A Proposal for Computer use in Mathematics Classes: Algebra Teaching in College

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Abstract

The present work is based on mathematical teaching, especially in the teaching of Algebra in math courses of the college. The theme mentioned is from the Algebraic Structure Group, there are researchers in Brazil who highlight the students' difficulty in understanding the topic. From this, students find it difficult to associate Algebra teaching with Dynamic Geometry software, in particular "GeoGebra". It is proposed a didactic session planning, in which the teacher takes hold of the "GeoGebra" for teaching the Group Algebraic Structure. To accomplish this teaching proposal, the Sequence Fedathi is used. The use of "GeoGebra" in Algebra classes is suggested. With the Fedathi Sequence is possible to use the software on the geometric visualization of Algebraic Structures.

Keywords: *Algebraic Structure Group; Fedathi Sequence; GeoGebra; Math teaching.*

1. Introduction

From 2010 there was an acceptance of technologies in the classroom. Private or public educational institutions use digital resources to complement the teaching process. Could be computer, notebook, tablet and smartphone.

With these devices, the teacher can use many software to help him. If the teacher is trained at a distance, you can use *Moodle*, *Solar* or *TeleMeios*. In classroom teaching, *GeoGebra*, *WolframAlpha*, *CabriGéomètre* or PDF files.

This work is delimited in the Mathematics Education, at undergraduate level, with a focus on bachelor and bachelor degree, because the theme is recurrent in these courses. We use the help of *GeoGebra* software to carry out this teaching approach.

The authors Franco and Soares (2013) talk about the difficulty of using visualization resources for teaching Algebra. In fact, the concept of the group algebraic structure does not have geometric visualization, but it is possible to give examples that collaborate in this aspect.

Elias and Saviolli (2013) say that the difficulty in teaching algebra happens because it is a content that requires students to know interwoven sets, functions and axioms. And when they are questioned, the absence of answers is commonplace.

Franco and Soares (2013) state that content demands from the student a high capacity for abstraction, this can be a difficulty in teaching the content or in understanding the concepts by students.

The authors Nogueira and Pavanello (2008) report that as a way of circumventing this high abstraction, it is possible to teach algebra through memorization, which also leads to the difficulty of understanding the concepts by students.

Alves and Araujo (2013) bring Maple software to assist in the teaching of symmetrical groups, but they do not use a teaching methodology to plan a didactic class with the theme.

Given this, this work is based on the teaching of Algebra with the help of *GeoGebra*, with the help of the Fedathi Sequence. With this, we seek to present the teaching of Algebra linked to a software of Dynamic Geometry, as well as with the support of a teaching methodology.

For this work, we have the question: Is it possible to connect the Dynamic Geometry software, especially *GeoGebra*, in the teaching of Algebra, in particular the Group Algebraic Structure?

We propose a didactic session plan for the teaching of Algebra using the Dynamic Geometry *software GeoGebra* with complementary character under the support of Fedathi Sequence.

The Fedathi Sequence is a teaching methodology that suggests a teacher's stance in the classroom, and as a consequence of this stance, the student takes on an investigative role that mimics a mathematician's steps in problem solving. This methodology has steps that are performed in the didactic session and fundamentals that the teacher must use to achieve the objectives of the class.

The writing of the work is developed in the Group Algebraic Structure, the mathematical content that delimits the study, plan of a didactic session where the Fedathi Sequence is presented and shows how to use it in the teaching of the Group linked to *GeoGebra* and in the Final Considerations we show the existing limitations and suggestions for future research.

2. Algebraic Structure Group: definition and teaching

The Group Algebraic Structure is defined in Gonçalves (2013, p. 119) as follows:

> Let G be a nonempty set where an operation between pairs of G is defined, denoted by, (*: G x G → G) | ((x, y) → x * y). We say that the pair G, * is a group that the following properties are valid:
> G1) a * (b * c) = (a * b) * c a, b, c, G
> G2) ∃e ∈G such that a * e = e * a, ∀ a ∈G
> G3) ∀a∈G, ∃b∈G such that a * b = b * a = e.

Property G1 is the associativity, where we can associate the elements of the set as desired and the result of the operation will be equal. The G2 property is called the neutral element of the operation, that is, any element operated with the neutral element will result in the chosen element. And the property G3 is based on the inverse of an element by the given operation, that is, every element of the set has an inverse element, which when operated results in the neutral element. When a set with a given binary operation has these three axioms, we call the set with the group operation.

