CONTRIBUTIONS OF FIGURES NUMBERS IN THE DEVELOPMENT OF GEOMETRIC THOUGHT

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Abstract

This study was constituted with the purpose of promoting reflections on Mathematics of basic education, from a transdisciplinary view of teaching and learning processes. To do so, we aim to analyze the contributions of figures in the development of geometric thinking. We characterize this research in empirical-exploratory, because for Lakatos and Marconi (2017), this type of research distinguishes itself as a scientific process of investigation that allows the researcher to formulate questions, with three purposes: to raise hypotheses, to increase the familiarization of the researcher in order to research, modify or clarify concepts, based on a qualitative and quantitative approach, according to the depth of the discussion about the object in question. For this, we look for information in other researches, databases of universities and virtual libraries, periodicals. We hope that the results contribute to the critical and ethical awareness from views of the importance of the development of mathematical thinking, but specifically of geometric thinking, aiming at non-rupture with arithmetic thinking, in order to interweave with algebraic thinking . We consider this research relevant because mathematics teaching is based on abstract content that often makes no sense to the student, and here we show a part of mathematics that is formal but can be fun when well crafted in the classroom. Finally, we present pedagogical tools of innovation, aimed at contributing to the emancipation of the knowledge of this science, without ruptures.

Keywords: Geometric thinking. Figurative numbers. Mathematics Teaching.

INTRODUCTION

At the end of the twentieth century, and in the first years of the 21st century, in order to subsidize educational activities in the area of mathematics, especially in higher education institutions (IES), several educational actions were developed in Brazil in order to fulfill articles set forth in the Federal Constitution of 1988. Among these actions, we mention: the Law of Directives and Bases of National Education (LDBEN) No. 9.394 / 96; the National Curricular Parameters (NCPs); published in 1997 by the Ministry of Education and Sport, and the National Education Plan (PNE), to be developed from 2014 to 2024.

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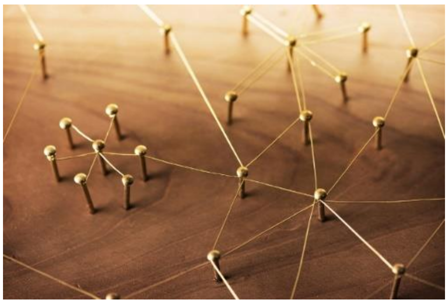
These documents aim at pointing out new perspectives for teaching, collaborating to the work involving the reality of the student, in order to promote in the construction of a political, critical and creative being. It is important to articulate debates in the school, also considering in this way the ideas of Basil Bersntein (1998) when he emphasizes that the pedagogical discourse is not a discourse in itself, but it is an announcement of the principle of recontextualization and is adjusted to other discourses to give the debate on curriculum. Although Mathematics courses provide quality teaching, we still see very low rates in external evaluations such as SPAECE and PISA (Santos, 2018). Does this come about because teaching is about reproduction rather than transformation? The trajectory of Mathematics Education in Brazil presents some obstacles that still have to be faced in the initial formation, among them we highlight the dispute for the hegemony between the knowledge, that is to say, the hegemony of the knowledge of the powerful and not of the powerful knowledge, thus, the time insufficient to contemplate an extensive curriculum during a brief time load, besides the impossibility of the professor and student being heard in the debates on the curricular reforms, this teacher voice is lost in the vacuum, of the disputes by power. Figure 1 below is an attempt to impel us to rethink these studies, under the prism of hybrid contextualization in pedagogical practice.

We want to contribute to a more meaningful Mathematics, bringing to the debate the importance of the development of mathematical thoughts, and the non-rupture of this area between the subareas. We are concerned about an area of mathematics that will dialogue with other areas of knowledge, but that does not fall into mere integrations, devoid of meaning. It is necessary to think of the teaching of mathematics, from a critical, reflexive approach, of an education committed to diversity, cultural equality, citizenship, a critical mathematics, guided by a dynamic, interdisciplinary, transdisciplinary curriculum. On D'Ambrosio's curriculum (2011, p. 121) he warns that it is about ... a Napoleonic illusion that an obligatory curriculum that covers the entire country has no effect on improving education. The author's reflection reveals that in order to overcome obstacles in order to improve mathematics education, we need many more variables, especially that which impedes the valuation of the education professional.

