Enhancing the Teaching and Learning of College Algebra with ColAlgMap

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Abstract

Teaching and learning lower level mathematics courses on the college level can be challenging and overwhelming. This paper will demonstrate how to design and develop a maplet package to aid in the teaching and learning of mathematics, to serve as a private tutor with infinite patience for college algebra students, and as a useful pedagogical tool for instructors. By using interactive worksheets and animated graphics in maplets, students will find the opportunity for numerous experiments that will foster their development of mathematical conceptual understanding and of computational skills.

Keywords: College algebra; teaching mathematics; learning mathematics; Maple; maplet;

1. Introduction

The Maple software is a computer algebraic system, a program that is able to manipulate information in a symbolic or algebraic manner. The symbolic capacity system makes Maple able to calculate exact analytical solutions of many mathematical problems. This makes Maple a powerful software program that can be used to solve general-purpose mathematical problems. While Maple math libraries are powerful, they do not explain the procedures and steps of solving the problems, and students need to learn the new and unfamiliar Maple language which may take away their precious studying time.

Maple has a package called maplet (see e.g. in Monagan M. B., et al., 2009, ErnicKamerich, 2011, Monagan M.B., 1998, Robert J. Lopez, 2012), which offers an ergonomic tool in the application. This package allows

the possibility of creating windows, buttons, dialogues and other visual interfaces which interact with the users. The maplet package is one of the effective ways to develop Graphical User Interface (GUI) applications. In short, a maplet allows a Maple software user to combine Maple math libraries and user procedures with interactive windows and dialogs. The authors use Maple as a support for the creation of a maplet package, namely ColAlgMap, which is an effective didactic tool to assist students and instructors in learning and studying College Algebra.

ColAlgMap creates both algorithmically-generated and user-entered problems while students are guided through the solution process. Students can find hints, receive immediate feedback with detailed explanations, and step-by-step checking of their responses with infinite patience. It is as effective as a private tutor. Instructors can also use ColAlgMap as an effective demonstration making use of the 2D/3D graphics and animations, and the ability to launch a maplet with a specific example or use it to create exam problems.

In short, ColAglMap (i) allows students to find their own errors, to correct themselves, and to understand the question topic better; (ii) proves to be a powerful motivating force for encouraging students to take their homework seriously; (iii) provides collaboration among students in the completion of their assignments and projects. Therefore, ColAlgMap can balance the development of understanding and technical skills.

2. Material and Methods

Design and develop maplets to solve problems in a College Algebra course.
A complete maplet should be able to do the followings:

1. Allow students to enter or automatically generate new problems.
2. Describe problems graphically.
3. Allow students to check whether an answer is correct or not.
4. Allow students to solve problems step-by-step with instructions for each step.
5. Allow students to view solutions step-by-step or all steps at once.
6. Display meaningful messages together with colorful status bars to interact with the students' actions.
7. Provide hints.

Design a GUI which contains the following sections and buttons:

1. The "Problem" section describes a problem or a set of problems. Students can enter a new problem or generate a random problem by using the "Similar Problem" button.
2. The "Answer" section lets students input the final answer. Students can then use the "Check Answer" button to check for correctness.
3. The "Solution Window" section displays students' answers in correct mathematical symbols and detailed solutions step-by-step. Students can click on the "Solution" button to start the solutions. They will then use the "Next Step" or "Undo" button to navigate to the next step or to the previous

one. The "All Steps" button is used to show all steps at once.

4. The "Interaction Window" section displays meaningful messages to tell students if their answers are correct or not, or detailed instructions for each step when the students try to solve the problems step-by-step. At each step, students will read the instructions in the "Interaction Window" and input their answers in the boxes below the "Check Step #" button. These boxes are dynamic fields which will display differently depending on each different step. Students will use the "Check Step #" button to check for the correctness of each step and corresponding messages will be displayed in the "Interaction Window". The maplet only goes to the next step if the current step is correctly evaluated and the correct step is appended to the "Solution Window". At each step other than the final one, students can also use the "Next Step" button to view the correct answer in the "Solution Window".
5. The "Plot Window" section describes the problem graphically. Students will check the box "Show Graph" to view the pictures. This check box will only be activated when the "Solution" button is activated.
6. The "Help" button will provide the necessary material and techniques for solving the problem.
7. Two status bars at the bottom use many different colors to give additional response to the students' answers such as green for a correct answer, red for a wrong answer, orange for a wrong input, or yellow for choosing "Next Step" and letting the maplet finish the step automatically. The number of short bars on the right is the number of steps students are expected to go through to finish the solutions of the problem. The black color of the short bars indicate the remaining steps. Each short bar will change color to green, red, or yellow corresponding to the response to the students' answers for each step.

All the sections above can be implemented using Maple components and packages such as Label, TextField, TextBox, Button, RadioButon, DropDownBox, CheckBox, Plotter, MathML and MathMLViewer. Moreover, since the "Solution Window" contains both English and symbolic expressions, MathML encoding has to be rendered manually in order to display correct indentations and spacing.

For example,
"The following expression is 10 pixels below:

$a^2 + b^2$ " can be represented semantically by the Presentation MathML as

```
<math>
    <mtext>The following expression is 10 pixels below:</mtext>
    <mspace linebreak='new line'/> <mspace height = "10px"/>
    <mrow>
        <mrow>
            <msup>
                <mi>a</mi>
                <mn>2</mn>
            </msup>
        </mrow>
        <mo>+</mo>
        <mrow>
            <msup>
                <mi>b</mi>
                <mn>2</mn>
            </msup>
        </mrow>
    </mrow>
</math>
```

3. Result

To be able to demonstrate the usefulness of ColAlgMap, the following maplet had been used to solve a class of circle problems during a College Algebra course at the University of North Georgia. The maplet in figure 1 contains five problems and the first problem is currently selected.

