure out the solution to a difficult mathematical solving problem.	3.33	SA
5. I prefer my life to be filled with puzzles that I must solve.	3.31	SA
6. I prefer mathematical task that is intellectually difficult and important than the one which is somewhat important but does not require much thought.	3.30	SA
7. Learning new ways to think excites vey much.	3.28	SA
8. I prefer something that requires much thought that challenges my thinking ability.	3.26	SA
9. Complex mathematics problems excite me.	3.19	A

Legend SA - Strongly Agree A –Agree

The pre-service mathematics teachers cited that they really enjoyed mathematical problem solving that involves coming up with new solution to the problems and did not quit in trying to figure out solution to a difficult mathematical solving problem. These two cognitive styles had a weighted mean of 3.33. These were strongly agreed on by the respondents. It is clear to the pre-service mathematics teachers that in problem solving, there is no single strategy to solve a problem and since they are those mostly good in mathematics, they find it interesting to solve and discover new techniques in arriving at the answer. According to Jaworski (2000) if teachers are to recognize that there are many problems in mathematics that have more than one answer and usually many different paths to arriving at an answer, then they need to meet problems of this nature. Teachers should provide students with experiences in problem solving that require use of all the mathematical resources, including cognitve styles of learning and other metacognitive skills. To understand how children learn and how that learning can be facilitated, teachers must give them a context to observe these processes in action.

The pre-service mathematics teachers agreed as shown in weighted mean of 3.31 that they preferred life filled with puzzles that must be solved. Pre-service mathematics teachers believe that puzzles help boost learning of important math-related skills and in teaching mathematics, puzzles are one of the instructional aids that enhance learning and positive attitude toward mathematics.

The pre-service mathematics teachers preferred mathematical tasks that are intellectually difficult and important than the one which is somewhat important but does not require much thought. This had a weighted mean of 3.30 indicating strong agreement among the pre-service mathematics teachers. This is true among the pre-service mathematics teachers because they need many activities which provide exposure in different mathematical tasks and advanced spatial skills. More important those which encourage thinking.

Learning new ways to think excite the pre-service mathematics teachers very much; preferred something that requires much thought that challenged their thinking ability; and perceived complex mathematics problem brought them excitement and thinking mathematical concept and problem was their idea of fun were the cognitive learning styles of the pre-service mathematics teachers reflected in weighted means ranging from 3.19 to 3.28 and an affirmation of agree to strong agreement. These show that pre-service mathematics teachers have good outlooks to learning mathematics problem solving; these further show that they will be good mathematics teachers. To be good mathematics teachers in the future, they need sufficient knowledge of mathematics, need to have a profound understanding of basic mathematics and are able to perceive connections between different concepts and fields. These results also show enjoyment while learning mathematical problem solving.

Metacognition. By making students aware of which strategies can be used for different tasks and then letting them try out what works best for them, mathematics teachers can assist them by providing a framework for meta-cognition based on assessment. They can encourage students to take active initiatives in their own learning process. Being aware of own thought process, knowing to go about problem solving, and performing decision making and interpretation of the written word are some examples of the activities involved. Since learning uses the self as the subjects reflection is a prerequisite for, as well as a result, of learning. Table 5 presents the description of pre-service mathematics teachers in terms of metacognition.

It can be noted from the table that pre-service mathematics teachers strongly agreed that they learned best when they were interested in problem solving which had a weighted mean of 3.56. This affirms that interest stimulates the pre-service mathematics teachers to obtain knowledge.

The pre-service mathematics teachers slowed down and consciously focused when they encounter important information. The weighted mean of 3.53 infers and the pre-service mathematics teachers strongly agreed on this. This shows that the pre-service mathematics teachers have the skills and strategy sequences used to process information more efficiently. The pre-service mathematics teachers likewise strongly agreed that they translated new information into their own words. This had a weighted mean of 3.5

illustrating the importance of metacognition in bringing about the success in mathematics problem solving. This thinking style enhanced understanding of the respondents in dealing with difficult problems and is also an indication that they understood what they read.