The definition of this mathematical object is abstract, with no geometric visualization in its definition. Franco and Soares (2013) found in their research that students have difficulties understanding algebraic concepts due to the geometric limitation characteristic of the mathematical object.

But if it is difficult to see this mathematical content, why not remove it from the Higher Education curriculum? Franco and Soares (2013, p. 161) state: “[...] the study of algebraic structures can be seen as a basic foundation for the exercise of mathematics teaching, regardless of the segment that the future teacher may act in.” In fact, the mathematics teacher in the final grades of elementary school will come across the process of teaching the game of signs, regarding the multiplication and division between integers, which is later seen in the set of real numbers. In fact, for example, the number -2 times -2 results in 4, and the product between -2 and 2 is -4, and this is based on the Group Algebraic Structure, which can be transposed to the Ring Algebraic Structure. So the subject in higher education is necessary for the teacher to understand why such multiplication between numbers is negative or positive.

Kluth (2007, p. 109) says: “[...] Algebra structures become a theme of Education and Mathematics Education, because they are interconnected with the teaching and learning objectives that have as support material the mathematical content. ”It is observed in the course of Mathematics, disciplines such as Differential and Integral Calculus, Linear Algebra, Real Analysis, Complex Variable Calculation and Numerical Calculus, for example, that use the Algebraic Structures, namely Groups, Rings, Bodies or Domain of Integrity.

Regarding teaching, Nogueira and Pavanello (2009) state that “[...] in mathematics classes, the predominance of activities that emphasize the memorization of definitions, formulas and rules, intensive training in algorithmic procedures, to the detriment of knowledge building [...]”. That is, the teaching is characterized by "mechanized", where the student only replicates the visa in class, whether in studies or in the resolution of activities. The incorporation of creativity and a conception of valuing error in favor of learning are set aside.

Thus, there is the Fedathi Sequence methodology, which minimizes this teaching through the mechanism and provides an environment for students to use their previous knowledge to find a solution to the question that is proposed. However, this activity should be widespread so that the student does not see only particular cases, which makes teaching confusing and extensive. The Fedathi Sequence provides this creative environment for the student, becomes a research environment, and Sousa (2015, p. 15) points out: "[...] it is a challenge to make the classroom a research environment that takes students' hypotheses and strategies, followed by verification of the results found." The teacher who uses the Fedathi Sequence in the classroom has to consider the strategies students use to solve the activity, and these strategies are generalized and synthesized with the formal mathematical language, by the teacher, and therefore the importance of the activity being generalizable.

According to Sousa (2015, p. 17), "[...] the essence of the Fedathi Sequence is the teacher's posture as a mediator in the classroom." There is a change in the teacher's posture in making the classroom an environment of research, where all answers are valid, whether right or wrong, but the teacher must mediate these errors to become part of the learning process. This change proposed by the Fedathi Sequence is also based on its foundations, where they are positions in which the teacher must assume in the classroom, using Fedathi Sequence as a teaching methodology (BORGES NETO, 2018).

3. Plan for a Didactic Session

The Fedathi Sequence is a methodological proposal that has the characteristic of transposing the scientific method for teaching (BORGES NETO, 2016). It is characterized by having four steps which are called Position Making, Maturation, Solution and Proof, and your fundamentals called Hand in Pocket Posture, Adidatic Situation, Mediation, Didactic Agreement, Counterexample, The Question and the Conception of Error (BORGES NETO, 2018).

Before entering the stages of Fedathi Sequence, the teacher must use the Didactic Agreement foundation, it says the rules that will be part of the didactic session, and the *plateau*, where are reminded the previous knowledge that students must have to continue the activity proposed in the Position taking. It is suggested that the teacher make, in the Didactic Agreement, the call for the use of *GeoGebra*, on the computer or cell phone, without giving up paper and pen (SANTANA, 2002). With the *plateau*, it is feasible for the teacher to recall the pertinent concepts in Sets, Functions and Equivalence Relationships, which are prior knowledge to construct the Group concept.