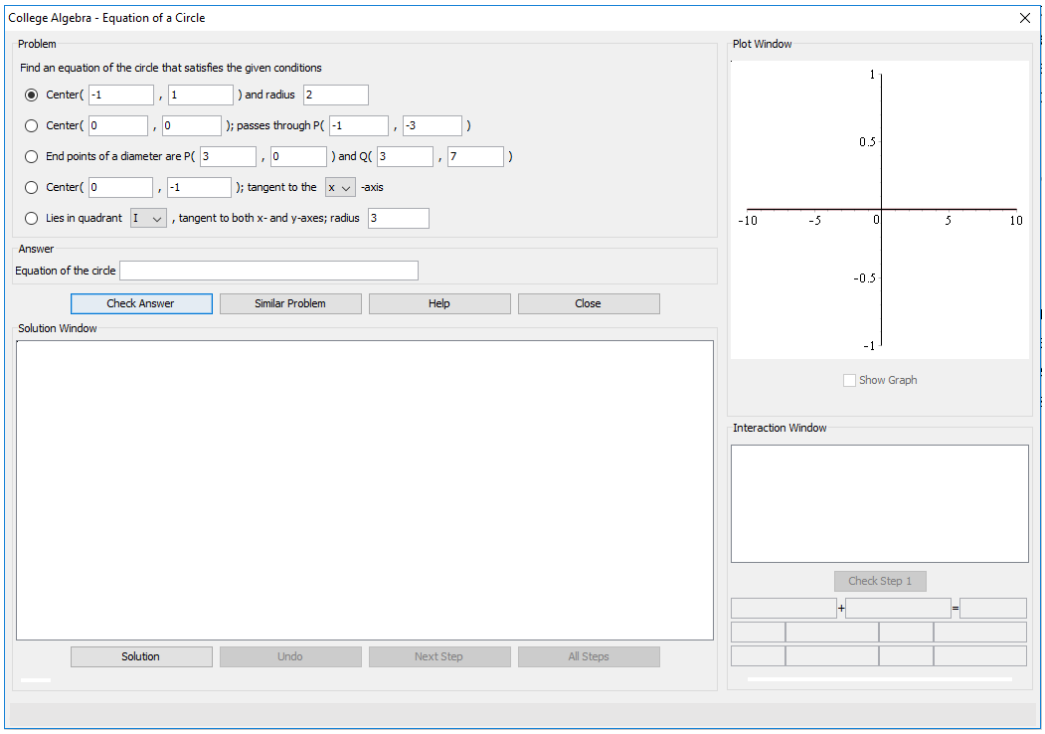

Figure 1. Find an equation of the circle that satisfies the given conditions: Center (-1, 1) and radius 2.

Figures 2 through 12 demonstrate how to use the maplet to solve the problem: *"Find an equation of the circle that satisfies the given conditions: End points of a diameter are P (3, 0) and Q (3, 7)".*

Figures 2 and 3 demonstrate how students check an answer. The maplet in figure 2 displays the student's answer in a readable form in the "Solution Window", instructive feedback in the "Interaction Window", and changes the status bar at the bottom right to red to indicate the answer is incorrect. The maplet responses to the correct student's answer in the "Interaction Window" of figure 3. Notice that the bottom right status bar changes from red to green.

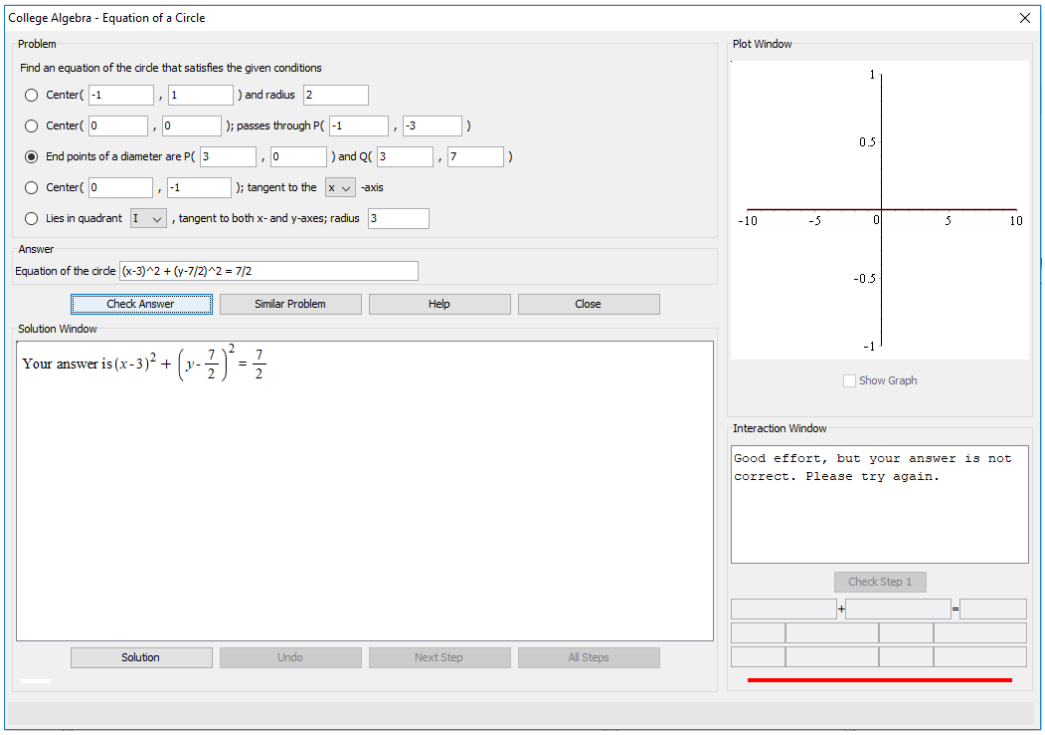

Figure 2. "Check Answer" is clicked for incorrect answer.

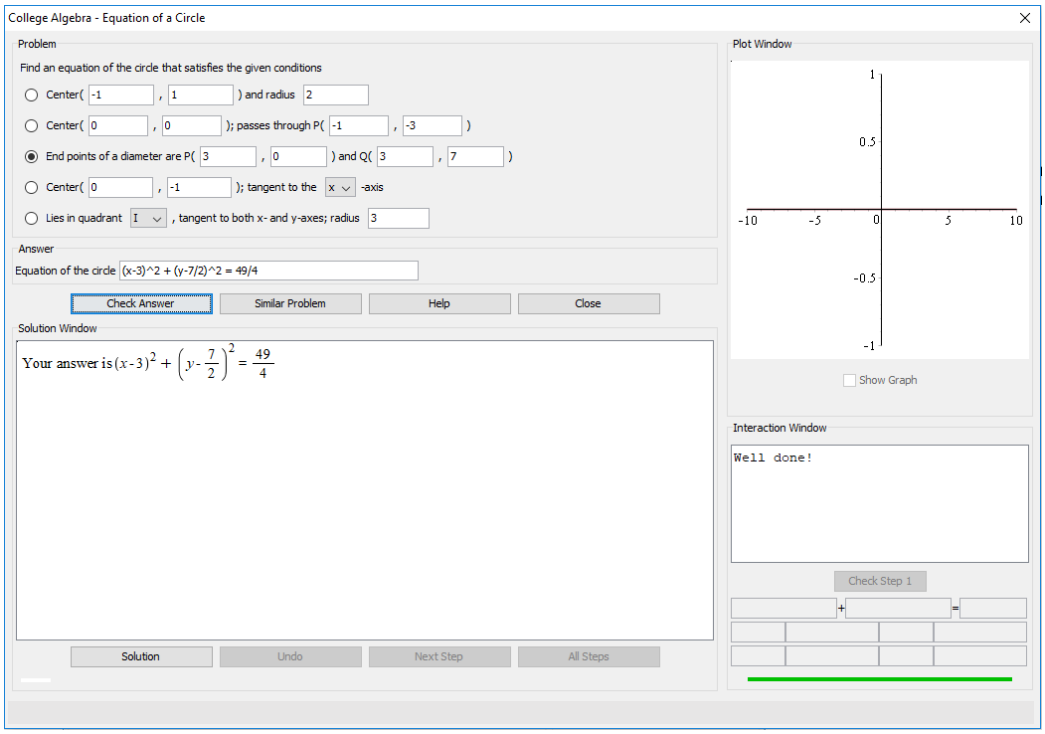

Figure 3. "Check Answer" is clicked for correct answer.