Metacognitive learning style of pre-service mathematics teachers were: drawing picture or diagram to help understand while learning is described; and reading the instruction carefully before beginning a task and stopping and rereading when confused which had same weighted mean of 3.44 indicating strong agreement. The results indicate the information management strategy used by the pre-service mathematics teachers in dealing with mathematical problem solving by using diagrams and strategy used to correct comprehension and performance error by re-reading and following instruction.

Table 5

Description of Pre-Service Mathematics Teachers in Terms of Metacognition

Metacognitive Learning Styles	Weighted Mean	Verbal Interpretation
1. I learn best when I am interested in mathematical problem solving.	3.56	A
2. I slow down and consciously focus when I encounter important information.	3.53	SA
3. I try to translate new information into my own words.	3.52	SA
4. I draw pictures or diagrams to help me understand while learning.	3.44	SA
5. I read instructions carefully before I begin a task.	3.44	SA
6. I stop and reread when I get confused.	3.44	SA
7. I can motivate myself to learn mathematical problem solving when I need to.	3.40	SA
8. I think of several ways to solve a problem and choose the best.	3.39	SA
9. I try t o use strategies that have worked in the past.	3.38	SA
10. I understand my intellectual strengths and weaknesses.	3.36	SA

This further shows the pre-service mathematics teachers have enough skills to monitor or regulate their learning in mathematics problem solving. Moreover, these also show the importance of metacognition in bringing about the success in mathematics problem solving. Scheonfeld (2010) noted that good problem-solvers constantly question their achievement and generate a number of possible options to the method of solution. By making careful moves such as pursuing productive leads and abandoning fruitless paths, they solve the problem successfully.

Level of Problem Solving Performance of Pre-service Mathematics Teachers

The problem solving performance of the pre-service mathematics teachers was also looked into by the researcher in study. This was measured through the instrument constructed by the researcher. Mathematical problem solving is a performance assessment test that requires judging a student's overall performance on a problem, making it more complex than simply marking an answer right or wrong. Rubrics were used as scoring guide and to reduce subjectivity in scoring mathematics problems answer .Table 6 presents the level of the problem solving performance of the pre-service mathematics teachers. It can be seen that there were 44 or 37.28 percent with the average performance. There were 28 or 23.73 percent of the pre-service mathematics teachers with above average while 24 or 20.34 percent had below overall performance. Seventeen or 14.07 percent were classified with outstanding performance; and although the respondents were pre-service mathematics teachers, there were five of them who had poor performance in problem solving. Generally, the overall performance of the pre-service mathematics teachers was average as shown in its mean of 27.12 and SD of 10.24.

Most likely from the pre-service description in the previous question of this study revealed that epistemological belief, cognitive, metacognitive learning style are contributing factors in the mathematics problem solving. This might be the reason why there only few had poor performance. Moreover, the study shows that pre-service mathematics teacher had adopted a learning style and know also how to regulate and monitor their learning to helps in coping with the difficulties and these could be the reason of raising performance from pool level to average level.

Table 6

Level of Problem Solving Performance of Pre-service Mathematics Teachers

Performance Level	Frequency	Percentage
Outstanding (41-50)	17	14.07
Above Average (31-40)	28	23.73
Average (21-30)	44	37.28
Below Average(11-20)	24	20.34
Poor(0 – 10)	5	4.24
TOTAL	118	100.00

Legend: $\bar{X} = 27.12$, $SD = 10.24$

From the interview conducted, some of the pre-service mathematics teachers performed poorly in mathematics problem solving due to poor comprehension, poor foundation in early mathematics specifically algebra and trigonometry, lack of computational skills and conceptual knowledge. These impeded their ability to solve problems. But as they moved to higher year where most of them became focused in to their learning, they quite improved their mathematics problem solving evidently observed in their class participation. They have become responsible in handling the difficulties in mathematics. This may be because from their belief that knowledge obtained from the authority but as they matured they believed knowledge can also be obtained from other sources as from books or from their own efforts of discovering.

Relationship of Determinants to Problem Solving Performance.

The study also looked into the relationship of the problem solving performance to epistemological belief, motivation, curiosity, cognitive and metacognitive learning style. Results are shown in Table 7.