The first step in the Fedhati Sequence is Position Taking, in which the teacher presents an activity in which it may be a game, a book issue, a list of exercises, a problem situation, or a generalized exercise. that is, the algorithm, the idea, to solve the activity serves to other problems of the same general character. For example, the teacher may ask the student to solve the quadratic equation $x^2 - 5x + 6 = 0$ without the aid of Bhaskara's formula, and the way he solves it may apply to any quadratic equation resolution. This is different if the activity is the resolution of the quadratic equation $x^2 - 9 = 0$, because the way to solve it is not characterized in all quadratic equations, which means that it is not a generalizable activity.

The Algebra Position Taking is presented, which is to verify that G, a set with the elements $f(x) = 2x + 4$,

$g(x) = 3x + 2$ and $h(x) = x + 1$, with the operation $\circ$, which is the composition of functions, is group.

In Maturation, after the Position Taking, the teacher stimulates the student's autonomy, in which she seeks the solution to the activity. If the student has any obstacles and requests the teacher's assistance, some Sequence Fedathi fundamentals should be used to address the student's doubt. As soon as the student asks a question, the teacher should not immediately respond or resolve it by the student, but instead use the question foundation to prompt the student to reflect on the subject. As the teacher will not answer the student how to do it, this is characterized by the Hand in Pocket Posture. If the student reflects after the teacher inquires and contains mathematical errors in the student's reasoning, the teacher must have the Conception of Error as something that can leverage learning, not just point out the error, and so we have to use another Fedathi Sequence plea. If the student still has errors, the teacher may suggest a counterexample for the student to realize that his idea, or reasoning, has misconceptions, and thus uses another foundation of Fedathi Sequence.

Still in Maturation, the student will look for ways to, with the elements of the set G, with the function composition operation, verify the three group properties, which are: associative, neutral element and inverse in relation to the function composition operation. As possible errors, the student uses the multiplication operation instead of function composition, both in paper and in *GeoGebra*, and generates the wrong result in relation to function composition, although the solution characteristic is approximate to the one proposed here. The student may also perform the composition of functions erroneously, and thus not proceed in *GeoGebra*, or paper, the verification that $(G, \circ)$ is group. As well as the student may make mistakes with the software for lack of experience in its use.

An important motto to consider is the use of technology only if what is done is better than without it, the use should be restricted to situations where knowledge could be better aggregated. Concerning the development of logical reasoning, Borges Neto and Capelo Borges (2007) point out that software helps the development of cognitive skills that need special attention by teachers so that they can identify and use them with their students in the classroom. it calls: random, trial and error, trial and error, and finally deduction.

When the student gives a random answer, he or she has not processed any reasoning to come up with an answer, disregarding any clues or clues that foster choice. Trial and error is very common among students who randomly test variables in order to find satisfactory possibilities, not necessarily having to formulate hypotheses. The trial and error is a controlled experiment with the testing of a predetermined hypothesis with the studied intention of finding an expected result. Deduction is the experimental process performed after the inference or analysis of attempts made from tests on other events or even this one.

The use of software as a didactic-pedagogical activity in the study group to solve problems related to the content of Related Fees has the purpose to analyze if the students were stimulated to develop answers directed to the cognitive skills of the essay and error and deduction, therefore, if the teacher does not have controlled mediation on the path taken by the student, it may focus on their solutions to chance and trial and error, thus not having generated a reflection on the experience of maturation.

Such mediation in the course of the meetings is not a fixed model, but the experience of a teacher who planned the didactic session according to the Sequence Fedathi teaching methodology. Students already had some degree of intimacy with the software and this helped to conduct the session. All the behavior of the teacher was guided by the principles of Sequence Fedathi, thus guiding the student work.

The third step provided by Sequence Fedathi is the Solution. Here the student presents to his classmates and the teacher how he solved the activity. If the student has an error, the teacher uses the fundamentals of Conception of Error, Posture Hand in Pocket, Question and Counterexample, so the student can reach the correct solution for the activity.