Figure 4 demonstrates when the student would like to see all steps of the solution at once by clicking on "Solution" and then "All Steps". The detailed solution is shown in the "Solution Window". The status bars changes to yellow to indicate the problem is solved automatically. Moreover, the five short status bars at the bottom right imply there are five steps to the solution.

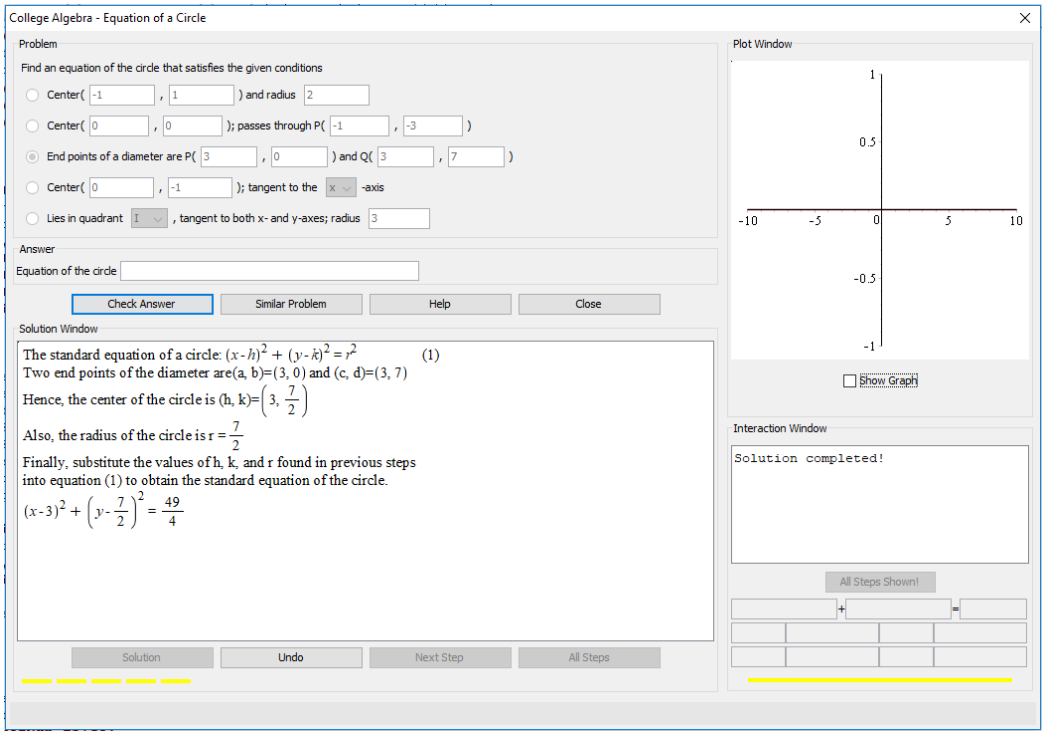

Figure 4. "All Steps" is clicked and the completed solution is shown.

Figures 5 through 10 demonstrate when the student would like to do the problem step-by-step by first clicking on the "Solution" button. The instructions for each step are shown in the "Interaction Window". The student can do the current step by entering the input in the boxes below the "Check Step #" button and then clicking on this button. Figure 6 shows that "Check Step 1" gives an incorrect answer. A message is displayed in the "Interaction Window", the first status bar on the bottom left, and the one on the bottom right, change to red to indicate the first step is incorrect. If an answer is correct, a congratulating message is received, and the instructions for the next step are displayed in the "Interaction Window" as seen in figure 7. Also, the status bars changes to green and the correct answer for step 1 is appended to the "Solution Window". If the student would like to see the step without doing it, he or she can click on the "Next Step" button. Figure 8 shows that step 2 is skipped. The answer to this step is appended in the "Solution Window" and the status bars change to yellow. Figures 9 through 11 demonstrate the rest of the solution step-by-step. In any step, the student can check the box "Show Graph" to see the graphical description of the problem as shown in figure 12.

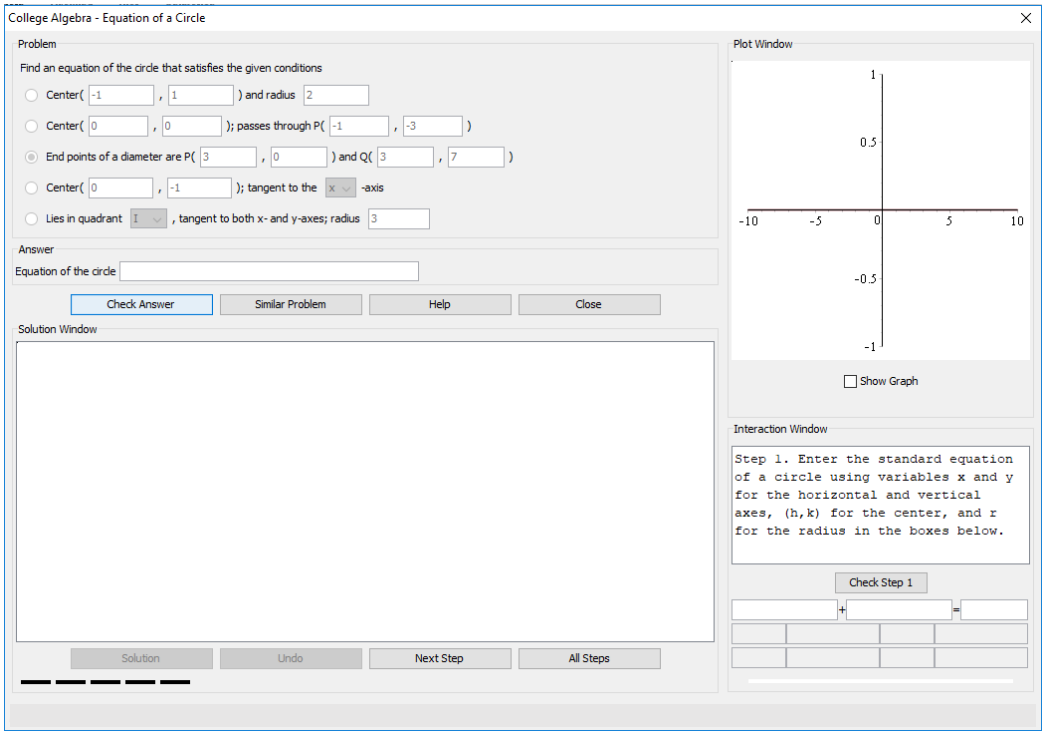

Figure 5. "Solution" is clicked and Step 1's instruction is shown.

