Distance Education in Brazil: An Experience with Mathematical Modeling with GeoGebra Aid

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Abstract

A mathematical modeling project was proposed for a student of a Distance Education in a Mathematics Degree course, which involved the use of solids of revolution. In this paper, we present and analyze the teaching-learning process for the development of modeling in this experience, based on the communication established between student and teacher. In order to facilitate student's self-learning, the GeoGebra software was chosen as an aid tool in computational implementation. In the teaching proposal, the student was asked to identify a building that had the shape of a figure of revolution and the Cathedral of Brasilia, located in the city of Brasilia, Brazil, was chosen, after verifying the feasibility for completion of this model in the GeoGebra environment. The analysis of the dialogues between student and researcher shows that the interaction was based on Prado and Valente's approach of "being virtually together" and that the process of the mathematical modeling took place in gradual steps, helping to meet the mathematical and coding needs. We conclude that GeoGebra proved to be a powerful software program, which improves the visualization of the rotation motion of the desired solid generating curve and facilitated the necessary adjustments in the parameters of the algebraic expression of the curve. Thus, this experience shows that it is possible for the teacher to work the solids of revolution from a real situation, which mobilized the student for meaningful learning.

Keywords: *Teaching and learning, Distance Education, Mathematical Modeling, GeoGebra.*

1. Introduction

Mathematical Modeling is a course delivered as part of the formation of the Distance Education (DE) students of Brazilian Federal University, which proposes the elaboration and application of projects of mathematical modeling. The course is a challenge in terms of the teaching-learning process, in comparison to face-to-face courses, where the students need to build their knowledge more autonomously. This is not unique to this subject, but to most of the subjects that make up the Distance Learning Mathematics curriculum.

As the first class of the course and in a reasonable number of students to organize "almost individual" supervision of project design and implementation, the course was designed to explore student profiles to objectively model modeling as an experience, tailored to individual interests and motivations.

Thus, some didactic resources were incorporated during the course development that would facilitate students' self-learning. The GeoGebra software was chosen as an instrument because of its simplicity in coding and the friendly computing environment. Video lessons guiding students on how to develop code in GeoGebra supported this initiative to introduce technology into a virtual environment.

The students resisted the facilitating technology believing it would add a level of difficulty to the already challenging distance learning course. The way to approach this initial resistance was through a proposal by the course coordinator to visit the hubs to present the software in a friendly and dynamic way to the students. This initiative was crucial for the technology not only to be accepted but enthusiastically received by the students.

Among the projects proposed using the GeoGebra software was the construction of mathematical mosaics, adjustment and curve graphs using the spreadsheet and statistical resources offered by the software, as well as a differentiated design involving the use solids of revolution.

In this project, the object of analysis of this article, the student was asked to identify in her city of residence, a building that had the shape of a figure of revolution. Brasília, the student's city, is full of modern buildings that presented several models for the student to choose from. The Cathedral was chosen, after verifying the possibility of realization and completion of the model. GeoGebra software and its fill features through sliders and figure tracking, was the tool the student used to develop her modeling.

The project used mathematical modeling capabilities of GeoGebra to teach surfaces of revolution, which is part of the content of the Analytical Geometry discipline. Among the difficulties of teaching this subject in Brazilian Higher Education was the visualization of geometric figures and the association of each figure with its corresponding algebraic equation ([1],[2]). In addition, students are generally not motivated to study this topic because it is considered too "abstract", far from their reality ([3],[4]).

Mathematical modeling, as a teaching strategy, has been advocated and practiced by many authors ([5],[6]) and can be a way to arouse a student's interest in mathematical content, simultaneously with learning the art of modeling mathematically at all levels of education. From this perspective, a "mathematical model" is a set of mathematical symbols and relationships that represent the object studied in some way ([7], p. 20) and mathematical modeling is the process for obtaining this model and its interpretation.

In the same conception, for Galleguillos and Borba [8], mathematical modeling is understood as a pedagogical strategy that allows for a choice of themes for students to work on, which enables students to understand how mathematical contents approached in the classroom are related to everyday issues.

The creation and development of dynamic geometry programs, such as GeoGebra, support the educational work of mathematical modeling, especially the teaching of geometry, in order to make the mathematical problem solving stage less arduous. Moreover, these programs allow the modeling of moving figures, a process that allows, according to Gravina and Contiero [9], the development of skills characteristic of mathematical reasoning:

- observe,
- establish relationships,
- make conjectures,
- unfold a problem into small problems.

Particularly in distance education, it is necessary to think about educational strategies that allows the student to move beyond the mere accumulation of online information, that can involve them, and arouse the motivation for learning [10]. In this way, the student will be able to develop their own personal resources and organizational means to learn with more meaning and autonomy.