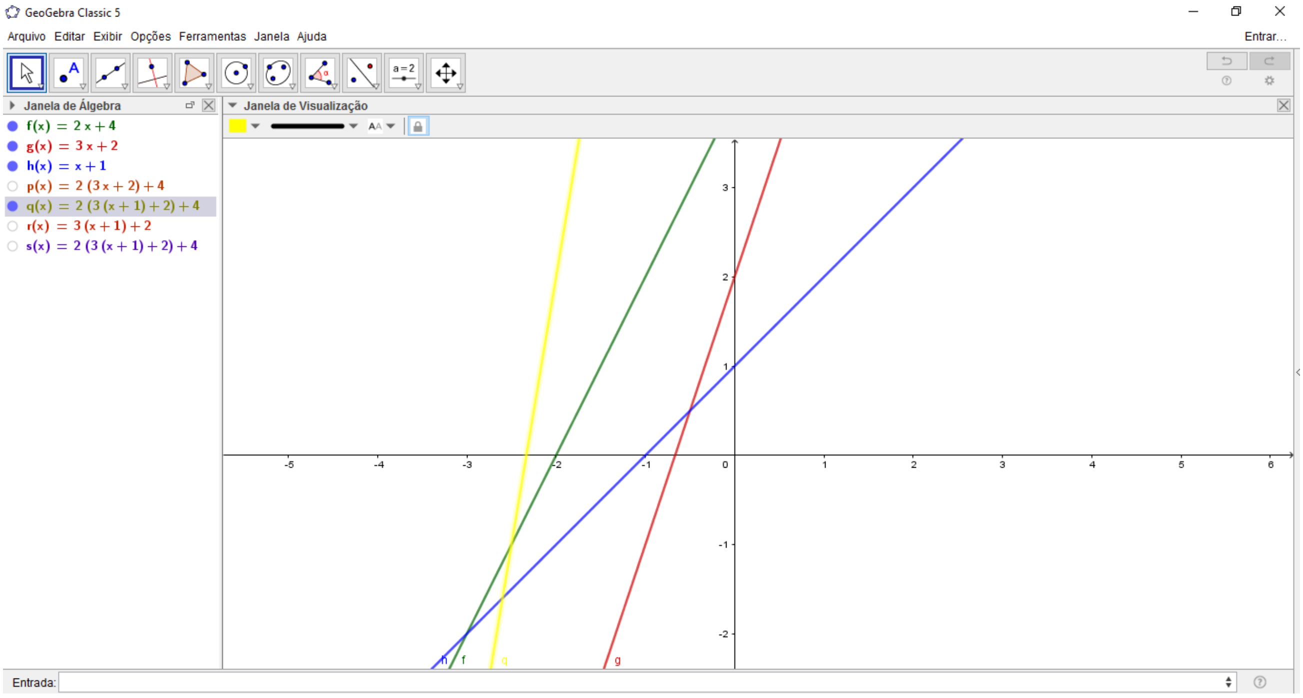
The solution the student may provide in *GeoGebra* is shown in figure 01.

Figure 01 - Composition of Functions (f ∘ g) ∘ h

Source: Prepared by the authors.

The green line is configured by the f function, the red line by the g function and the blue line by the h function. The purple line is characterized by the composition of functions (f∘ g) ∘ h. Figure 02 shows the composition of functions f ∘ (g ∘ h).

Figure 02 - Composition of Functions f ∘ (g∘ h)