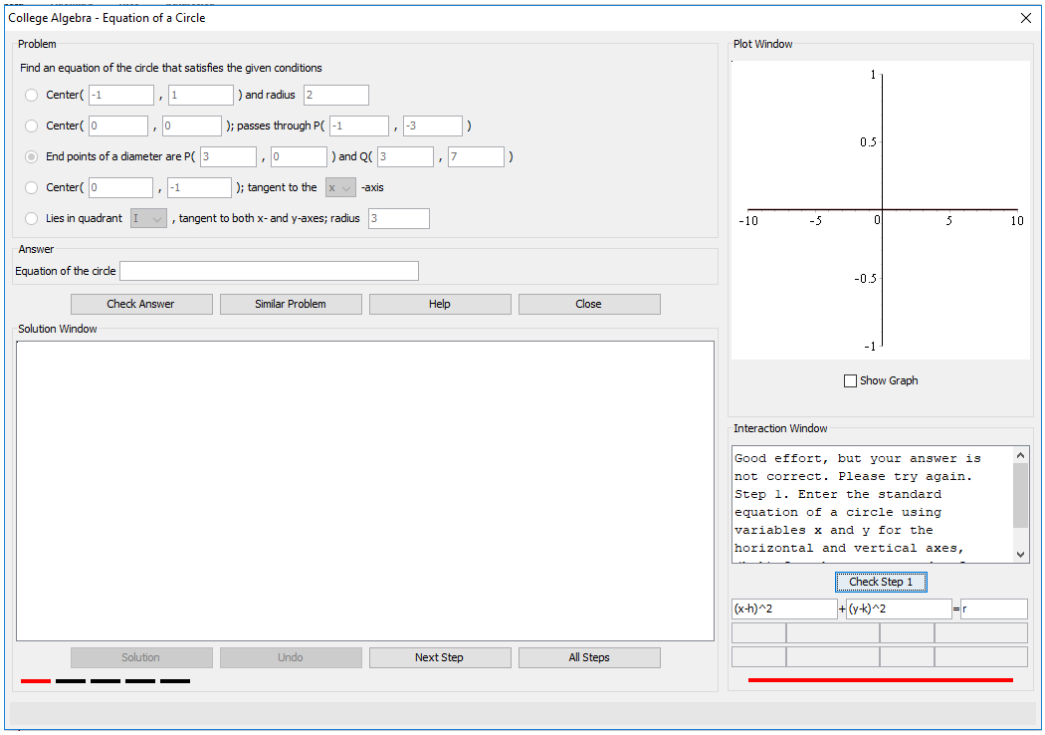

Figure 6. "Check Step 1" is clicked for incorrect answer.

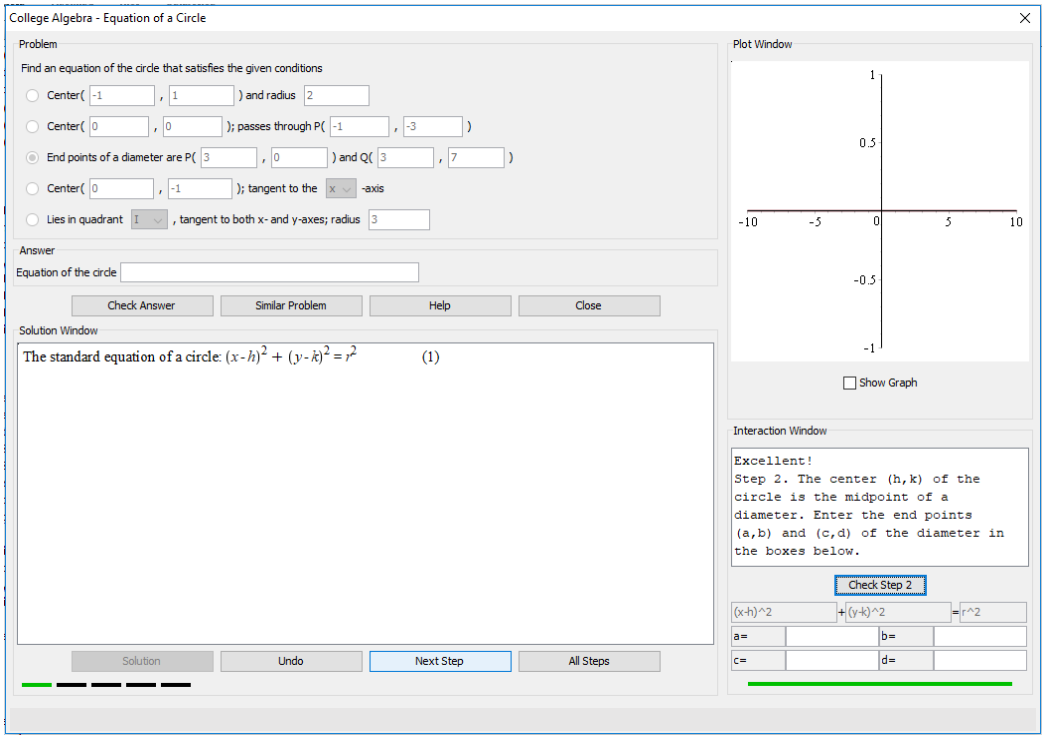

Figure 7. "Check Step 1" is correct and move to step 2.