Figure 1. Areas of knowledge in movement.



Source: Shutterstock[3].

It is in this emblematic scenario that we weave some questions: What knowledge is needed to be taught? How to contribute to the development of geometric thinking beyond breaks with algebra and arithmetic? How to work for connection between these areas? In order to better clarify these issues, we aim to dialogue

[3]Fonte: http://educacaointegral.org.br/noticias/como-alcancar-interdisciplinaridade-na-escola/. Acesso dia 12 de agosto de 2016.

with the purpose of minimizing obstacles and finding the unknowns that impede students' cognitive advancement.

In this way, these questions led us to the elaboration of general and specific objectives, which lead to the development of research. Thus, in general, we aim to understand the importance of the development of geometric thinking from the work with figures figured in the final years of elementary school.

More specifically, we aim to select material for the definition, foundation and mathematical presentation of the study object of this research; to relate the algebraic and arithmetical thoughts, avoiding the rupture with the geometric thought; understand the importance of data analysis as well as the implications of results in the construction of mathematical thinking; and, finally, to develop activities that collaborate and broaden the discussion and dissemination of geometric thinking, from the figures.

In order to achieve the objectives concretely, we rely on the methodological conception of investigative steps. Our investigative approach followed the assumptions of the exploratory empirical research and consisted of a theoretical-speculative essay, and aimed to articulate a series of questions that the subject can indicate us from within a general interpretative framework, through which it was possible to deduce explanatory hypotheses. Following Lakatos and Marconi (2017), we characterized this research as a scientific investigative process that allowed us to formulate questions, with three purposes: to raise hypotheses, to increase the familiarization of the researcher with the object of research, to modify or clarify concepts. We highlight these purposes as the methodological north of the research.

Because it was a theoretical-speculative research, it did not have subjects, but it followed the nortes of bibliographical research. And it covered some phases, namely: (1) We selected the bibliographical relationship of the theoreticians in which we support, all studied ethically, critically-reflective, exploratory and expository. The data found were precious, in order to understand the importance of the development of mathematical thinking, especially geometric thinking, from the perspective of figurative numbers. In this initial phase, the data were compiled, systematized, and socialized in the chapters that follow. After collecting and analyzing the data, the study object was better delineated; (2) In this phase, the relevant stage of the research, we carried out readings and selections of research results on our theme, databases were consulted, aiming for a greater critical-reflexive analysis. From the studies were collected data on several aspects of the problem. We emphasize that later we present activities, as a product of this research. We emphasize the importance of this phase with regard to the changes of posture of the researchers in relation to the teaching of mathematical contents, in question the figures figured contributions to the development of geometric thinking; (3) This phase was dedicated to the systematization of the data found, the activities elaborated, as well as subsidies for the discussion about the challenges faced in the teaching of mathematics, in school contexts. Still in this phase, we suggest integrative actions in the consolidation of teaching and research, as strategies that enhance the teaching and learning processes of mathematics, through not only discovery but also creation. The following are discussions about mathematics teaching, and mathematical thinking types.

2 Teaching mathematics: challenges and ruptures

The Common Base National Curricular-BNCC (BRASIL, 2017, p.23) presents Arithmetic, Algebra, Geometry, Statistics and Probability in thematic units in order to develop concepts and ensure that students relate real-world empirical observations to representations (tables, figures and schemas) and associate these representations with a mathematical activity, concepts and properties, making inductions and conjectures. We expect students to develop the ability to identify opportunities for using mathematics to solve problems. The deduction of some properties and the verification of conjectures, from others, can be stimulated, mainly so that the students enter the high school with ample knowledge of these areas of the mathematics. The expectation is that students solve problems with natural numbers, integers and rational, involving the fundamental operations, with their different meanings, and using different strategies, with an understanding of the processes involved in them. In order to deepen the notion of number, its interfaces, and in that sense, advance to other types of numbers, such as figured numbers.

We know that mathematics is not just symbolic manipulation according to certain obsolete rules, it is rather the conception of patterns. The passage of Arithmetic to Algebra is one of the great difficulties of students, and teachers should promote diverse learning situations, so that their students, develop not only algebraic thinking, but also geometric thinking, without ruptures.