Given these considerations, the aim of this paper is to present and discuss this teaching experience in DE, which used mathematical modeling and GeoGebra to construct a representative model of the Cathedral of Brasilia.

This paper is organized into four sections: section 2 presents the basic concepts of the mathematical modeling process and the particularities of GeoGebra software, as theoretical guidelines for the teaching experience analysis, discussed in section 3; Section 4 explains in detail the development of the routine in GeoGebra and the final considerations are set out in Section 5.

2. DE, mathematical modeling and GeoGebra: possibilities

Computer science is becoming increasingly relevant in the educational scenario. Its use as a learning tool and its action in different environments has been increasing rapidly among us. Working with Information and Communication Technologies (ICT), either in the classroom or in e-learning, is a form of pedagogical support, as it is a tool that is already part of the current student scenario [11].

Driven by technological advances in the transmission of information and communication between people since the late twentieth century, DE has been widely disseminated in higher education without a single pedagogical model. Prado and Valente [12] suggest a classificatory scale of the pedagogies adopted in this teaching modality, according to the degree of interaction between those involved in an educational process. Among the approaches of this scale are the extreme, which would be broadcast and “being together virtual”, i.e., being together in classrooms at the interface of the physical and virtual and an intermediate approach, called virtualization of the traditional school.

The broadcast approach is characterized by the absence of teacher-student interaction and student interaction. It consists in the organization of information in a certain order, sent to the student by some technological means and without return of the student to the teacher.

In the intermediate approach, also called by the authors “virtual school”, we try to implement, from technological means, teacher-centered educational actions present in traditional teaching. In this case, there is some interaction between teacher and student, which most often boils down to checking whether the learner has memorized the information provided in an assessment.

At the other end of this classificatory scale is the "being together virtual" approach characterized by monitoring and advising on the process of knowledge construction mediated by technology [13]. In this proposal, the teacher is placed next to the apprentice, which allows the teacher to know who their student is and what they are doing, being able to propose challenges, experience situations with them and then help solve any problems.

According to Almeida [10], distance education is generally characterized by the student's time management, the development of autonomy in carrying out the proposed activities, the dialogue with peers to exchange information and the development of joint productions. For this author, being together virtually, also called computer-assisted learning (AAC), explores the interactive potential of information and communication technologies (ICT) provided by multidimensional communication, which brings emitters closer to the receivers of courses, allowing the creation of learning and collaboration conditions ([10], p. 4).

In the sense of this last approach, mathematical modeling is presented as a teaching strategy that allows a greater interaction between teacher and students and a more active attitude of students during the development of the educational process, as long as it is viewed by the teacher in this perspective.

Mathematical modeling, as a teaching-learning strategy, works around a mathematical theme or model, guiding the *syllabus development* and guiding the student to the realization of their own model. Biembengutt and Hein [14] suggest five steps for the implementation of this process: *diagnosis*, *choice of theme or mathematical model*, *development of program content*, *modeling orientation* and *process evaluation*.

The *diagnosis*, initially made by the teacher, should gather information about the students' socioeconomic reality, their availability of extra-class time and the degree of mathematical knowledge they have.

In *choosing the theme or mathematical model*, the teacher must consider a broad and interesting theme: enough to make it possible to develop a particular topic for the syllabus and to motivate students to work. There is also the possibility for students to choose the theme, which allows greater involvement of them in the process, although it may bring inadequacy, or even incompatible complexity to the content to be worked.

During the *syllabus development*, the teacher begins the work with the recognition of the problem situation and the necessary familiarization of the students with it, including preliminary research. Then, the problem is formulated in mathematical terms, from which the teacher develops the necessary *content.* Among other activities, similar examples are presented, so that the content is not restricted to the model under discussion. Then the question is reviewed and finally resolved. Finally, teacher and students interpret the model in the reality considered and verify its suitability, making modifications and adjustments, as necessary.

The authors suggest, for modeling guidance, that the teacher predicts moments of orientation interspersed with the stage of development of the syllabus, ending with an oral or written presentation of the work by each student (or group of students).

Finally, the last step is teacher evaluation of the process, which can be done by considering two main aspects, [14]:

- the student's degree of learning;
- the redirection of the teacher's work.

Of course, all the previous steps will provide pointers that can assist the teacher in student evaluation so that it is procedural rather than just at the end of the assignment.

Regarding the quality in the teaching-learning process, both in classroom and distance education, for Cyrino and Baldini [15], the insertion of technology can induce significant changes in mathematics teaching. It is necessary to propose that the training courses for mathematics teachers should also be based on practical and theoretical classes about didactic-pedagogical resources that can help in the teaching process, that is, an ICT-oriented mathematical education.