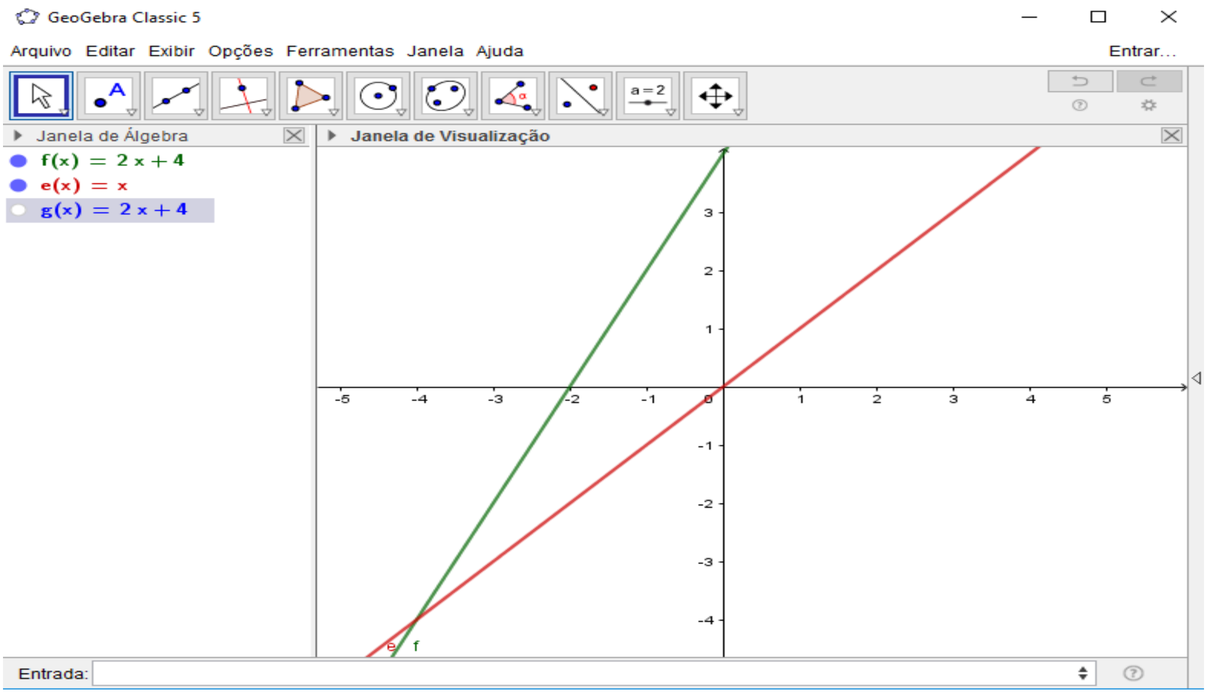


Source: Prepared by the authors.

The yellow and purple lines have the same geometric place, which characterizes that the associative holds true for this set with the function composition operation.

Note, in Figure 03, the verification of property G2 where the neutral element between the composition of functions is the function e (x) = x.

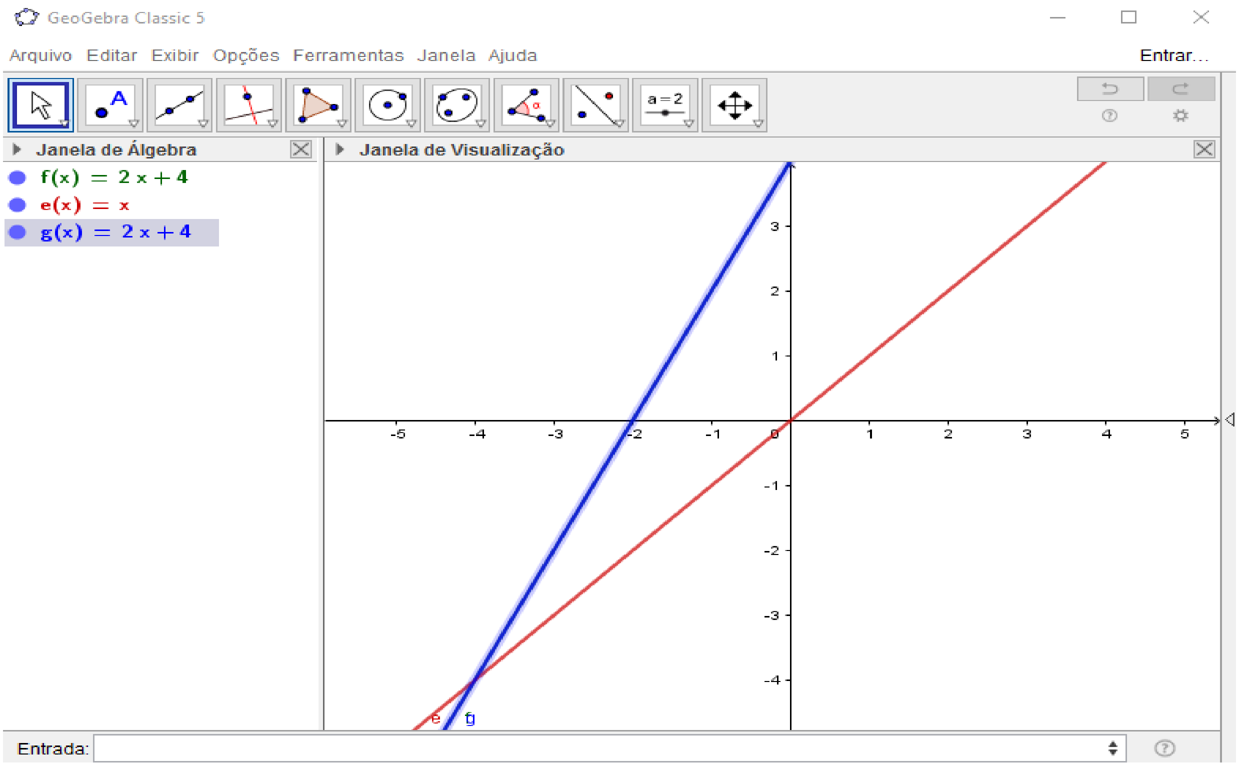
Figure 03 - Visualization of functions f(x) e e(x).