Results show that problem solving performance has significant relationship to epistemological belief, curiosity, cognitive and metacognitive learning style as shown in computed Pearson product Moment correlation or r values ranging from 0,312 -0.818, leading to the rejection of the null hypothesis. For epistemological belief r_c was 0.818 , for curiosity r_c 0.312, for cognitive r_c 0.533 and for r_c metacognitive, which all had p values less than $\alpha = 0.05$ suggesting for rejection of the null hypothesis.

On the other hand, motivation was not significantly related to the problem solving performance as shown in the r_c of 0.177 and p value of 0.056 which is greater than $\alpha = 0.05$ suggesting acceptance of

Table 7

Relationship of Determinants to Problem Solving Performance

Determinants	R	P value	Decision	Decision
Epistemological Belief	0.818	0.000	Reject	Significant
Motivation	0.177	0.056	Accept	Not Significant
Curiosity	0.312	0.001	Reject	Significant
Cognition	0.533	0.000	Reject	Significant
Metacognition	0.566	0.000	Reject	Significant

$a = 0.05 \; df = 116$

the null hypotheses. Results of the present study supported the study of Schoemmer Aikins and Duell (2013) that other aspects of cognition indicate strong relationship to problem solving performance which means poor problem solvers lack the ability to identify structure of mathematical structure. Similarly, the epistemological belief has a significant relationship to mathematical performance.

The relationship between metacognition and problem solving is shown in the steps in problem solving as : understanding the problem devising a plan; carrying the plan and looking back . This further shows metacognitive strategies would enable problem solvers to monitor and improve progress and develop an understanding of how to apply knowledge in new situations.

Best Problem-Solving Predictor when Taken Singly or in Combination

The study looked into the best predictor of the problem solving performance when taken singly or in combination. Before the stepwise multiple regression was conducted, the assumption for the said statistical test was first established. These were testing for multi-collinearity among the independent variables, normality, heterodastisity and minimum sample. All these assumption were satisfied prior to conducting the stepwise multiple regression. The predictors or the determinants were the epistemological belief, motivation, curiosity, cognitive and metacognitive learning styles. Best predictor when taken singly or in combination was determined through the method of stepwise regression. Stepwise multiple regression analysis was performed to determine the best predictor.

Table 8 displays the summary of the regression analysis performed with the problem solving performance test as the dependent variable.

Tabular values show that the best single predictor of the problem solving performance was epistemological belief indicated in computed beta weight 0.589 with p value of 0.000 which is less than $\alpha = 0.05$ and found to be very significant at 0.05 as supported by the computed f-value of 235.27. This means that there is direct positive relationship between problem solving performance and epistemological belief. Higher level of epistemological belief means higher level of performance in mathematics problem solving.

The multiple correlation coefficient was 0.818 indicating approximately 67 percent of problem solving performance could be accounted for by epistemological belief, that is about two thirds of the variance of the problem solving performance of the pre-service mathematics teachers can be attributed to the pre-service mathematics teachers' epistemological belief as measured by the epistemological belief inventory.

The second best predictor of the problem solving performance of the pre-service mathematics teachers was curiosity with f value 137.04 with p value of 0.000 which is less than 0.05 level of significant. With the inclusion of curiosity as predictor of problem solving, the computed proportion overlap increased to 0.704, that is, about 70.4 percent of the variance in the problem solving performance of the pre-service mathematics teachers may be explained or accounted for by the joint effect of the epistemological belief and curiosity. The computed proportion overlap represents a marginal increase of about 3.4 percent from the previous proportion of 0.67 or 67 percent.

Table 8

Regression Analysis Table

Regression of Problem Solving Performance with Predictors Variables

Index	Name	Mean	Standard Deviation
1	Epistemological Belief Inventory	144.2712	14.42
2	Motivational Inventory	66.4153	5.28
3	Curiosity Inventory	67.661	7.67
4	Cognitive Learning Inventory	48.822	5.46
5	Metacognitive Learning Inventory	83.3729	8.51
Dependent Variable	PSPT	27.1186	10.39

Step 1	Variable EBI Entered				
	Dependent Variable	PSPT			
	Variable	Regression Coefficient	Standard Error	F(1,116)	Prob.
	EBI	0.589	0.038	235.27	0.000
	Constant	-57.913			