It is important to emphasize that, in addition, technology should not replace the educator at any time, but the educator becomes the mediator of learning. The teacher enjoys the technological resources as another pedagogical alternative, and source of motivation and stimulus for the work, diversifying and innovating the strategies to be adopted. It is also the role of institutions to provide and manage human and material resources for learning to take place. For this, a perfect interaction between the teacher and the resources is necessary, that is, it is necessary not only to manipulate properly, but, above all, to know how to use it in a pedagogically correct way, seeking methodologies that are more consistent with the reality of the institution and the institution’s student context.

In the same way, the National Curriculum Guidelines for the Mathematics, Bachelor Degree courses ([16], p. 6), state that “From the beginning of the course, graduating students should become familiar with the use of the computer as a working tool, encouraging its use for teaching mathematics, especially for problem formulation and problem solving.” Technologies aimed at teaching mathematics have evolved continuously and rapidly, whether with calculators, programs for virtual environments, or software, many of which are freely distributed, which make it possible to improve the quality of the teaching-learning process.

Particularly, software contributes greatly to the development of modeling projects, in which data are handled that connects to predict future situations, geometric figures are associated with existing buildings, or equations are formulated that relate observed variables, among many other situations.

In the experiment to be reported, the software chosen was GeoGebra, an interactive math program for learning and teaching math and science from elementary school to college level. Its creator Markus Hohenwarter started the project in 2001 as part of his master's thesis at the University of Salzburg, continuing its construction at the University of Florida Atlantic between 2006 and 2008, followed at Florida State University between 2008 and 2009. Hohenwarter's team is currently based at the University of Linz, Austria, and is made up of open source developers and translators located in virtually every country in the world, including contributions from Brazilian experts. GeoGebra source code is licensed under the GNU General Public License (GPL), and all other non-software components are under Creative Commons BY-NC-SA [17].

GeoGebra brings geometry, algebra, spreadsheets, graphs, statistics and calculations together in one easy-to-use package to the point that it has become the leading provider of dynamic math software, supporting education and innovations in science, technology, engineering and math. The program is available on various platforms with its desktop applications for Windows, MacOS and Linux, its tablet applications for Android, iPad and Windows, and its web application based on HTML5 technology. In 2013, Bernard Parisse's Giac was integrated into GeoGebra's CAS vision, which gives the possibility of use in more advanced problems [17].

The most outstanding point of the software is its dynamics, in which constructions can be made with geometric and / or analytical elements and all of them can be dynamically altered in interaction with the algebra window, where the analytical formulas change with each click. Elements can be inserted and modified directly via mouse, touch, or via the input bar.

Its key features are: 2D and 3D interactive geometry environment, embedded spreadsheet, embedded CAS, integrated statistics and calculation tools, enable scripting and a large number of interactive learning and teaching resources shared on the web page. GeoGebra applets can be uploaded directly to the platform which is an official GeoGebra cloud service and resource repository.

For Amado et al. [18], there are numerous benefits that this technological resource can offer: the bi and three-dimensional interpretation in geometry (figure construction), the possibility of formulating conjectures, relating geometric properties, the opportunity to perform trigonometry research tasks since the construction , the visualization and creation of mathematically valid arguments, among others.

Research on the teaching of Analytical Geometry using GeoGebra indicates that visualization and manipulation of figures were facilitated with this type of software. They support ways of thinking that go beyond oral or written speech and static design, enabling students to interact with dynamic systems of representation, externalize and internalize new thoughts, in a continuous process of action / reaction between subject and tool [19].

All of the considerations in this section have contributed to the mathematical modeling experience in DE, pedagogically grounded in “being together virtually”, choosing easy-to-manipulate software, and a concern for the quality of the teaching-learning process. Such an experience will be reported in the next section.

3. The Teaching Experience in Distance Education

As part of a course entitled “Mathematical Modeling”, in a distance learning initial course for mathematics teachers, the teacher - one of the authors of this paper - proposed that students work on a project to model some real problem. The experience reported here refers to the project developed by one of the students, resident of the city of Brasilia (DF), whose chosen theme was the mathematical modeling through GeoGebra, of the metropolitan cathedral of Nossa Senhora Aparecida, known as the cathedral of Brasilia, which was designed by Brazilian architect Oscar Niemeyer, famous for many buildings around Brasilia during its construction.

Figure 1. Brasilia's Cathedral. Picture source: http://www.copa2014.gov.br/pt-br/galeria/brasiliaturismooutubro2013

Based on the five steps for implementing a mathematical modeling process as a teaching strategy by Biembengutt and Hein [14], we will analyze this experience.

