# Fuzzy Modeling of the Weight - Length Allometric Relationship of the Fish Species *Plagioscion Squamosissimus*

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## Abstract

*As part of a project to iniciate an Enviromental Engineering undergraduate student in the scientifict world of advance researchers, the purpose of this study is to model the allometric relationship between weight and length of the species Plagioscion squamosissimus, a Brazilian Cerrado region fish, commonly known as Cerrado croaker. This fish, among other species, is the reason for the research project "Peixe Vivo" (Fish Alive), launched by a Brazilian company, whose data is the source for this research. The motivation to use the fuzzy set theory comes from the fact that, when the objective is to determine the curve that defines the allometric relation of the fish, the path between the measured weight-length measurement until the simulation of the model is reached, is full of inaccuracies. In order to solve this problem and make the modeling more consistent with reality, an important tool of the fuzzy theory is used: the Zadeh Extension Principle. In this way, a pertinence to the deterministic allometric relation is incorporated, inserting the variations that actually occur in the real data and in the course of the modeling process. As a computational tool, it is used the free software GeoGebra that provides a simple way to develped the model and a simultaneous its graphical interpretation.*

**Keywords:** Allometric relationship weight-length; Fuzzy Theory; Zadeh Extension Principle; GeoGebra.

## 1. Introduction

The variety of shapes and sizes plays a key role in the ability of organisms to inhabit extremely different locations. Probably, size is the most important feature of an organism, and basically determines its lifestyle [1]. Allometry is the part of biology that studies scale relations for morphological, physiological or ecological parameters.

This study intends to model the allometric relationship between weight and length of the *Plagioscion squamosissimus* fish species, commonly known as Cerrado Croaker, aiming at the expansion and creation of more effective measures for the conservation of the ichthyofauna in the hydrographic basins where a Brazilian company's plants are installed, favoring communities that use water resources as a development factor. A set of data collected by the Peixe Vivo project, launched by an energy company of the state of Minas Gerais - Brazil, in 2007, that was kindly assigned to model this proposal.

The motivation to use the fuzzy set theory comes from the fact that, the path between the measured weight-length measurement until the simulation of the model is reached is full of inaccuracies. In order make the modeling more consistent with reality, the Zadeh Extension Principle is used. In this way, a pertinence to the deterministic allometric relation is incorporated, inserting the variations that actually occur in the real data and in the modeling process.

For the fuzzy modeling simulations, the GeoGebra software [2] was used, whose dynamic geometry solves, in a simple and elegant way, the derivation of the deterministic allometric relation curve, through its statistical tools, in a first instance. Then, with the coding of the parametric curves of the pertinence functions depending on the length, the fuzzy surface of the allometric relation of the croaker is constructed.

## 2. Mathematical Background

### *2.1 Fuzzy Theory*

The fuzzy set theory, formalized by Zadeh [3] in 1965, has been extensively developed, influencing many fields of application. One of the most important tools in the theory is the Zadeh extension principle, which allows to define new fuzzy sets from a fuzzy set and a data function. A fuzzy set is the graph of a function $\mu_A$, called fuzzy set $A$ pertinence function, that is, $f$ continuous real function, with $U \subset R$ as its domain and $A$ a fuzzy set defined in $U$. A new fuzzy set can be defined through the function $f$ and the fuzzy set A, called by $\hat{f}(A)$, whose support is the image set $f(U)$ and the pertinence function is defined as

$$\mu_{\hat{f}(A)}(y) = \sup_{x \in f^{-1}(y)} \mu_A(x),\ y \in f(U), \text{ and 0 otherwise.} \tag{1}$$

From the definition (1), you can also build a function $\hat{f}$ of the set $£(R)$, of fuzzy sets with support in R and image in R which matches to A the set $\hat{f}(A)$, known as the Zadeh extension function of $f$. In general, construct the extension function $\hat{f}$ is a complex task, except when $f$ is monotonous, as is the case in this study, because from the definition (1), there is