Std.Error of Estimate	5.99818
R squared	0.670
Adjusted R Squared	0.667
Multiple R	0.818

Analysis of Variance

Source	Sum of Square	DF	Mean Square	F ratio	Prob.
Regression	8466.112	1	8466.112	235.27	0.000
Residual	3464.156	116	35.985		
Total	12640.337	117			

Variable not in Equation

Name	Partial Squared	Tolerance	F to Enter	Prob.
MI	0.0256	0.770	-0.837	0.404
CI	0.1049	0.822	-0.785	0.000
CLI	0.0237	0.684	2.144	0.034
MLI	0.0471	0.679	2.214	0.029

Step 2 Variable: CI Entered

Variables	Regression Coefficient	PSPT Standard Error	F(1,116)	Prob.	Partial
EBI	0.652	0.04	16.184	0.000	0.834
CI	0.278	0.076	3.672	0.000	-0.324
Constant	48.1				

Standard Error of Est.	5.6998
Squared r	0.704
Adjusted r Squares	0.699
Multiple R	0.839

Analysis of Variance

Source	Sum of Square	DF	Mean Square	F ratio	Prob.
Regression	8904.225	2	4452.126	137.04	0.000
Residual	3736.087	115	32.458		

Total	12640.339	117

Variable not in Equation

Name	Partial Squared	Tolerance	F to Enter	Prob.
MI	0.000576	0.626	-0.251	0.802
CLI	0.061009	0.649	2.727	0.007
MLI	0.0729	0.671	0.2991	0.003

Step 3 Variable: MLI Entered

Dependent Variables PSPT

Variables	Regression Coefficient	Standard Error	F(1,116)	Prob.	Partial
EBI	0.584	0.45	12.98	0.000	0.772
CI	0.303	0.074	4.109	0.000	-0.359
MLI	0.219	0.073	1.991	0.003	0.27
Constant	-48.1				

Standard Error of Est.	5.51247
Squared r	0.726
Adjusted R square	0.719
Multiple R	0.852

Analysis of Variance

Source	Sum of Square	DF	Mean Square	F ratio	Prob.
Regression	9176.183	3	3057.728	100.658	0.000
Residual	3464.337	114	30.387		
Total	12640.337	117			

Variable not in Equation

Name	Partial r Squared	Tolerance	F to Enter	Prob
MI	0.000361	0.652	-0.197	0.844
CSI	0.03386	0.534	1.987	0.044

Step 4 Variable: CLI Entered

Dependent Variables PSPT

Variables	Regression Coefficient	Standard Error	F(1,116)	Prob.	Partial
EBI	0.555	0.047	11.855	0.000	0.745

CI	0.331	0.074	4.479	0.000	0.0388
MLI	0.175	0.075	2.325	0.022	0.214
CLI	0.237	0.119	1.987	0.049	0.184
Constant	56.628				

Standard Error of Est.	5.44253
Squared r	0.735
Adjusted R square	0.726
Multiple R	0.857

Analysis of Variance

Source	Sum of Square	DF	Mean Square	F ratio	Prob.
Regression	9293.145	3	2323.286	78.433	0.000
Residual	3347.194	113	29.621		
Total	12640.337	117			

The third best predictor of the problem solving was the metacognitive learning styles with f value of 100.658 and p value of 0.000 which is less than 0.05. With the inclusion of metacognitive as predictor of problem solving performance, the computed proportion overlap increased to 0.726 or 72.6 percent, that is, 72.6 percent of the variance in the problem solving performance may be explained by joint effect of epistemological belief, curiosity and metacognition. The computed proportion overlap increased by 2.2 percent from the previous computed proportion overlap of 70.4 percent.

The last best predictor among the predictors in the present study was the cognitive learning styles with f value of 78.433 and p value of 0.000 which is still lower than 0.05. With the inclusion of the cognitive predictor of problem solving performance, the computed proportion overlap increased to 0.735 or 73.5 percent. This implies that 73.5 percent of the variance in the problem solving performance of the pre-service mathematics teachers could be accounted to by the joint effect of epistemological belief, curiosity, metacognitive and the cognitive variables. There was only 0.9 percent increase from the previous computed proportion overlap of 0.726 or 72.6 percent. This was only a very minimal contribution in the enhancement of the predicting power of the regression equation model as shown in the proportion overlap of 0.009 which is less than 1 percent. It is further surmised that when the last variable, motivation, is added it will give insignificant contribution to the model, The pattern is very clear as given by the increases of 3.4, 2.2 and 0.9 percent respectively after each step. The pattern is decreasing that it can be concluded that motivation will not add significantly to the proportion overlap.