The first step, *diagnosis*, was made instinctively by the teacher, which was based on the knowledge acquired about the class in a previous course discipline and a visit to the pole, in which she held a GeoGebra workshop and could have a closer contact with this student.

The instructions given to the students initially requested a work development plan and were released by the course tutor, as we can see in the following message in Moodle:

> *Re: Doubts- Activity 4*
>
> *by* [redacted] *- Thursday, May 14 2015, 21:56*
>
> *Hello!* [redacted]*,*
>
> *You will choose a theme that can be applied either in Primary or Higher Education and will write a development plan with the details of the application in teaching.*
>
> *In the project you can use one of the resources used during the Modeling discipline.*
>
> *The handout provided some examples for you to reflect on. If you use a theme in this material the methodology has to be totally different.*
>
> *The plan must contain: Introduction, Theoretical Rationale, Methodology, Conclusion and References.*
>
> *Have fun with your tasks!*
>
> *Sincerely*
>
> [redacted]

In these initial instructions, the student choose a first subject, as shown in the following message, starting a negotiation with the teacher about the choice of the modeling theme, which characterizes the second step of this process.

Re: Doubts- Activity 4

by [redacted] - Sunday, May 17 2015, 14:04

Good afternoon, thank you [redacted], the theme I have chosen is modeling the "blood flow".

This idea was rejected by the teacher for not being original, since it was among the examples suggested by the teacher, without modifications in the methodology that was used in the example. From there, the student needed several conversations with the teacher to decide on the theme of the cathedral of Brasilia, with approach made through GeoGebra. It is evident the benefits brought by the visit of the teacher to the pole, since she could arouse in this student an interest in the use of GeoGebra, besides being aware of her city of residence, facts that favored the negotiation by choosing the theme.

Thus, starting the implementation of the third step, the student did bibliographical research, with the objective of learning about the theme, culminating with a characterization of the structure of the Cathedral of Brasilia: history, design and constructive technology.

As the teacher conducted individual modeling projects, according to each student's proposal, the *syllabus development* was made by each one, with her guidance. Thus, the student from Brasilia developed, at the same time, a study on solids of revolution and their coding in GeoGebra. At this point the student discovered that the two most important elements of the construction are the generating curve and the axis of rotation. With these elements, teacher and student began introducing the first codes in GeoGebra for creating curves in the 3D environment and later for rotating these curves around an axis, as shown in the following email dialogs:

From: [redacted]

To: [redacted]

Subject: RE: Modeling project

Date: Thu, 28 May 2015 14:42:30 -0300

Very well done [redacted]! There you are the student getting to work!

Ok, there was a coding error in the first two.

The first code does not make sense because they must be concrete formulas of x (t), y (t) and z (t) and still with the word "Curve" and also with the variable variation, in this case t. I think that was just a theoretical part.

The second correct code is:

Curve [cos (t), 0, sen (t), t, -pi, pi]

Missing variable name (t in this case) from curve on fourth entry.

Try again to enter this code and tell me you saw or send a png file.

I need the answers to some questions I submitted earlier.

Sincerely

[redacted]

From: [redacted] *[mailto:*[redacted]*]*

Sent:Thursday, May 28, 2015 1:31 PM

To:[redacted]

Subject: Modeling project

Good afternoon, I would like you to take a look at what went wrong with the turn commands you sent me.

When I put the first command where you say that the point of the trajectory appears that and right after pressing create slider a message on the screen says that the command is invalid.

At the second point where the first curve is created, it worked. But when I put the second curve that message appeared on the screen.

And after launching the last curve in the bar code the image obtained was following even with the errors mentioned above.

NOTE: The attached images are in the order I described.

From: [redacted] *[mailto:*[redacted]*]*

Sent:Friday, May 29, 2015 7:12 AM

To:[redacted]

Subject: RE: Modeling project

Very well congratulations [redacted]*!!*

Ok, open a new window and insert the following code:

Curve[sec(t), 0, tg(t) + 2, t, 0, 2pi]

And here is another task: click on "rotate around a line" (as shown in the attached figure) and click on the curve (better click on the equation in the Algebra Window) and immediately on the z axis. ONLY CLICK ONCE EVERYTHING! A window will appear asking for the rotation angle. You accept the value that is already entered by clicking ok.

Tell me if you did it, what you saw, and the difficulties in accomplishing the task. The way you did very well with the previous steps.

Looking forward for your answer!