$$\mu_{\hat{f}(A)}(y) = \mu_A\left(f^{-1}(y)\right). \tag{2}$$

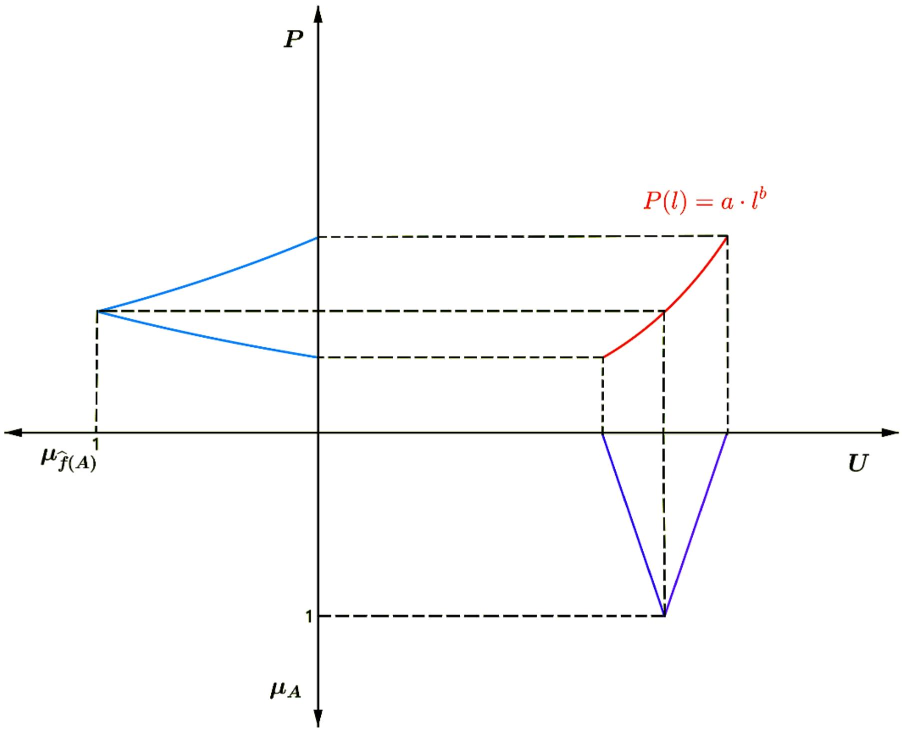


Figure 1. The Zadeh extension, $\hat{f}(A)$, of a fuzzy number A.

### *2.2 Weight - Length Allometric Relation*

Mathematically, the so-called Principle of Allometry [4] is represented by the following equation:

$$\frac{1}{p}\frac{dp}{dt} = \rho\frac{1}{l}\frac{dl}{dt}, \tag{3}$$

where x(t) and y(t) are the measurements of the distinct organs or parts of the same individual at a given time t, positive values other than zero. Furthermore, ρ is the relative growth rate of proportionality (coefficient of allometry). When integrating both members of Equation (3), we have

$$\int \frac{1}{p}dx = \rho\int \frac{1}{l}dy\text{, or equivalent, } \ln(p) = \rho\ln(l) + \ln(c), \tag{4}$$

where $\ln(c)$ is a constant and $c > 0$. Therefore, from (4), we have

$$\ln(p) = \ln(cl^{\rho}),$$

or equivalent

$$p = cl^{\rho}. \tag{5}$$

which represents the allometric relation weight – length.

## 3. Methodology

For this research, we used 99 data of weight and length of the species *Plagioscion squamosissimus*, known as croaker. These data collected by the Peixe Vivo project team, whose objective is to perform morphometric measurements of fish that inhabit hydroelectric plants in the brazilian cerrado region. In this study, morphometric information was defined in the laboratory after treatment with formaldehyde, and it is emphasized that the fishs were collected in the Araguari and Paranaíba rivers located in the state of Minas Gerais - Brazil.

From the data obtained from the Peixe Vivo source, a curve fit was performed in GeoGebra software through its bivariate analysis tools. The deterministic allometric relation of the weight - length of the croaker, result of the bivariate analysis with 92.26% of correlation coefficient, is given by

$$P(l) = 2.4637 \cdot 10^{-6} \cdot l^{3.28}, \tag{6}$$

where $l$ is the length of the fish and $P(l)$ is the weight as a function of its length. The value obtained $b = 3.28$ can not be considered as an exact measure, since the measures are subject to many abiotic and biotic factors that can interfere in its respectives values. Thus, the parameter $b$ of the allometric relation is defined by means of a triangular fuzzy set, this being $\hat{b} = [3.00; 3.28; 3.56]$, in which the value 3.28 has maximum pertinence 1. The fuzzy set is shown in Figure 1 with membership function $\mu_A$, supported in the x-edge.
Considering $a = 2.4637 \cdot 10^{-6}$ in the Equation (5), the pertinence function $\mu_A(b)$ is given by

$$\mu_A(b) = 0, \text{ if } b < 3.00,$$

$$\mu_A(b) = \frac{b}{0.28} - \frac{3}{0.28}, \text{ if } 3.00 \leq b \leq 3.28, \tag{7}$$

$$\mu_A(b) = -\frac{b}{0.28} + \frac{3.56}{0.28}, \text{ if } 3.28 \leq b \leq 3.56,$$