Source: Prepared by the authors.

When composing f (e (x)) in *GeoGebra*, figure 4 is compiled.

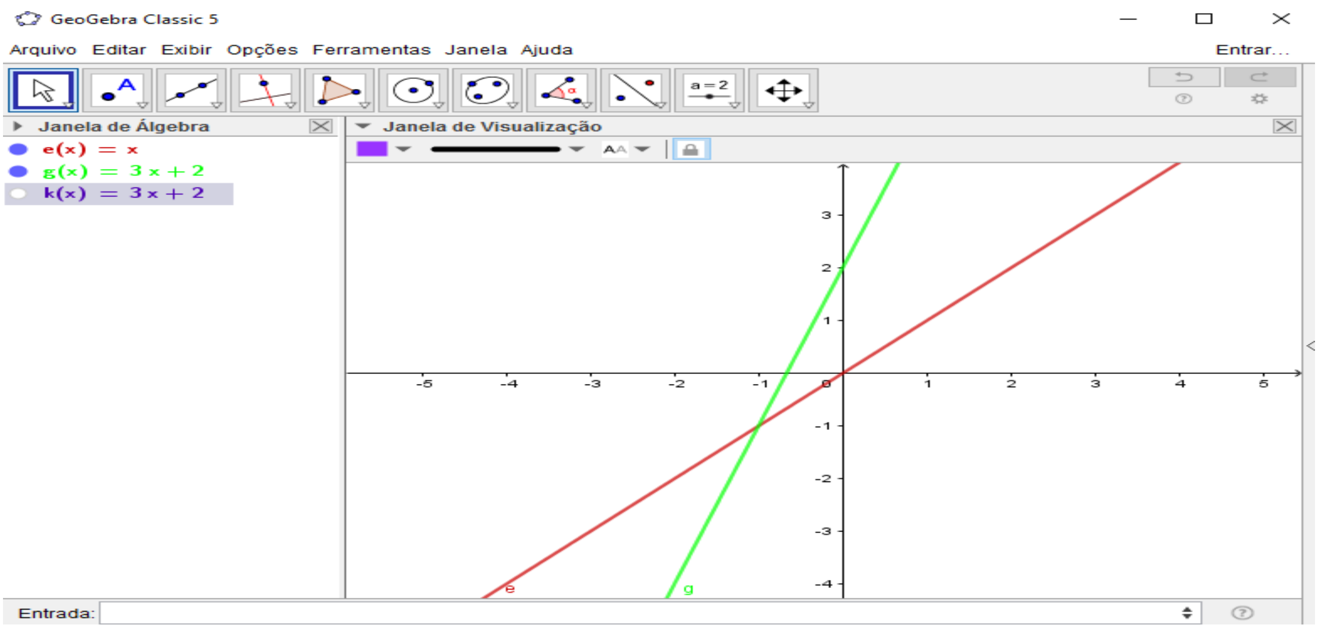
Figure 04 – composition of functions between f (x) and (x).



Source: Prepared by the authors.