Sincerely

[redacted]

From: [redacted] *[mailto:*[redacted]*]*

Sent:Thursday, June 11, 2015 7:26 PM

To: [redacted]

Subject: RE: Modeling project

Good night dear teacher,

I entered the value in the code box and in this case the curve d was formed, as shown in the example I can't find "rotate around the screen". Then I clicked on the "rotate preview screen" option and then the z axis equation. follows attached model

Note: the images are in the order of explanation.

From: [redacted]

To: [redacted]

Subject: RE: Modeling project

Date: Fri, 12 Jun 2015 12:14:40 -0300

Oi [redacted]*!*

It is not "rotate around the screen" but "rotate around a line"

Start a new ggb file and enter the curve code:

Curve [sec (t), 0, tg (t) + 2, t, 0.2pi].

Next, CLICK THE 3D VIEW WINDOW, and then look for the spin button around a line that I described in my explanation.

Looking forward for your answer!

Sincerely

[redacted]

Once the project is discussed and approved, the fourth step of the modeling process, which is the orientation of modeling, is intensified. This was based on the decision to use another communication tool that would allow a more dynamic and precise orientation between student and teacher via the SKYPE feature. The most used feature of this tool was the screen sharing, which facilitated the development of the project, mainly due to the need for visualization of figures generated in GeoGebra, both by the teacher and the student, and adjustments in the codes from what they were producing.

Thus, it is noted that there was an evolution in this *orientation process* that became more interactive and agile. The communication established between student and teacher, as can be seen from previous dialogues, and the results obtained during this modeling process, more detailed in the following section, reveal that the teaching-learning process conducted by this teacher was guided by the approach of the teacher. “Being together virtually”, as discussed in Valente [13].

Finally, in the fifth step of the mathematical modeling process used in teaching, the assessment is made, both from the point of view of student learning and from the point of view of project work carried out for the subject. As a conclusion of the project, there is the opinion of the student herself, taken from her written work, delivered at the end of the course:

> It is concluded that when it comes to technology there are no borders, every day we know and create something new, so this challenge in the teacher's life, be inside this new technology that gains space every minute. GeoGebra came to add and enriching the life of the teacher as well as the student makes a monotonous class into something wonderfully differentiated and attractive, leading the student to create through mathematics.

The teacher, in turn, evaluated the student's learning process as positive: her gradual evolution with mathematical theory and Geogebra, especially her satisfaction and surprise to see the model she built herself. From the point of view of modeling work in teaching, she realized that her insistence and support to the student throughout the process were fundamental to the successful implementation of the model.

4. Mathematical Model with GeoGebra Aid

The mathematical modeling of Brasilia's cathedral followed the process of “being together virtually” in some gradual steps that were planned to meet the mathematical and coding needs in GeoGebra. These steps are listed below.

Step One: The student was introduced to the concept of solids of revolution, in which a bibliography was suggested for the student to understand what are the basic elements of these geometric figures. The student should conclude that the axis of rotation and the generating curve are the two basic building blocks of a solid of revolution. This was done quite quickly and the teacher tried not to go into mathematical details that would distract the student from the main goal.

Second step: the study of parametric plane curves was studied, a topic not studied in Euclidean geometry from this point of view. As soon as the student showed sufficient knowledge, submitting tasks in the Moodle environment questions forum and through e-mail exchanges with the course teacher, the classical curves of analytical geometry were proposed: circumferences, parables, ellipses and hyperboles. The student, still not understanding the purpose of this study, describes in her project the codes used to trace such curves:

> [...] I know that a curve is the trajectory of a point (x (t), y (t), z (t)), when a parameter t varies in a segment. So I put in the GeoGebra code box the following curves in space:
>
> 1) Curve [t, 0, t2, t, -10,10], which is a parable in the xz plane
> 2) Curve [cos (t), 0, sin (t), t, -pi, pi] which is a circumference in the xz plane
> 3) Curve [1 / t, 0, t, t, 0.2,3] a hyperbole in the xz plane