$$\mu_A(b) = 0, \text{ if } b > 3.56.$$

Calculating the inverse of the parametric function $f_l(b) = a \cdot l^b$, where l is a value set in the interval $[l_i, l_f] e, l_i > 0$, where $l_1$ e $l_f$ represent the initial and final length of the croaker, respectively, obtains the expression

$$f_1^{-1}(b) = \frac{\ln(b) - \ln(a)}{\ln(l)} \tag{8}$$

Thus, using Equation (2) it is possible to code in GeoGebra the expression of the pertinence function of the Zadeh extension of A, given by

$$\mu_{\hat{f}(A)}(b) = 0, \text{ if } b < 3.00,$$

$$\mu_{\hat{f}(A)}(b) = \frac{1}{0.28} \cdot \left( \frac{\ln(b) - \ln(a)}{\ln(l)} \right) - \frac{3}{0.28}, \text{ if } 3.00 \leq b \leq 3.28, \quad (9)$$

$$\mu_{\hat{f}(A)}(b) = -\frac{1}{0.28} \cdot \left( \frac{\ln(b) - \ln(a)}{\ln(l)} \right) + \frac{3.56}{0.28}, \text{ if } 3.28 \leq b \leq 3.56,$$

$$\mu_{\hat{f}(A)}(b) = 0, \text{ if } b > 3.56.$$

The membership funcion of the Zadeh Extension $\mu_{\hat{f}(A)}$ is shown in Figure 1 supported in the possitve ray of y edge.

The simulation of the construction of the fuzzy model for the allometric relation (2) is performed considering the value of l as a delineating control, which facilitates the construction of the dynamics of Figure 3 of the results section.

## 4. Results

The result of the adjustment of curves of Peixe Vivo data is shown in Figure 2. The Zadeh extension of the fuzzy set $\hat{f}(A)$, by the function $f_l(b)$ for the fixed value of l, is shown in Figure 1.

From the deterministic allometric relation, formulas (7) and (8) are used to construct the corresponding pertinence functions with the "parametric curve" command, as shown by the following encodings:

```
j(x)=(-(ln(x) / ln(l) - ln(2.4637 * 10^{(-6)}) / ln(l))) / 0.28 + 3.56 / 0.28;
u=2.4637 * 10^{(-6)} l^3.28;
v=2.4637 * 10^{(-6)} l^3.56;
Curva(l, t, j(t), t, u, v);
```

The variable l is considered as a slider to obtain the dynamics shown in Figure 4: each blue color parametric curve in Figure 3 represents the Zadeh extension of the fuzzy number $\hat{b}$ corresponding to the value l. Thus, the z-axis represents the pertinence of the points of the plane $xy$. The x-axis represents the length l and the y-axis corresponds to the weight P(l).

The GeoGebra enviroment is shown in Figure 3. The Peixe Vivo data is shown in the spreadsheet at the left. The elements of the fuzzy allometric relation is shown in the GeoGebra algebra window, and the construction of the fuzzy surface representing the memberships of the allometric relation on time in shown in the GeoGebra visualization window.