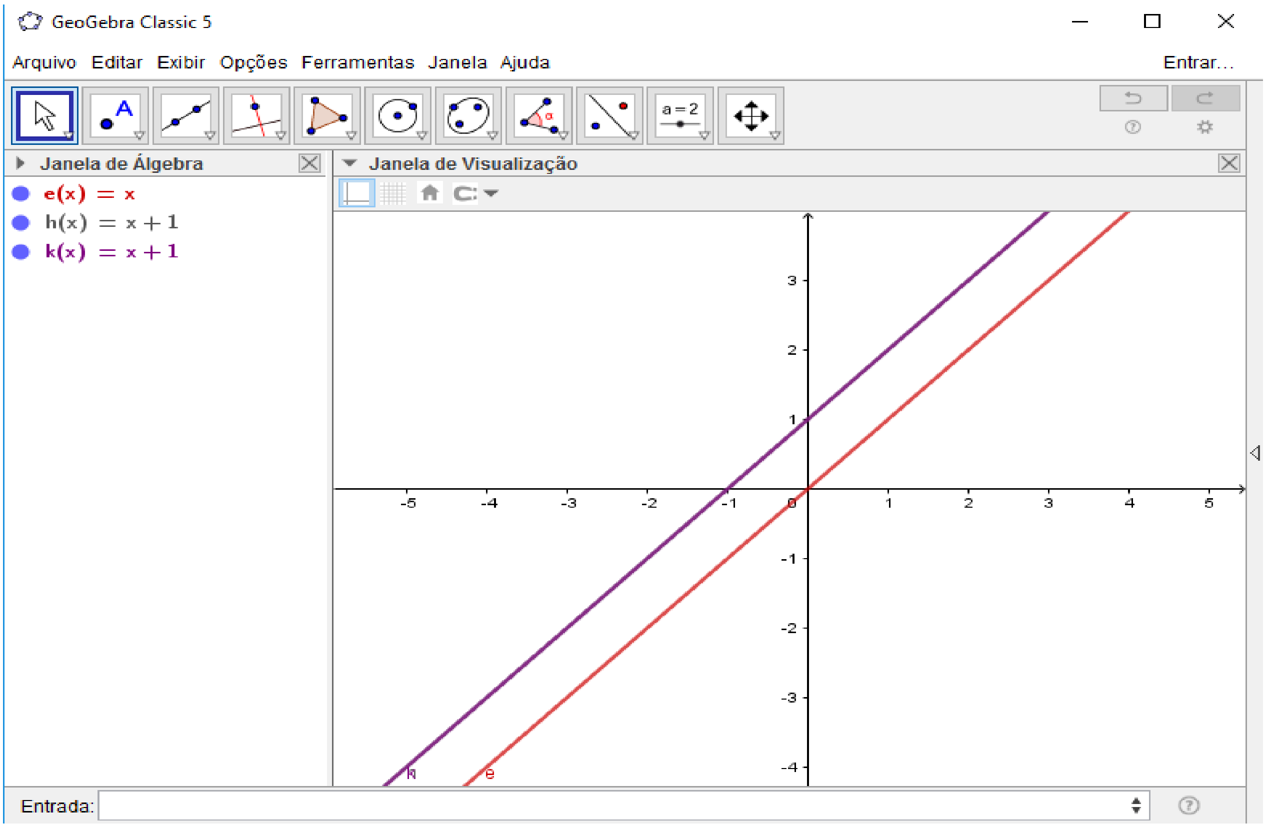
The line of function e (x) overlapped over the line of function f (x). Thus, we characterize e (x) the neutral element of G. We can see in figures 05 and 06 the composition of the function e (x) with g (x) and h (x).

Figure 05: Visualization of functions g(x) and e(x).



Source: Prepared by the authors.

Figure 06: composition between the functions h (x) and (x).



Source: Prepared by the authors.