2.1 Arithmetic thinking

Arithmetic involves the study that classifies and determines numbers and operations in all natures, and thus is defined in a simplistic way, without richness of details, without constraints that allow the construction of a foundation for the definition and development of arithmetic thinking. It is necessary, seek to mean the teaching of arithmetic, and the principle of a numerical sense, collaborates with cognitive actions, systematized in the recognition of techniques, not limited to the execution of the algorithm thinking. And for this, it becomes fundamentally important to highlight the innumerable experiences that the student can have in the street and in the school and to seek the connection between them, thus offering a more complete learning, with the search for meanings and as consequence, with meaning.

Arithmetic thinking is constituted as a process that depends on intuitive and figurative reasoning, in addition to relative and absolute thoughts juxtaposed to estimates, and additive and proportional thoughts. Thus, starting from these thoughts, we arrive at a teaching of an arithmetic based on the production of meanings. Arithmetic, as this most basic thought of mathematics, if taught more productively, based on the production of meanings, is the basis of the whole content of mathematics.

To develop well the teaching of arithmetic is of great relevance in mathematics, for the valorization of the intuitive and figurative thought, because it collaborates so that the teachers prioritize in the classroom, the more formal aspects of teaching. Mattos, Puggian and Louzano (2011, p.11) agree that ... arithmetic can not be reduced to school rules or the arithmetic of natural numbers, it must be developed in schools in order to solve problems.

These reflections, converging to the teaching of arithmetic in the quota of meaning production would effectively promote the more complete development of arithmetic thinking with the consolidation of the numerical sense, and since this is undoubtedly the foundation for teaching mathematics, the ideal would be that a to improve the standard of teaching and learning processes in mathematics, not only in the idea of a National Curricular Common Base-BNCC (Brasil, 2017), but also of an effective establish meaningful relationships, interlace poles, and narrow distances between areas, specifically arithmetic, algebra, and geometry here.

2.2 Algebraic thinking

For Fioretini and Miorim (1993) the teaching of algebra is based on the assumption that algebraic thinking only manifests itself and develops from literal calculation or from the manipulation of the symbolic language of algebra that results in a special form of thought. For theoreticians like Ponte (2005) algebraic thinking, as well as geometric thinking, has been constituted a transversal orientation of the curriculum. This type of thinking is about symbolization, the study of structures and modeling, that is, it implies knowing, understanding and using symbolic instruments to represent the problem mathematically, using formal algorithms to obtain a result, in order to interpret and evaluate this result.

One of the most complex thoughts, algebraic thinking implies,

> (...) to develop not only the ability to work with algebraic calculus and functions, but also the ability to deal with mathematical structures, relations of order and equivalence, applying them to different domains of mathematics (interpreting and solving problems), and to others Ponte (2005). According to Day and Jones (1997), students only begin the domain of algebraic thinking when they acquire the ability to perceive and construct relationships between variables. (Borralho and Barbosa, 2011, p.)

It is important to emphasize the importance of the teacher, since mastery of different strategies can help in the development of algebraic thinking. The National Curricular Common Base - BNCC (Brasil, 1997) argues that the development of algebraic thinking can occur already from the first years of schooling. It is sufficient to understand that what we teach in arithmetic and the way we teach it have strong implications for the development of algebraic thinking.

We understand that conducting discovery activities that lead students to more generic thinking helps them to perceive regularities and patterns from mathematical structures or expressions that lead them to think analytically, and this can be an important alternative for development of thinking and algebraic language in the student.

We agree that algebraic thinking can be developed gradually even before the existence of a symbolic algebraic language, but rather from a reflection on practices involving relations / comparisons between numerical expressions or geometric patterns, and this is important when the student is able to transform a arithmetic expression in a simpler one, and still develop some kind of generalization process, besides perceiving and expressing regularities or invariance.

The evolution of algebraic thinking starts from a pre-algebraic phase - when the student uses some other element considered algebraic, for example, the letter, but still can not conceive of it as any generalized number or as a variable, passes through a phase of transition - when he moves from arithmetic to algebraic thinking, it is at this stage that the student accepts and understands the existence of any number, establishes some processes and generalization, whether or not to use symbolic language, and at that moment, that he finally moves on to a more developed algebraic thought-presenting the ability to think and express himself generically, especially when the student accepts and understands the existence of numerical quantities open or variable within a numerical range, being able not only to express it, but also to operate them. Not necessarily, at this stage - the third phase of algebraic thinking, the student has already attained, and knows how to make use of a strictly algebraic-symbolic language. (Fiorentini & Miorim, 1993).