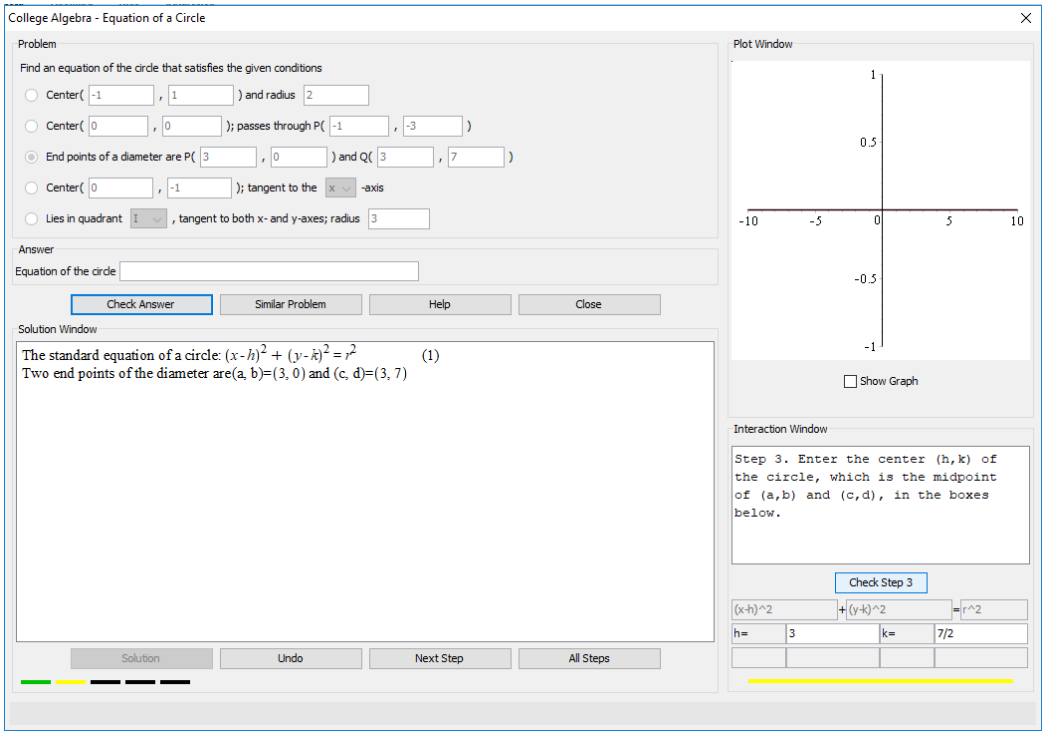

Figure 8. "Next Step" is clicked and move to step 3.

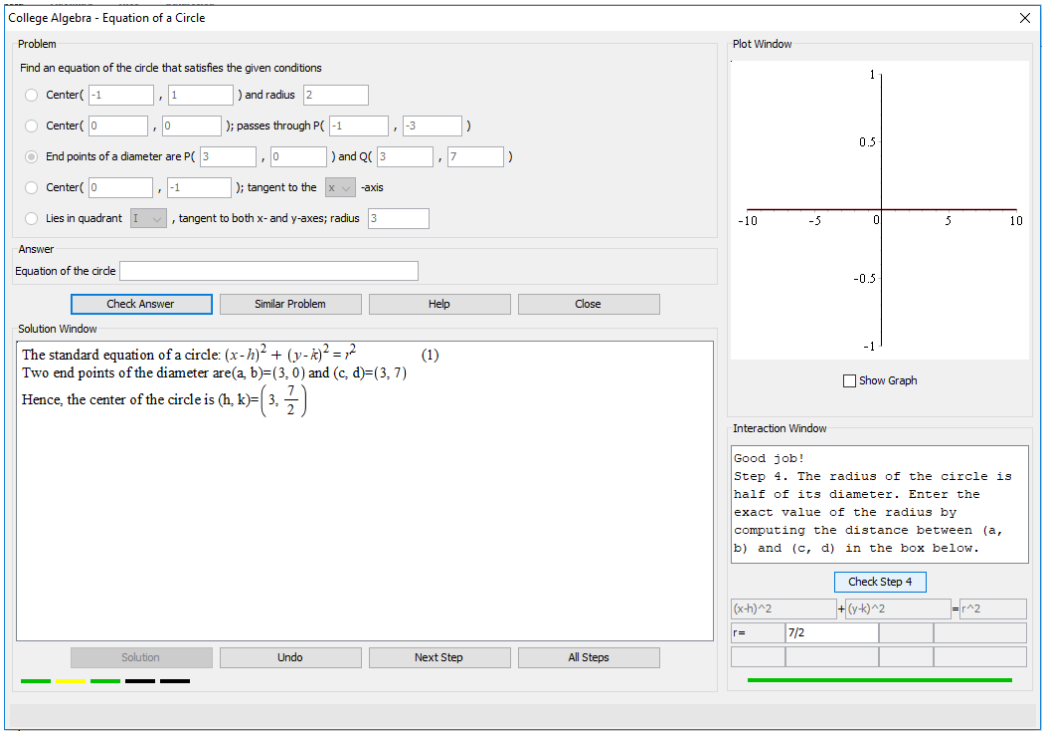

Figure 9. "Check Step 3" is correct and move to step 4.

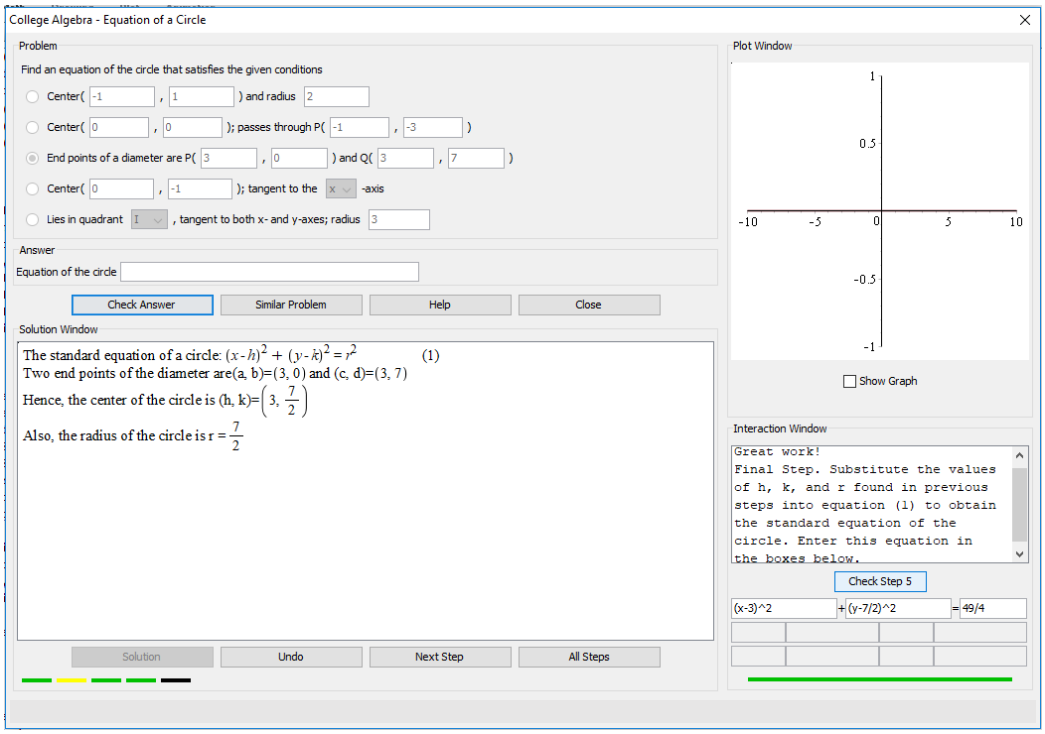

Figure 10. "Check Step 4" is correct and move to step 5.