In summary, it was found out that the best single predictor of the problem solving performance of the pre-service mathematics teachers was the epistemological belief. On the other hand, the best combination of predictor of the problem solving performance were the combined effects of epistemological belief, curiosity, metacognition and cognitive variables.

Mathematical Model on Enhancing Problem Solving Performance of Pre-service Mathematics Teachers

The study looked into the mathematical model that would enhance the problem solving performance of pre-service mathematics teachers. Models describe beliefs about how the world functions. In mathematical modeling, those beliefs are translated into the language of mathematics. Path analytic (PA) models (Khine et. al., 2013) which are conceived in terms of observed variables. Although they focus only on observed variables, they form an important part of the historical development of Structural Equation Model and employ the same underlying process of model testing and fitting as other structural equation models

Model specification or model formulation is concerned with formulating a model based on a theory and/ or previous studies in the field. The existing theories and previous research served as bases for the hypothesized model of the determinant of problem solving performance of the pre-service mathematics teachers. This is shown as:

$$y = c + x_1b_1 + x_2b_2 + x_3b_3 + x_4b_4 + x_5b_5 + \varepsilon$$

Y= problem solving performance; X_1=Epistemological belief score;X_2= Motivational Inventory score,X_3= Curiosity inventory score;X_4=Cognitive Learning Inventory Score;X_5=Metacognitive Leaning Inventory score; c= constant ;ε = error

The hypothesized model was based in the study conducted by Aikins and Duell, on domain general epistemological belief AVQUICK had indirect effects on cognitive depth and mathematical performance. The indirect effects were mediated by students' mathematics background and by their domain specific mathematical problem solving belief of MTUSE; Marcou and Philippou on Performance = 0.476 X Self-efficacy beliefs concerning MPS, constitutes the one and only indicator of the performance in MPS (Beta=0.32, t=4.631, p=0. 00); Abedalaziz and Akmar (2012) that the five scales significantly predict mathematical achievement to a different extent. The strongest predictor was belief regarding the role of effort in increasing mathematical ability; by Sami (2009) that knowledge of cognition, regulation of cognition, innate ability, and quick learning significantly contributed to the model. This model explained the 14 percent of the variability in the students' science achievement. Also finding revealed that metacognition influenced the students' science achievement more than epistemological beliefs. This is the hypothesized model of the study.

The second step in a structural equation modeling is model identification which is concerned with whether one can derive a unique value for each parameter. In the present study, there were six observed variables: epistemological belief, motivation, curiosity, cognitive metacognitive and the problem solving performance 6(6+1)/2= 21 data points. There were also 21 parameters to be estimated; therefore its degree of freedom is equal to zero as computed using the SPSS AMOS (Analysis of Moment Structures). When degree of freedom is zero, model can be identified but cannot be evaluated using fit indices (Khine, 2013).

Step three of the model development of the present research was parameter estimation. Mathematical model specifies the relationship among the variables in mathematical terms based on the hypotheses so that the relationship would be clearer and would have information on coefficient (such as the sign and magnitude of the effect of explanatory variables on explained variable) parameter that should be estimated. To specify the model, the path diagram in Figure 2 shows the relationship between the problem solving performance and predictor variables, epistemological belief, metacognitive learning and cognitive learning. Unidirectional arrows in the path diagram represent one-way structural influence from one variable to another known as recursive model.

From the path diagram of the determinants of problem solving performance, the single arrow signifies the contribution of the variables in the problem solving performance which is the standard coefficient. This means when EBI goes up by 1 standard deviation, PSPT goes up by 0.56 standard deviation; when CI goes up by 1 standard deviation, PSPT goes down by 0.33 standard deviation; when CL goes up by 1 standard deviation, PSP goes up by 0.24 standard deviation and when ML goes up 1 standard deviation, PSP goes up 0.18 standard deviation and when curiosity goes up 1 standard deviation, PSP goes down 0.333 standard deviation.