The algebraic thinking that should be initiated in the early years of Elementary Education (BRAZIL, 2017) needs to be developed by the end of elementary school. Thus, it is necessary to propose activities to the student that helps him in the gradual development, until he replaces the usual language by the mathematical language, reaching the abstraction.

How do the relations of this type of thought come to geometrical thinking? The following reflections on geometric thinking.

2.3 Geometric thinking

Studies on geometry point to situations involving shape, dimension and direction. More specifically, we also find geometry linked to the teaching of the sense of localization, the recognition of flat and non-planar figures, as well as the establishment of relationships between spatial figures and their planning and the manipulation of geometric forms using compositional and decomposition procedures, transformation, enlargement and reduction. Our view of the physical world is essentially three-dimensional, it becomes paramount the need to study the occupation, location, and displacement of objects in space, rather than focusing on the teaching of geometry in the plane. Geometry at BNCC (Brasil, 2017, p. 269),

> (...) involves the study of a broad set of concepts and procedures necessary to solve problems in the physical world and in different areas of knowledge. Thus, in this thematic unit, studying position and displacements in space, forms and relationships between elements of flat and spatial figures can develop students' geometric thinking. Such thinking is necessary to investigate properties, make conjectures, and produce convincing geometric arguments. It is also important to consider the functional aspect that must be present in the study of Geometry: the geometric transformations, especially the symmetries. The fundamental mathematical ideas associated with this theme are, mainly, construction, representation and interdependence.

According to BNCC (Brasil, 2017), the teaching of Geometry, from the final years of elementary school, must be seen as consolidation and expansion of previous knowledge. It is at this level that activities that analyze and produce transformations and extensions/reductions of flat geometrical figures among other concepts, of which we emphasize the geometric progressions that contribute to the development of geometric thinking, should be emphasized. These reflections contribute to the formation of a type of reasoning important to Mathematics, the hypothetical-deductive reasoning, and so, it is important to highlight the approximation of Algebra with Geometry, from the beginning of the study, as well as the approximation of Arithmetic thinking, it is important to also observe the arithmetic progressions, aiming the aid in the development of these thoughts. It is in this context that we do not want to reduce the mathematical contents to the mere application of calculation formulas, but to expand and consolidate the development and correlation of these studies from the mathematical thoughts types.

The following figures are presented as important pedagogical support in the development of geometric thinking.

3. The Numbers Figure: Contributions to the development of Geometric Thought

The Pythagoreans were historically known for their interests in the mysteries that involved numbers, especially geometry. It was they who created the figure numbers, which are represented by the gathering of points in a given geometric region. The figure numbers may have been the first manifestation of Arithmetic Progression (PA) of the second order (Roque, 2012). But it is also likely that the Pythagoreans already knew the expression of the general term as well as the sum of the terms of the sequences of figurative numbers. Considering this truth, then we can admit that the Pythagoreans also knew the Arithmetic Progressions (PA) of the third order. Nicomachus of Gerasa also developed studies on odd numbers, and found that cubic numbers could be obtained from them by making the following sums: (i) 1; 3 + 5; 7 + 9 + 11; 13 + 15 + 17 + 19; ... these sums represent a PA of order 3 given by (1, 8, 27, 64, ...). (Boyer & Merzbach, 2012).

With this understanding, the "Precious Mirror", which is a Chinese work, which in addition to the polynomial sums, it brings the arithmetic triangle that in the west became widely known under the name of Pascal's Triangle. In China, Jia Xian's Mathematical Handbook, written around the year 1050, also dealt with some concepts related to the arithmetic triangle. The arithmetic triangle presents a collection of infinite PA, containing for each order arranged in its columns, a representative, from the order zero to the order n. It was between the years of 1280 and 1320 d. C., who historians point out that lived the last and greatest Chinese mathematician of the Golden Age, Chu Shih-Chieh, who for twenty years taught for various regions of China. In 1303 he wrote an important mathematical treatise entitled Ssu-yu-ü-chien or Precious mirror of the four elements. We emphasize that this work marked the peak of the development of Chinese algebra. Tartaglia claimed to create the arithmetic triangle for him, and in some countries of Europe, the arithmetic triangle is known as the Triangle of Tartaglia. The French Blaise Pascal, in the years of 1623-1662, had his first contact with the Arithmetic Triangle motivated by the resolution of a problem that involved the probability of playing two dice, if it obtained a double 6.