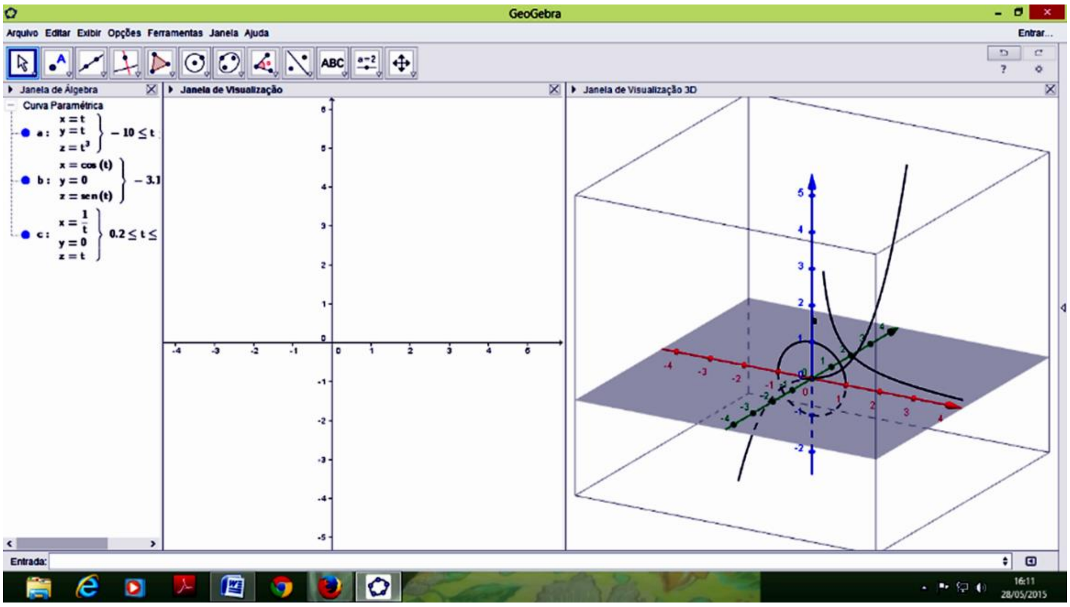

Figure 2. Curves 1), 2) and 3) in GeoGebra environment

Although, following the study, the teacher knew that only one generating curve would be used to construct the model of the Cathedral of Brasilia, at this stage of the work the orientation leads the student to experiment with various curves in GeoGebra. This proposal to develop analogous examples, besides exercise resolution, is suggested by Biembengut and Hein [14], as a way of not restricting the content to the studied model. In this way, the student learned about the parameterizations and visualized the tracing of all conical curves, not just what would be required for modeling in focus.

Third step: The objective of this stage was to rotate around the chosen axis, in this case the z axis, of the generating curve. This step was more difficult for the student, even though she was emailed some code ready to implement. Avoiding frustrations on the part of the student, the teacher suggested other alternatives that the student could understand and apply. The most understandable way for the student was to create a slider to understand the curve's movement. This proposal was understood and developed after using another important technological resource: SKYPE, as described above. The student can finally see the first sketch of her project when she visualized on her own computer the movement of the generating curve.

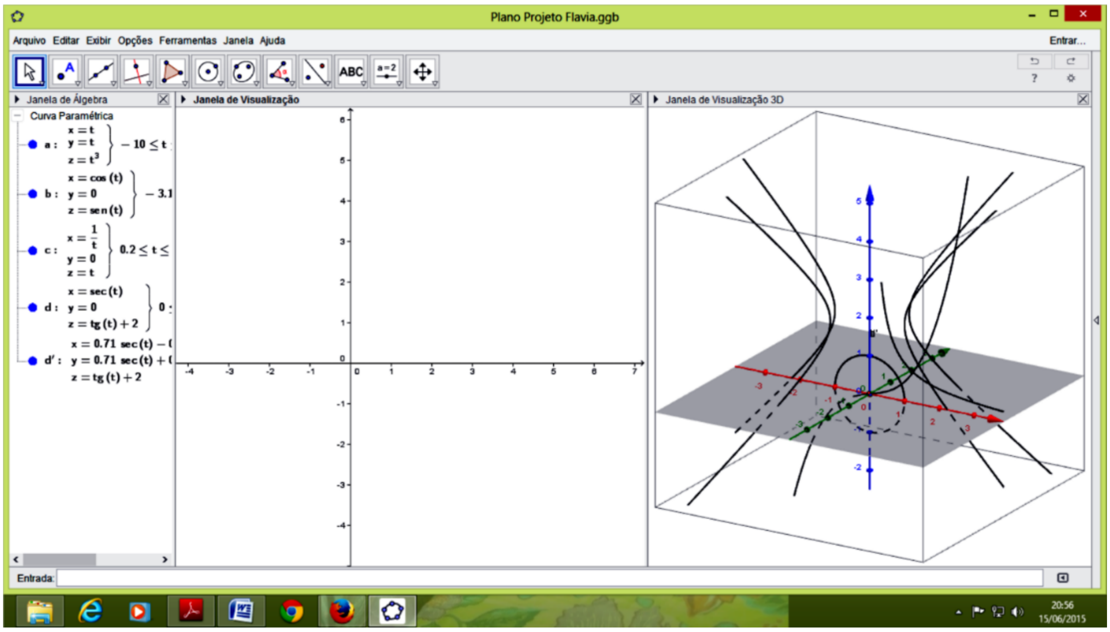

Figure 3. Hyperbole 3) and its rotation of a fixed angle around Oz

Fourth step: The student was asked to decide what would be the best curve to be rotated around the z axis in order to obtain "the windows" of the cathedral. This stage presented the greatest difficulties for the student, as it required several attempts compared to the already large number of trials and errors from the previous steps. The teacher's initial idea was to use curve 3), as shown in Figure 2, from the second research stage. But the student herself came up with the idea that hyperbole would be the generating curve of the solid of revolution that represents the cathedral's glass. This is described by the student as follows:

> In the input bar I typed: Curve [sec (t), 0, tg (t) + 2, t, 0.2pi]. I clicked to rotate around a line on the toolbar - reflection about a line. And then I clicked on the equation that is in the algebra window and immediately on the z axis, where a window appeared asking for the angle of rotation and I agreed with what was 45 °, and the image was as in Figure 2.

After some attempts to find the shape that best fits the original model, the equation curve was chosen.

$$(1.1 \sec(t), 0, 1.3 \operatorname{tg}(t) + 4), \text{ with } t \text{ between } 0 \text{ and } 2\pi. \qquad (1)$$

Step five: The student, already more confident in using the software and its coding, went to study the sequence commands and the syntax for rotation about an axis. This step took place in SKYPE and its coding was done on time and without frustrations by the student. The only question left to decide was how to distinguish between the "glass" and the white concrete columns that characterize the architecture of the Cathedral of Brasilia. The teacher's suggestion was to use the equation curve (1) for both the glass and the concrete columns, coloring in the tones closest to reality, blue and white respectively. At the same time, she suggested using different values for gyration, determined by a new parameter, as described by the student in the following report: After making the curve adjustments for the next step. We set the slider n to values from 0 to 100 and at the command bar I typed the following command in the input: String [Rotate [d, i * π

/ n, Z-axis], i, 0, n], where d is the curve generator chosen for the cathedral glasses. The result was surprising when I asked for the d-curve trail and animated the slider. We did it, with the same d-curve, but we call it the spinning leaping degrees of spin to create the white cathedral supports. The result is shown in Figure 4.

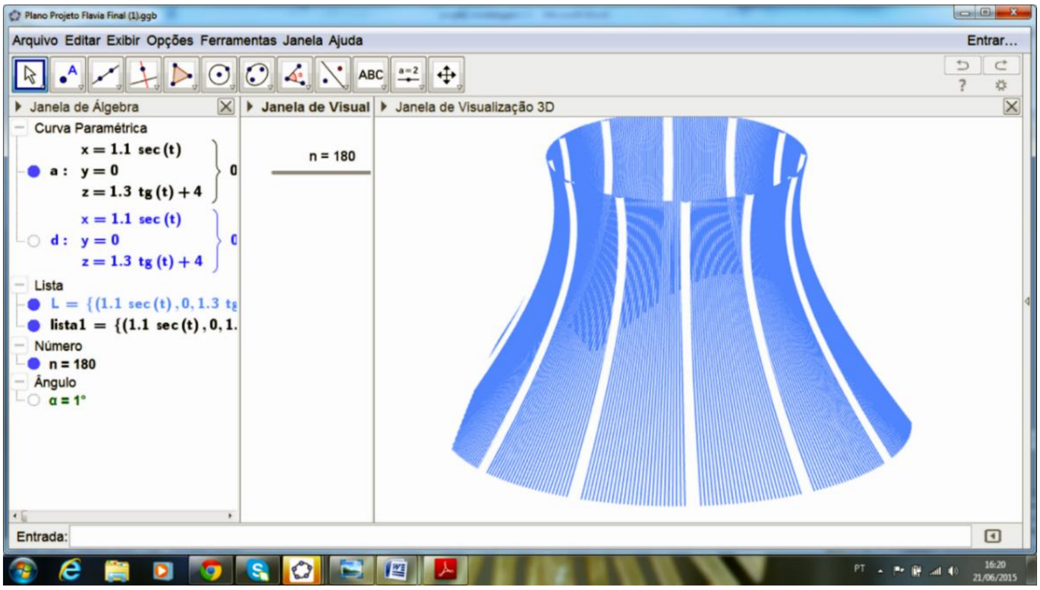

Figure 4. First Brasilia's Cathedral model obtained by the student in GeoGebra.

To finish the figure, in order to obtain the most accurate sketch possible with the model to be reproduced, the challenge was to extend the parameter t of the white curve (1). This was finally resolved by the teacher, as it would take a long time for the student to realize this part of the modeling, inducing the student to use a different t parameter value than determined for the glasses. Figure 5 presents the result of this last modification made in GeoGebra.