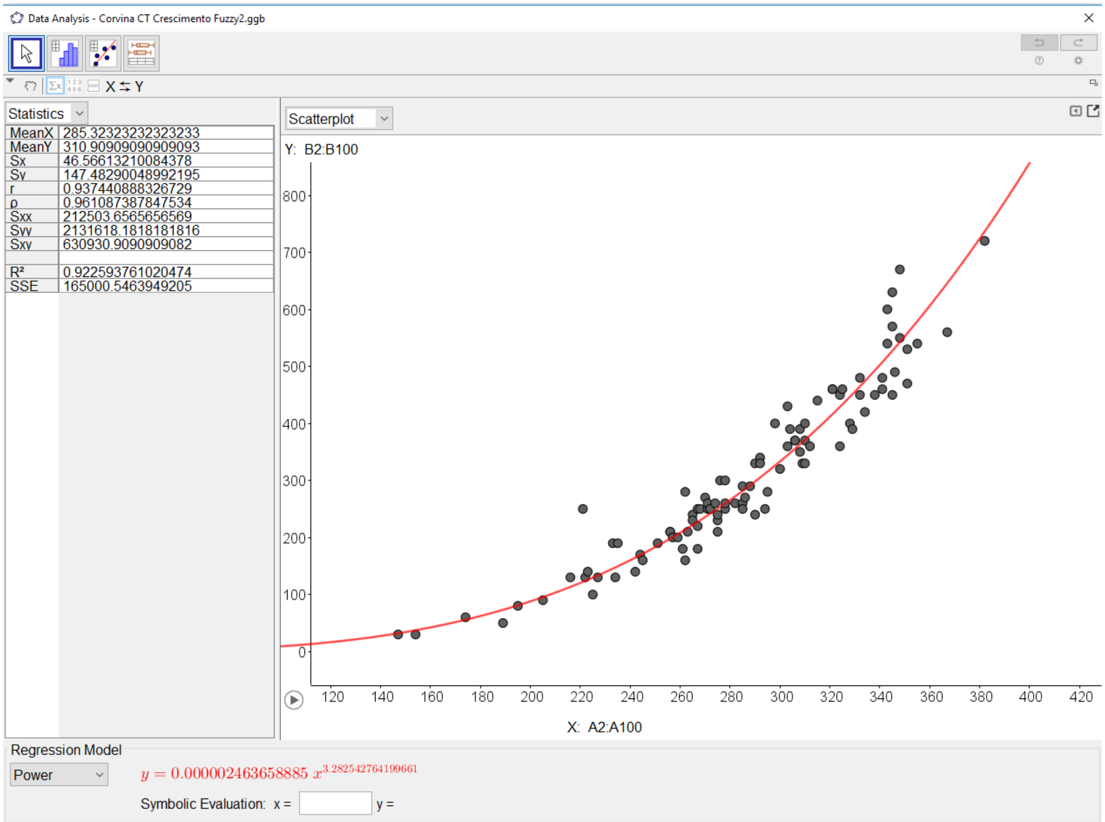


Figure 2: bivariate analysis of Peixe Vivo data to obtain Equation (2) in GeoGebra.

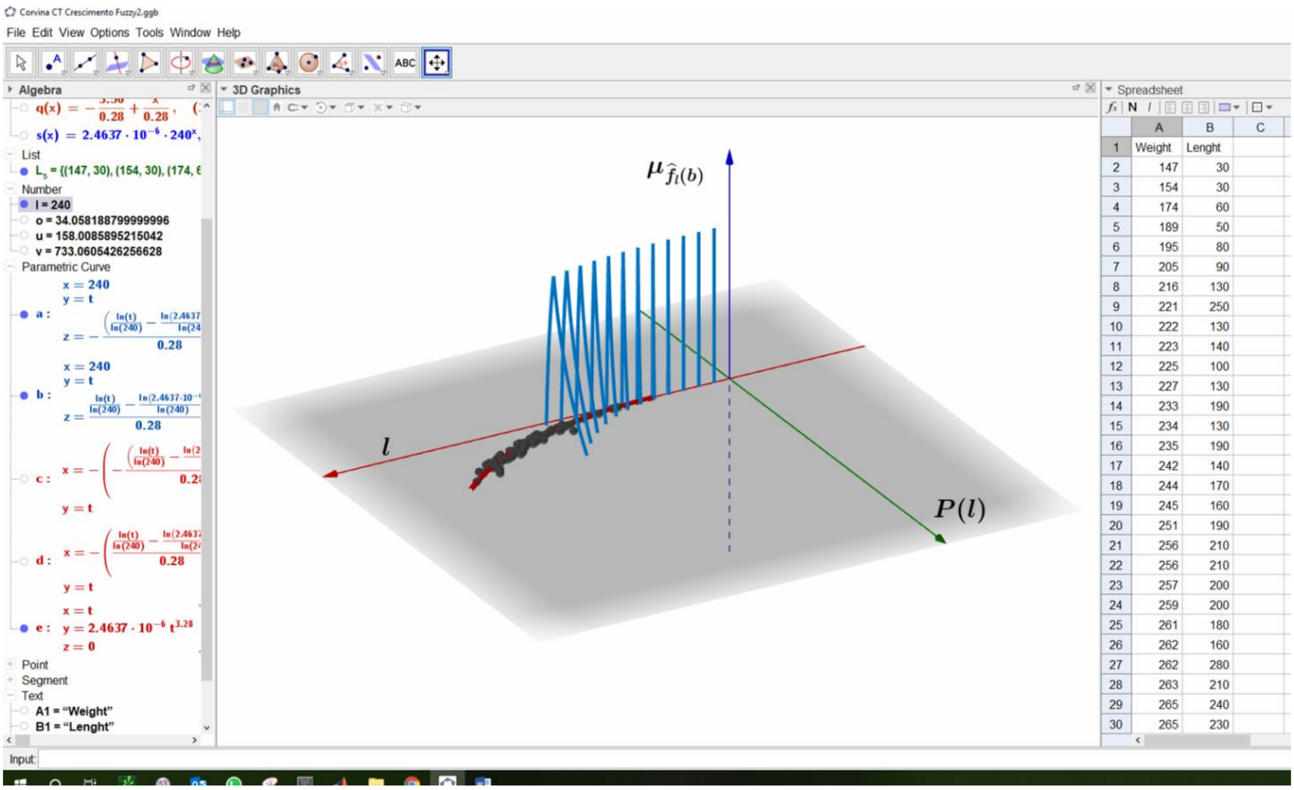


Figure 3. The GeoGebra enviroment of the simulations.