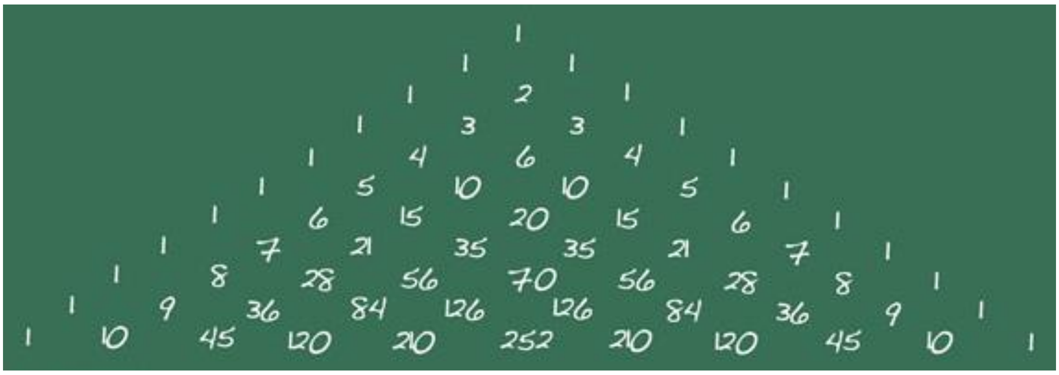
Pascal proposed a new model for the triangle, and studied its properties more thoroughly than its predecessors, proving several of them. The consecration of the current denomination, Pascal's Triangle was due to the fact that in 1739, De Moivre (Abrahan de Moivre, 1667-1754) published a work that reached great repercussion at the time, in which he used the denomination "triangulum, arithmeticum pascalianum" for the Arithmetic Triangle. Let us look at the representation of the arithmetic triangle, in figure 2, below.

Figure 2. The Chinese Arithmetic Triangle.

Source: Boyer & Merzbach (2012)

The figure that presents the arithmetic triangle, shows that not only the traditional PAs were known of the ancient peoples, but also those of orders 2, 3 and 4. It is worth emphasizing that the arithmetic triangle is well known historically because it presents many numerical patterns involving sequences, sums, combinations, binomial coefficients, among other patterns. Thus, to produce the sequence, a generator element is taken, which is generally number 1, but not necessarily number 1, but any other number. And in this sense, each element of the sequence must be the sum of the two elements that are just above it, if there are not two elements it must be equal to the generating element. Let's look at Figure 3, below.

Figure 3. Arithmetic triangle.



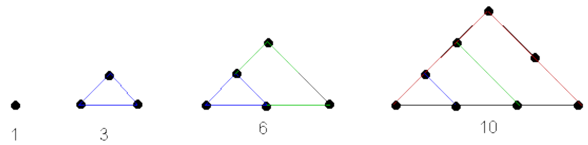
Source: Proec (2015)[4]

3.1 Figurative numbers: geometric representations

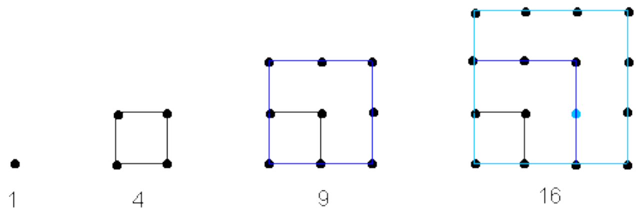
We know that the figures are numbers expressed as points, and these points are grouped by suggesting the idea of geometric shapes. In basic mathematics we find the PAs of order 2, other interesting manifestations of this type of PA are the sequences of the figures that will be in focus next. Let's look at the following representations.

Figure 4. Figure numbers.

Triangular numbers.