The double arrows denote the correlation between each pair of the variables. It can be noted that 0.52 is the estimated correlation between CL and ML; 0.408 is the estimated correlation between CLI and CI; 0.422 is the estimated correlation between CI and EBI 0.32 is the estimated correlation between MLI and CI; 0.562 is the estimated correlation between CL and EBI and 0.57 is the estimated correlation between ML and EBI.

Thus, after the delineation of the model, the Structural Equation Model that would enhance the problem solving performance of pre-service mathematics teachers is :

$$\text{PSP} = \alpha + 0.331\text{C} + 0.237\ \text{CL} + 0.175\ \text{ML} + 0.555\ \text{EB} + \text{e}.$$

In the structural equation model, PSP is the problem solving performance, CL is the cognitive learning, ML is the metacognitive learning, and EB is the epistemological belief, It can be seen that PSP is a function of Curiosity (C), cognitive (CL), metacognitive (ML) and epistemological belief (EB) of problem solving performance is influenced by variables from inside the model. This shows the direct effect of the independent variables on the dependent variable and also shows the structural coefficient linking the variables, epistemological belief, cognitive, metacognitive learning and the cognitive learning. Curiosity is in the structural equation model or mathematical model although it is one of the variables significantly related to the problem solving with regression coefficient of 0.25.

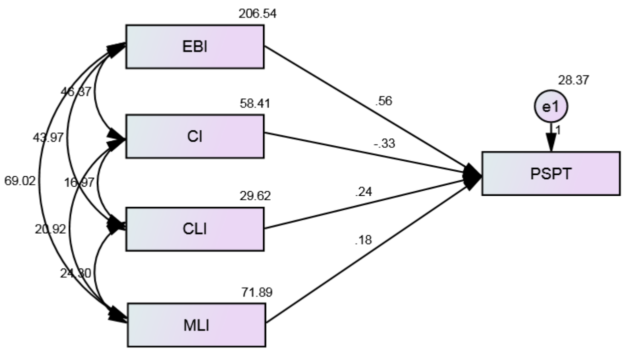

Figure 2

Path Diagram of Determinants of Problem Solving Performance

The structural coefficient of regression coefficient from the model indicates the following: regression coefficient of 0.237 for CL which means that there is a positive relationship between CL and PSP and that in every unit change in CL, there is a corresponding increase of 0.237 in PSP; the regression coefficient of 0.175 implies there is also positive relationship between ML and PSP and that in every unit change in ML there is corresponding increase in PSP; and regression coefficient of 0.555 implies positive relationship between EB and PSP and for every unit change in EB there is a corresponding increase of 0.555 in PSP. Model fit and model re specification were not conducted because from the model identification, the degree of freedom is zero. Therefore, the model is $\mathrm{PSP} = \alpha + 0.331\mathrm{C} + 0.237\ \mathrm{CL} + 0.175\ \mathrm{ML} + 0.555\ \mathrm{EB} + \mathrm{e}$

Conclusion and Recommendations

Based from findings of the study, the following conclusions are drawn: pre-service mathematics teachers have strong agreement on epistemological, motivational, curiosity, cognitive learning and metacognitive learning belief on learning of mathematics problem solving; pre-service mathematics teachers have average performance in the problem solving performance test.; epistemological belief, cognitive learning, metacognitive learning and curiosity are all significantly related to problem solving performance; the best single predictor of the problem solving performance is the epistemological belief, while the best combination predictors comprise epistemological belief, metacognitive learning and cognitive learning; and the mathematical model developed for enhancing problem solving performance is $\mathrm{PSP} = \alpha + 0.331\mathrm{C} + 0.237\ \mathrm{CL} + 0.175\ \mathrm{ML} + 0.555\ \mathrm{EB} + \mathrm{e}$

Similarly based on the findings and conclusions of the study, the following recommendations are offered : further testing of the model is recommended ; future research addressing the same issue may be done but to be participated in by more teacher education institutions; and an intervention studies that incorporate mathematical belief components which are found to be beneficial for improving mathematics performance are recommended.

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