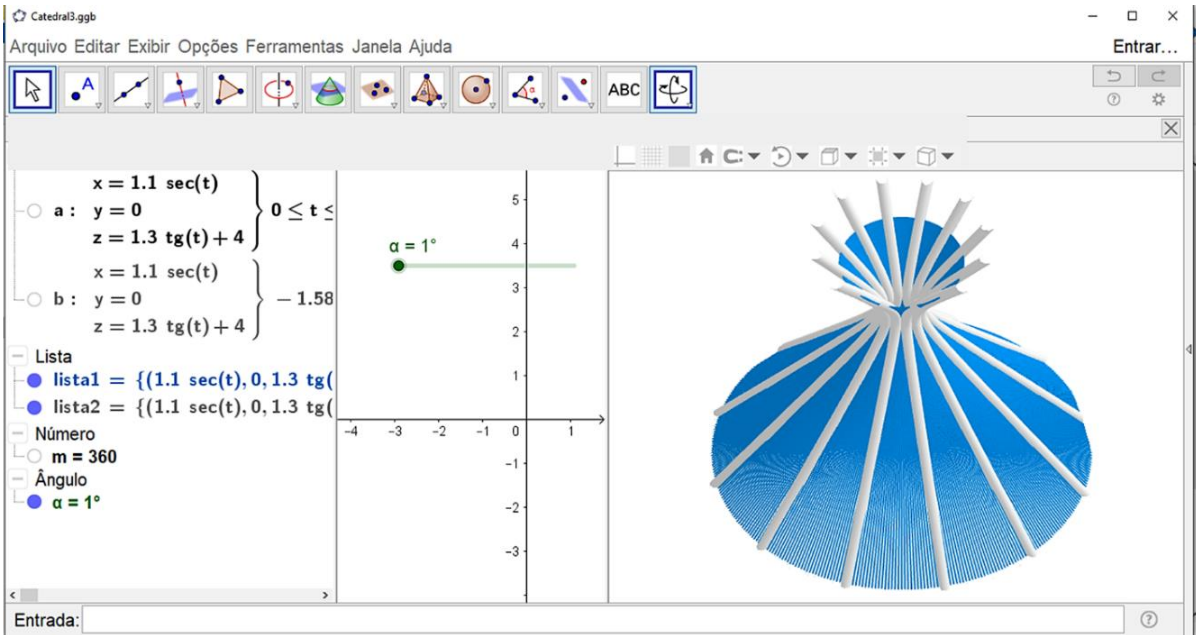

Figure 5. Last version of the model with the codification on the left

Finally, the model in GeoGebra took on a closer look to the actual building, as you can see in Figure 6, comparing it with a real photo of the cathedral.

Figure 6. Result of the modeling on the left and the Brasilia's Cathedral picture that inspired the mathematical modeling.

4. Conclusions

The teaching experience in the Mathematical Modeling course of the Distance Education Mathematics course, presented and discussed in this text, showed that it is possible for the teacher to work the intended mathematical theory from a real situation. Thus, learning mathematical concepts becomes more motivating for the student. As Bassanezi [5], said, based on his experiences as a teacher and teacher trainer, application-oriented pedagogical processes, as opposed to formalistic procedures, may lead the student to better understand mathematical arguments, incorporate concepts and results more meaningfully and create a predisposition for learn math because it has somehow come to understand and value it.

With ICT support and teacher guidance, students can test and execute mathematical models, improving their knowledge. In the specific case analyzed in this text, GeoGebra proved to be a powerful software, which favored the visualization of the rotation motion of the desired solid generating curve and facilitated the necessary adjustments in the parameters of the algebraic expression of the curve. We have found that programs such as GeoGebra can make concepts a little more understandable and a little less intimidating, making reading more enjoyable, since mathematical texts can be abstract and technical. Another key technology in this experience was Skype, which allowed for more accurate and faster student-teacher communication.

Moreover, as it is an initial training course for mathematics teachers, experiences such as this may favor the future professional practice of these students, as it is possible to adapt the mathematical modeling strategy to the teaching of elementary content [20]. There are drawbacks, as pointed out by Biembengut and Hein [6], that teachers need to consider in order to make the necessary adjustments in their school context: unpredictability of the mathematical tools to be used for a given theme or model initially chosen;

difficulties in adapting to the legally established curriculum; difficulties in the simultaneous monitoring by the teacher of the themes chosen by the students.

Given all these considerations, it ends with what is an essential condition for a good work of mathematical modeling as a teaching-learning strategy: the role of the teacher as the student's advisor. Putting themself “virtually” with the student, with this approach the teacher follows their development, provokes reflections from their mistakes and stimulates an active production posture of the student.

5. References

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