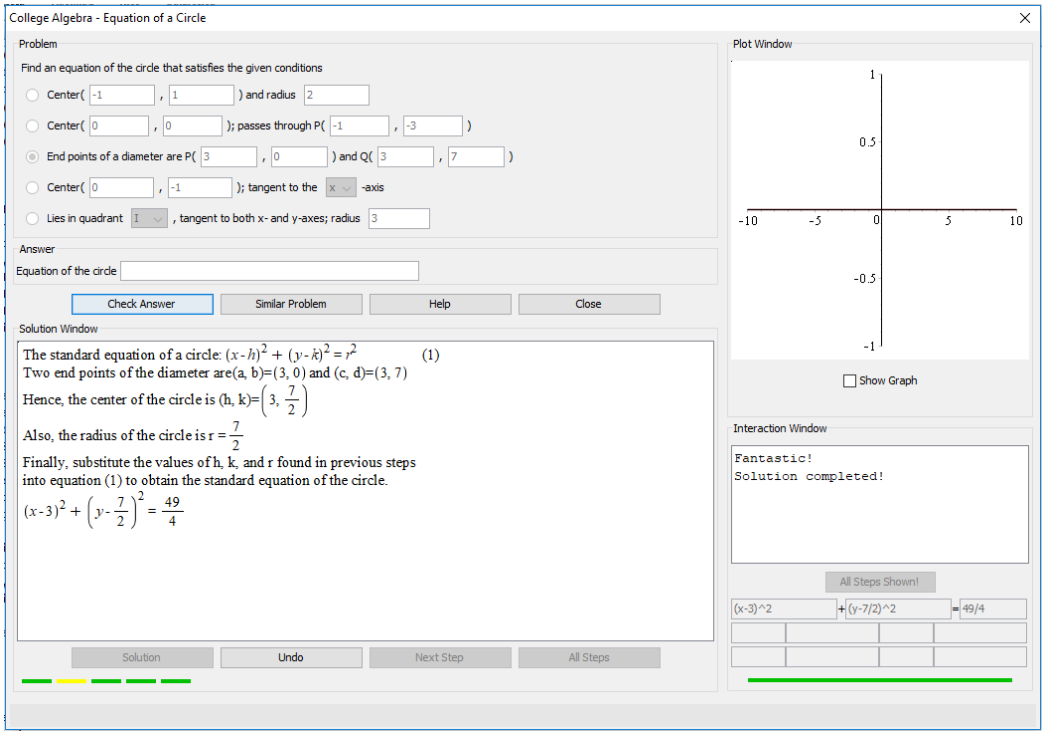

Figure 11. "Check Step 5" is correct and the solution is completed.

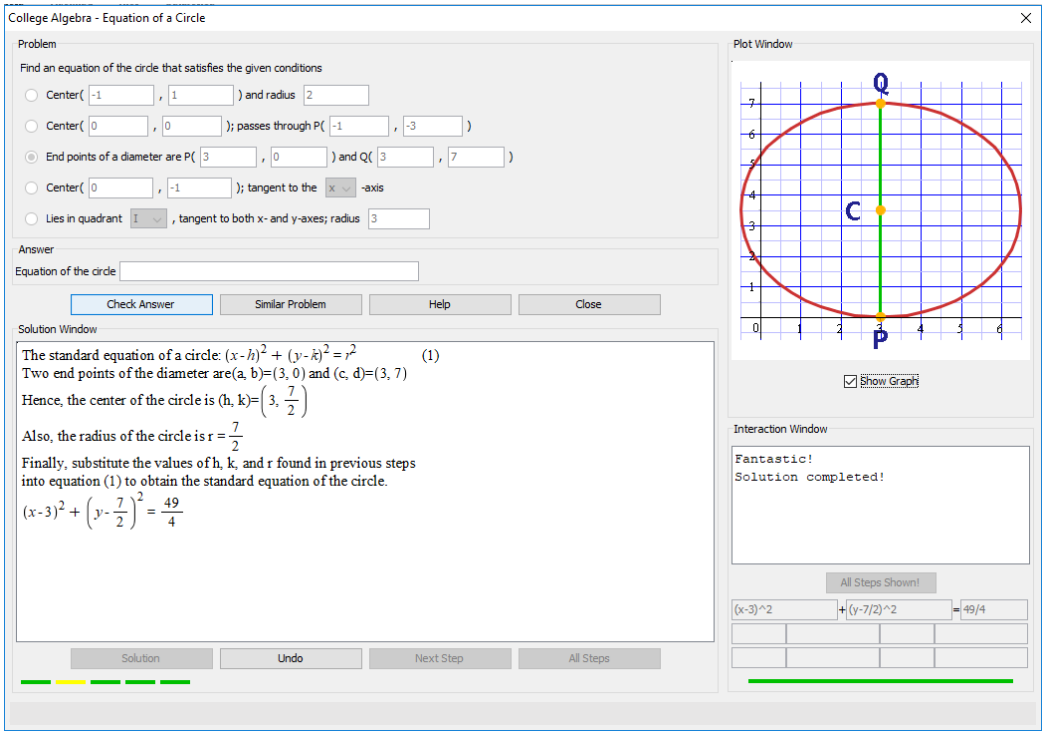

Figure 12. "Show Graph" is checked and the graph is displayed.

The student can also practice more problems by clicking on the "Similar Problem" button. The maplet will generate a new version of the chosen problem as shown in figure 13.