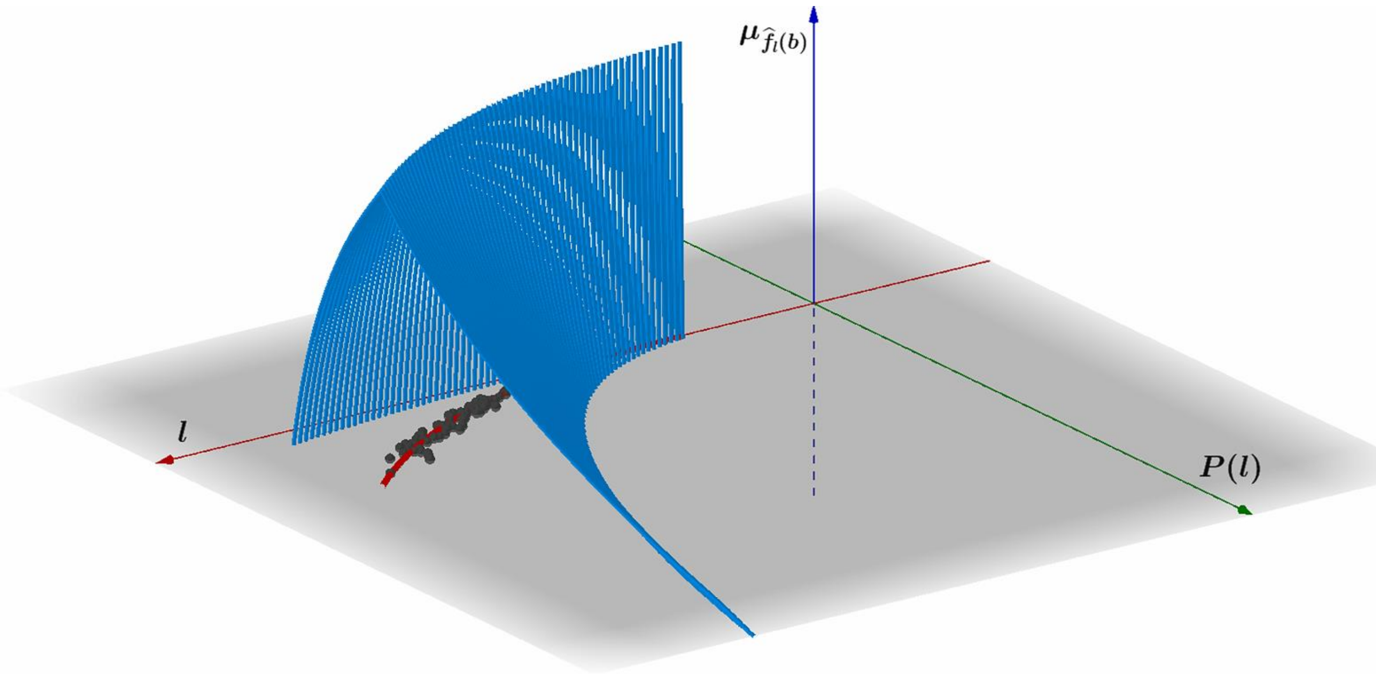


Figure 4. The fuzzy allometric surface obtained after refining the scale of the slider.
The deterministic curve is shown in the xoy plane together with the data. The blue surface represents the memberships of the fuzzy allometric model obtained at each *l* value by the Zadeh extension principle.

## 5. Conclusion

The fuzzy modeling of the allometric weight - length ratio of the *Plagioscion squamosissimus* fish species (Cerrado Croaker) was done through data provided by an electrical Brazilian company project, and the use of GeoGebra software as a computational tool. The modeling estimates, with degrees of pertinence, the values corresponding to the allometric weight - length ratio of the deterministic model, obtained by a curve fitting in the same software. This allows you to insert the variations that actually occur in the actual data and in the course of the modeling process, imparting greater reliability to the solution obtained. For a next one research, it is intended to carry out the same study for other species of fish from the Cerrado and fish treated without formaldehyde, and to analyze the mathematical behavior of the modeling in these cases.

## 6. Acknowledgement

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## 7. References

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