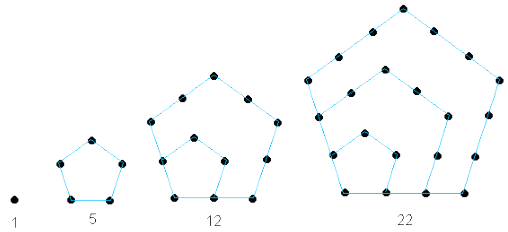


Square *number*



Pentagonal Numbers

[4] http://proec.ufabc.edu.br/a-proec/divulgacao-cientifica/ufabciencia/o-triangulo-aritmetico. Accessed on November 4, 2018.

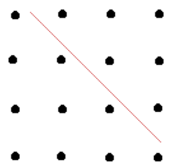


In the following we have listed and proved some theorems concerning the figure numbers, based on what was done by the Pythagoreans.

Theorem I: The triangular number T_n is equal to the sum of the n first positive integers.
Figure 5. The triangular number.

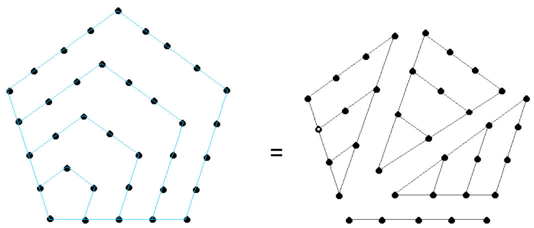
$$T_4 = 10$$

Theorem II: Every square number is the sum of two successive triangular numbers. Refer to figure 3, below.
Figure 6. Square number.



We observe that a square number in its geometric form can be divided as in figure 3, previous. Next, let us see the proof of the theorem algebraically.
Be the nth triangular number T_n , given by the sum of the arithmetic progression, $T_n = 1+2+3+..+n = \frac{n*(n+1)}{2}$, be the nth square number S_n equal to n^2. We have $S_n = n^2 = \frac{n*(n+1)}{2} + \frac{n*(n-1)}{2} = T_n + T_{n-1}$

Let us see next the pentagonal numbers, figure 4 below presents the theorem.
Theorem III: *the nth number pentagonal is equal to n plus three times the (n-1) -th triangular number.*
Figure 7. Pentagonal number.



Be the nth pentagonal number, P_n , given by the sum of an arithmetic progression

$$\begin{aligned} P_n &= 1+4+7+\ldots+(3*n-2) = \tfrac{n*(3*n-1)}{2} \\ &= n + \tfrac{(3*n)*(n-1)}{2} = n + 3*T_{n-1} \end{aligned}$$

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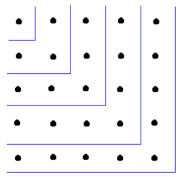
Theorem IV: *The sum of the first n odd integers, starting with 1, is the square of n.*

Figure 8. Square of n.



Calculating the sum of the arithmetic progression, we have:

$$1+3+5+..+(2*n-1) = \frac{n*(2*n)}{2} = n^2$$

As we have pointed out before, the Pythagorean school has always been interested in researching and discovering the secrets of geometry and numbers.

Figure 9. Number figured.

This is an infinite numerical sequence: 1, 3, 6, 10, 15, 21, 28, 36 ... Each term of the sequence of triangular numbers can be obtained by using the general formula: T (n) = 1 + 2 + 3 + ... + n

Or

$$T(n) = \frac{n \cdot (n+1)}{2}$$

For example, if we want to know the 5th triangular number, just do: T (5) = 1 + 2 + 3 + 4 + 5 = 15. And the 8th triangular number will be given by: T (8) = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36. As for the square numbers, the number of points also represents a number that ends up in a square. We also have one more infinite sequence: 1, 4, 9, 16, 25, 36, 49 ... Thus, each sequence number of the square numbers can be obtained according to the general term formula below: $Q(n) = n^2$. But if for example, if we want to know which is the 3rd square number, we have: $Q(3) = 3^2 = 9$. And the tenth square number, what will it be? $Q(10) = 10^2 = 100$.