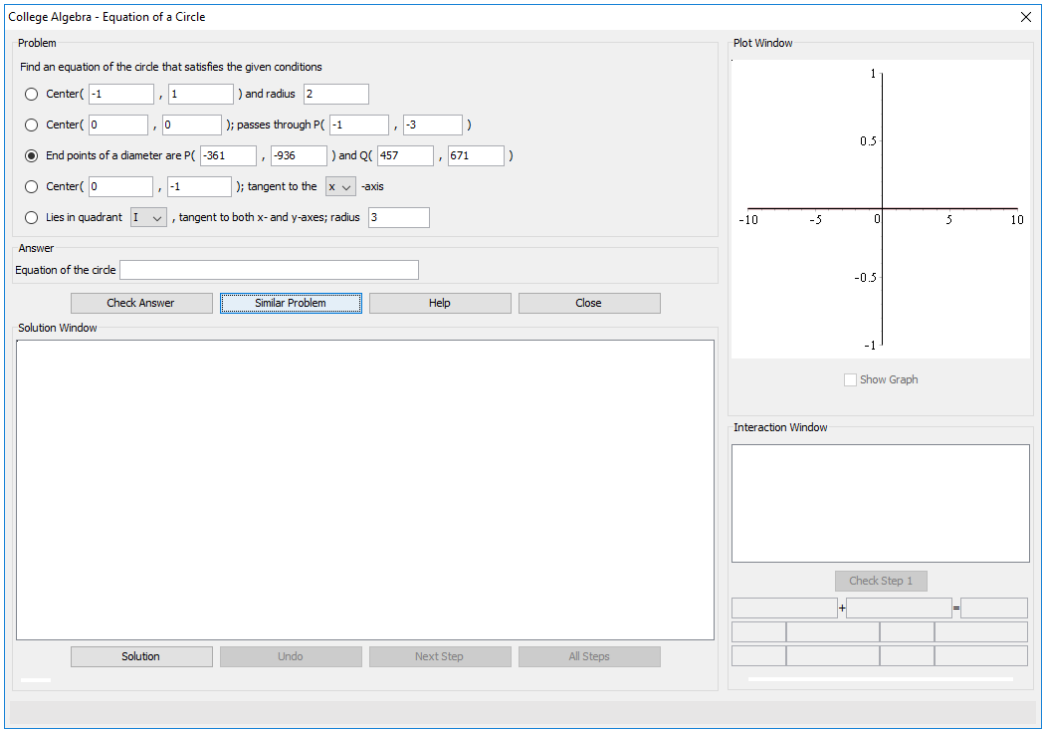

Figure 13. "Similar Problem" is clicked and new points are generated P (-361, -936) and Q (457, 671) for the selected problem.

Figures 14 through 17 show other problems in finding an equation of the circle that satisfies different given

conditions and their corresponding detailed solutions when "All Steps" is clicked and "Show Graph" is checked.

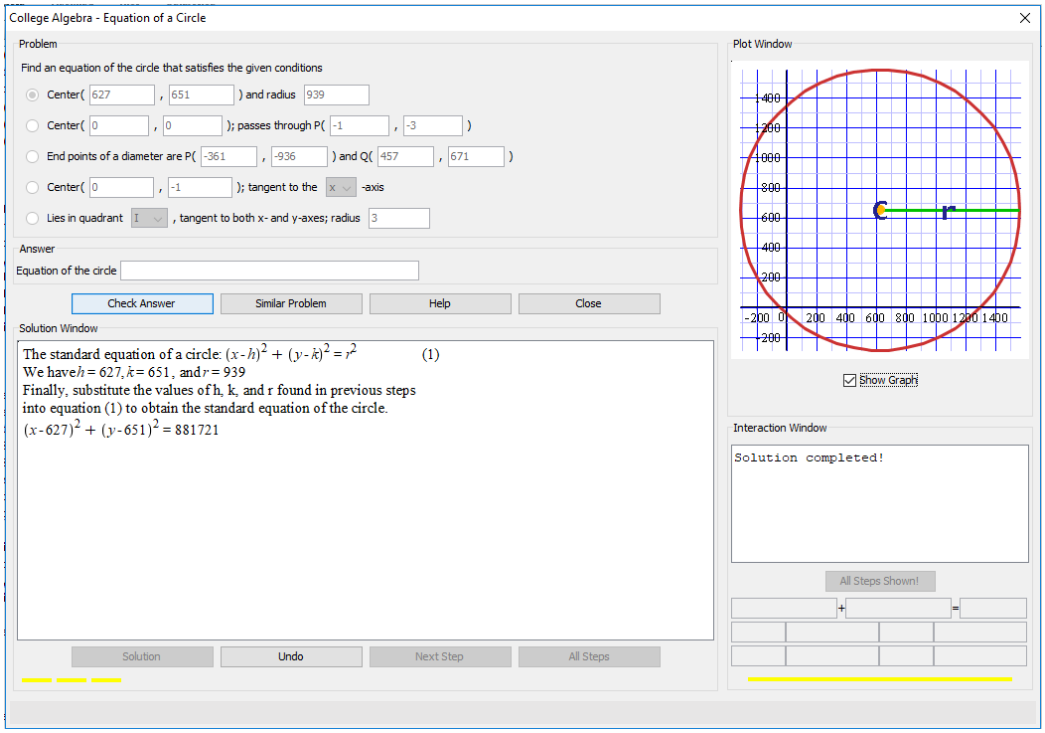

Figure 14. Center (627, 651) and radius 939.

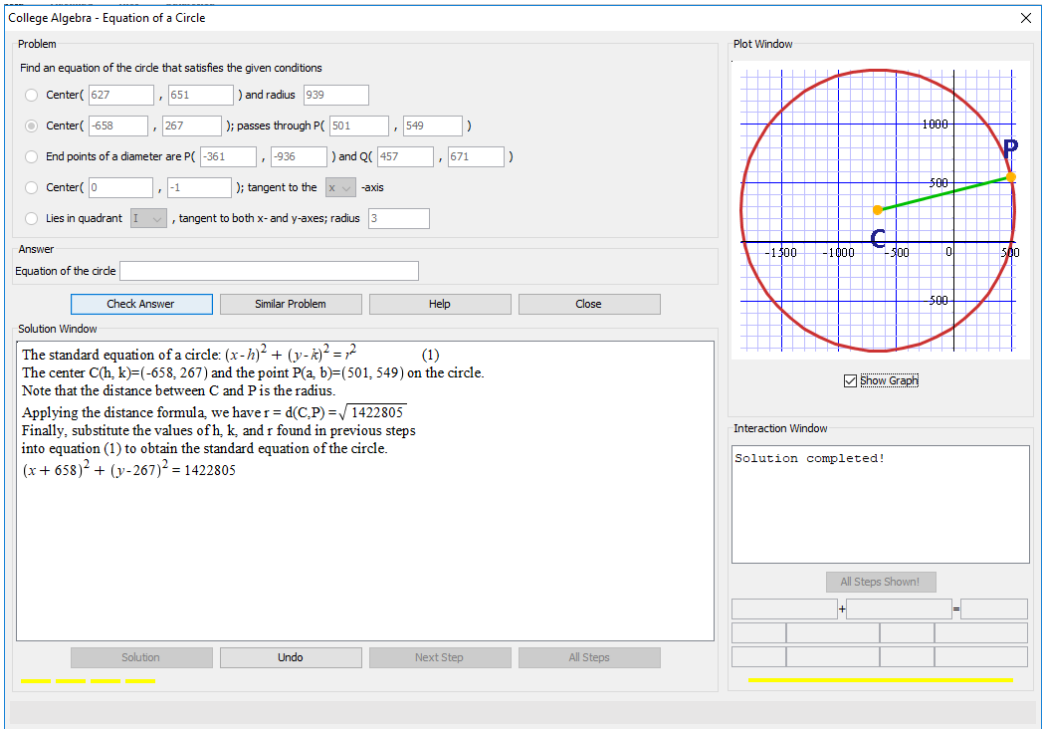

Figure 15. Center (-658, 267); passes through P (501, 549).