In the case of pentagonal numbers, the number of points represents numbers which, in turn, form pentagons. Each element of the sequence of pentagonal numbers can be obtained from the general formula:

$$P(n) = \frac{n(3n-1)}{2}$$

In this sense, to determine the 5th term of the sequence of pentagonal numbers, we will present:

$$P(5) = \frac{5(3 \cdot 5 - 1)}{2} = \frac{5 \cdot 14}{2} = 35$$

Thus, the 10th term of this sequence will be: $P(10) = \frac{10(3 \cdot 10 - 1)}{2} = \frac{10 \cdot 29}{2} = 145$

The sequence of pentagonal numbers is also infinite: 1, 5, 12, 22, 35 ... It is worth mentioning that working with the figured numbers is an interesting mathematical research activity that can be taken to the classroom at any level of learning, in this sense, the following we present some contributions of the figures figured for the development of geometric thinking.

3.1 Contributions of the figured flat figures (2D) in the development of geometric thinking[5]

Pythagoras was attributed to the arithmetic of dots, but we have no proof that it is a creation of this mathematician, and / or the members of the old school, called the Pythagorean. The fact is that to the Pythagoreans, there was no notion of pure numbers. But as for the spatial and concrete character, it is possible to affirm that the Pythagorean numbers were not the mathematical objects as we know them today, but rather, abstract beings.

As for the figured figures of the Pythagoreans, they were made up of a multiplicity of points that were not mathematical that referred to discrete elements, they were organized pebbles, from a certain configuration, that is, these numbers represented figures formed by points. For example, it was not a cipher, like 3, that serves pictorial representation for a number, but the delimitation of an area consisting of points, such as a constellation.

We present below some of the propositions, theorems and corollaries that determine fundamental relations between triangular, square, pentagonal and hexagonal numbers, as well as the existence of heptagonal and octagonal numbers, in order to interweave these concepts to the development of geometric thinking.Teorema: Given away $n \in \mathbb{N}$, com $n \geq 1$, a triangular number is given by the relation $T_n = \frac{n(n+1)}{2}$.

Demonstration: from an inductive reasoning, we have to, $T_{n+1} = T_n + (n+1)$, para $n \geq 1$. However, this formula is still inadequate, because to calculate the triangular number T_{100}, for example, it is necessary first to know the value of T_{99}. To overcome this problem, we rely on the sums, which gave rise to the first 10 terms of the sequence of triangular numbers. Let's see:

$$T_1 = 1;$$
$$T_2 = T_1 + (1+1) = 1 + 2 = 3;$$
$$T_3 = T_2 + (2+1) = 3 + 3 = 6;$$
$$T_4 = T_3 + (3+1) = 6 + 4 = 10;$$
$$T_5 = T_4 + (4+1) = 10 + 5 = 15;$$
$$T_6 = T_5 + (5+1) = 15 + 6 = 21;$$
$$T_7 = T_6 + (6+1) = 21 + 7 = 28;$$

[5] The formulas were extracted from the text by Marques (2014), which appears in the references of this work.

$$T_8 = T_7 + (7+1) = 28 + 8 = 36;$$
$$T_9 = T_8 + (8+1) = 36 + 9 = 45;$$
$$T_{10} = T_9 + (9+1) = 45 + 10 = 55;$$

From this observation, we saw that:

$T_1 = 1 = \frac{1 \cdot 2}{2} = \frac{1 \cdot (1+1)}{2};$

$$T_2 = 3 = \frac{2 \cdot 3}{2} = \frac{2 \cdot (2+1)}{2};$$
$$T_3 = 6 = \frac{3 \cdot 4}{2} = \frac{3 \cdot (3+1)}{2};$$
$$T_4 = 10 = \frac{4 \cdot 5}{2} = \frac{4 \cdot (4+1)}{2};$$
$$T_5 = 15 = \frac{5 \cdot 6}{2} = \frac{5 \cdot (5+1)}{2};$$
$$T_6 = 21 = \frac{6 \cdot 7}{2} = \frac{6 \cdot (6+1)}{2};$$
$$T_7 = 28 = \frac{7 \cdot 8}{2} = \frac{7 \cdot (7+1)}{2};$$
$$T_8 = 36 = \frac{8 \cdot 9}{2} = \frac{8 \cdot (8+1)}{2};$$
$$T_9 = 45 = \frac{9 \cdot 10}{2} = \frac{9 \cdot (9+1)}{2};$$
$$T_{10} = 55 = \frac{10 \cdot 11}{2} = \frac{10 \cdot (10+1)}{2};$$

Valid procedure for the following terms of the sequence of triangular numbers. We note that only the axiomatic rules that are allowed in the numerical sets were employed. And, therefore, we hold the validity of the following property, for n $\in$ IN any.