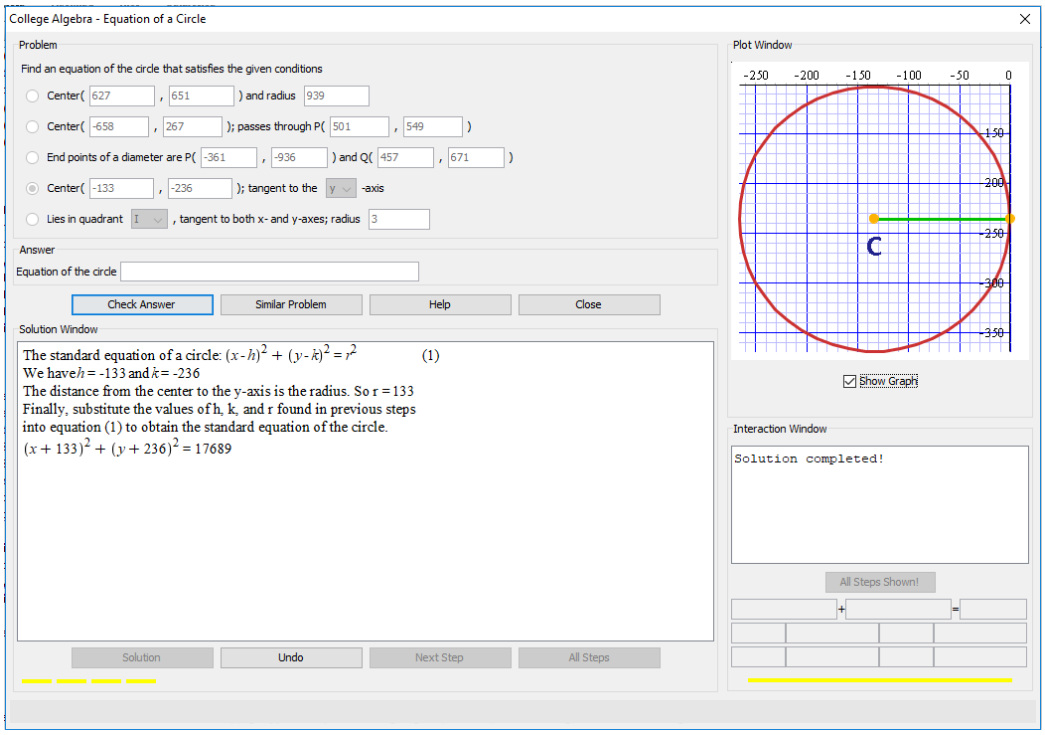

Figure 16. Center (-133,-236); tangent to the y-axis.

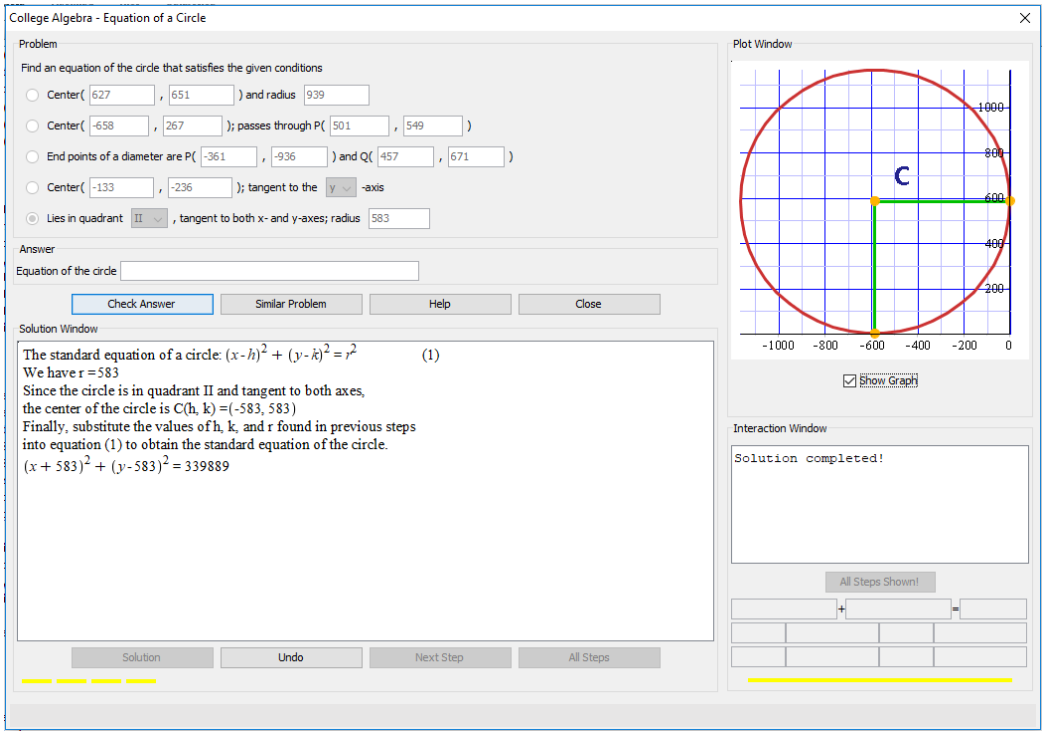

Figure 17. Lies in quadrant II, tangent to both x- and y-axis; radius 583.

4. Conclusion

ColAlgMap significantly helps instructors manage a large pool of students, and also provides students with adequate practice to prepare for their courses. It can be used for demonstrations in lectures to reveal a new

concept, as a group activity in a lab setting, or as a drill-and-practice tutor for individual students. The maplets support both randomly generated problems as well as user-entered problems. This allows students to enter a specific problem from another source. Also, the algorithmically-generated problems allow students to work on different problems and to use their experience to derive a general method for a class of similar problems. For example, the Equation of a Circle maplet provides a general idea for finding the equation of a circle by first identifying its Center and Radius.

ColAlgMap uses a combination of symbolic, graphic (2D/3D, sometimes with animation), numeric, and verbal devices to investigate problems. Moreover, immediate feedback and the program's infinite patience are two features students can find very helpful. ColAlgMap can guide students to work through problems step-by-step. Correct answers to earlier steps are required before moving on to later steps. Hints are available, and when errors are detected, instructive feedback is provided. All of this enables students to use ColAlgMap as a tutor without the physical tutor in the convenience of their dorms or at home, without unwarranted stress.

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