$T_n = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$So we have to:

By checking the same property, and its validity for the set of natural numbers, we define a set $\mathcal{H}$, as an induction hypothesis, where $\mathcal{H} := \{ n \in \mathbb{N} \mid T_n = \frac{n(n+1)}{2} \}$, as an induction hypothesis, where $\{1, 2, 3, \ldots\} \in \mathcal{H} \neq \emptyset$.

Thus, the next stage of this demonstration by mathematical induction needs verification that $n \in \mathcal{H}$, so too $n + 1 \in \mathcal{H}$.

To obtain the induction thesis, we present:

$$T_{n+1} = 1 + 2 + 3 + \cdots + n + (n+1) = \frac{n(n+1)}{2} + (n+1)$$
$$= \frac{(n+1)(n+2)}{2} = \frac{(n+1)((n+1)+1)}{2}$$

By the principle of mathematical induction, we conclude that

$$n+1 \in \mathcal{H} := \{n \in \mathbb{N} \mid T_n = \frac{n(n+1)}{2}\} \therefore \mathcal{H} = \mathbb{N} \qquad \blacksquare$$

Such relationships are prominent in order to obtain a generalized model. Thus, we also understand that these relations, little worked in the teaching of geometry specifically for the development of geometric thinking, collaborate for the evolution of hypothetical and fallible knowledge, because this is how science progresses, through problems, conjectures and refutations . It is in the creative process of the development of mathematics that the inductive slope plays a fundamental role. Thus, we have that learning mathematics has a strong investigative bias, in which discovery / creation of strategies, through trial and error, are presented as inherent and indispensable processes for learning.
Here are our considerations.

4 CONSIDERATIONS

We point out our considerations, highlighting some implications and impacts, from the actions to be carried out through this investigation. We start from the assumption that the Pythagorean Numbers, when being discovered and presented, started to subsidize the intuitive study, following a heuristic methodology, of the concept of boundary of a succession and of arithmetic progression.

We hope with this research that the reflections on the importance of a more meaningful teaching, considering the diversity that involves the types of mathematical thoughts, emphasized here, the arithmetic, algebraic and geometric thoughts, collaborate with the teacher (initial formation) of their difficulties in relation to basic mathematical concepts, and teaching strategies; that helps in resizing the look in / to the teaching of geometry, specifically, the figure numbers; collaborate with reflections on the [re] organization of the curriculum from the difficulties pointed out in the analysis of the proficiency in external evaluations, aiming at the transformation of teaching from a revolutionary perspective, based on methodologies that have proposals of innovation, in the conscious decision making by teachers, aiming at the quality of education, and finally, that it serves to reflect in a practice of a more humane, ethical, life-transforming teacher, and for that, aware of his creative and innovative pedagogical responsibility for the purposes of science, but also of culture, respectively, to be a scientist, and a solver of problems, to be pragmatic with problems of the society, however, always glimpsing a teaching ruled in the praxis.

The major challenge of this research was to find research that discussed the importance of types of mathematical thinking, specifically, geometric thinking, from a historical and practical perspective, from the figurative figures and the relation of teaching to learning in mathematics classrooms, in the final years of elementary school.

We searched for this discussion in the National Curricular-BNCC (BRASIL, 2017), and we did not find something related to this topic in the presentation of skills, competences, or descriptors. Thus, it was possible to identify a monograph (MARQUES, 2014), and an article by (ALVES, BORGES NETO & MAIA) addressing the perspective of the figure numbers, but we did not identify the relation with geometric thinking. From the observance of these challenges, we consider it necessary that this theme be expanded, seeking now subsidies in the classrooms in elementary school, with the purpose of collaborating with the passage from arithmetic to geometric thinking, without ruptures, reaching algebraic thinking, without major gaps and / or ruptures.

Given the relevance of the theme and the current discussion on the national scene, about the advances and setbacks in the evaluation indices in mathematics, we consider the construction of a new educational paradigm in a perspective of a more inclusive and meaningful education to be primordial, and this presupposes a movement of educational changes, with respect to [re] organization curricular that meets the innovative formative expectations, with ends in the discussions, on occasion, preliminaries.

We know the challenges that the area faces, so we want to expand those discussions, at another time.

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