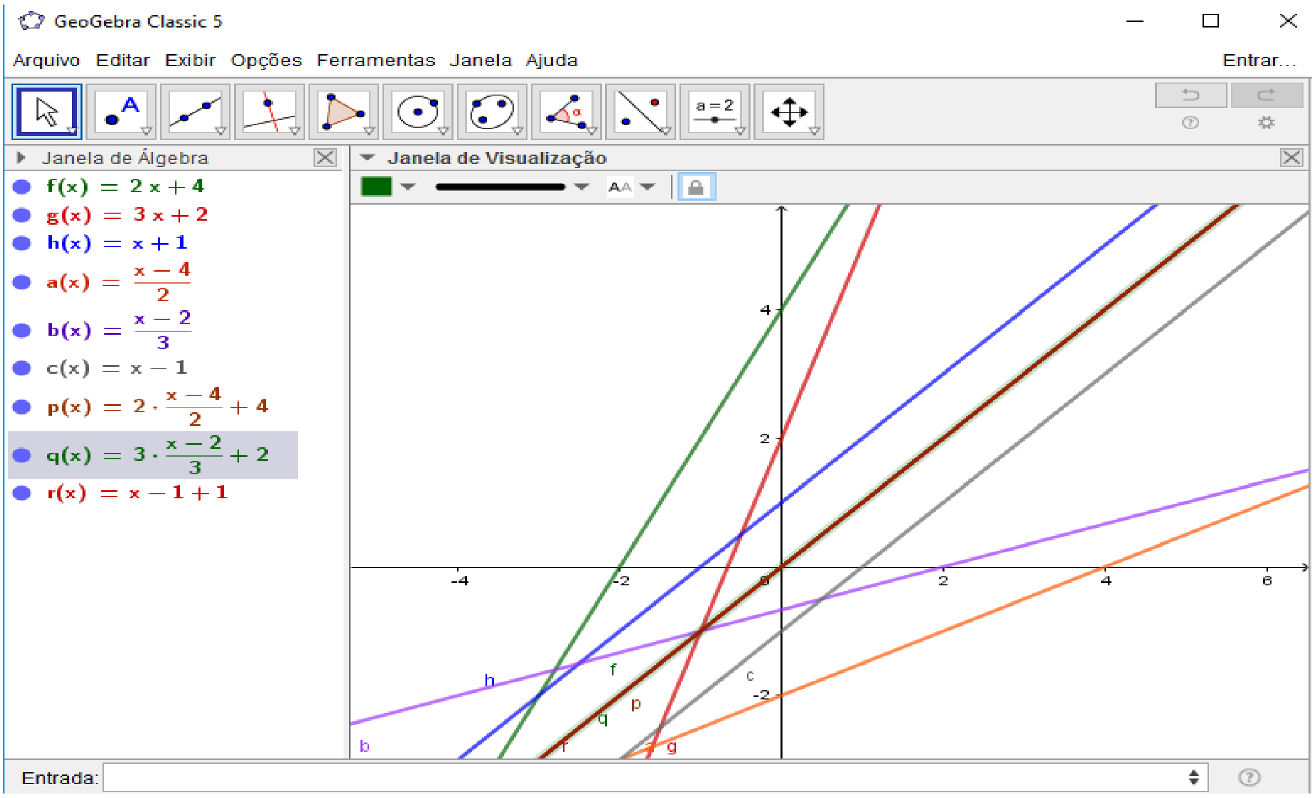
Then e (x) is the neutral element of set G with the function composition operation. Note that the line that expresses geometrically the composition of one of the functions, f (x), g (x) or h (x), with e (x), overlaps the original function, whether f (x), g (x), or h (x), this characterizes the visualization of the operation of functions, being the Position Taking used here.

And finally, a solution that the student may provide in class about the inverse element is followed by figure 07.

The functions a (x), b (x) and c (x) are adopted as the inverse functions of f (x), g (x) and h (x), respectively. Note that the functions p (x), q (x) and r (x) in the Algebra Window of figure 07 configure the identity function, that is, the neutral element in the composition of functions, and (x). Therefore, operating each function with its inverse compiles the identity function. And with that, we have the verification that the set G with the function composition operation is group.

One mistake that the student may have in solving the inverse element of this activity is to be mistaken in the inverse function of each function of the set G, and thus generate another result that does not corroborate those described in the Algebra Window of figure 07.

Figure 07 - inverse functions of f (x), g (x) and h (x).



Source: Prepared by the authors.

In the last phase provided by Sequence Fedathi, the Exam, the teacher takes the students' solutions and synthesizes the content worked. Here, too, one must generalize the problem. The generalization of the suggested Position Taking refers to Example 4, in Gonçalves (2013, p. 121), where given G the set of lines in the Cartesian plane with ≠ 0, that is, G = {f: R → R: f (x) = ax + b, 0 a, b R. As it is not possible to perform mathematical demonstration in *GeoGebra*, the teacher can use the blackboard, with brush and eraser, to demonstrate that this set G, with the operation composition of functions, is group. Such a demonstration may be used for any activity requiring the demonstration of a set with a given operation is group. The obtaining of the neutral element is given first by the assumption that this neutral element exists, and then a value is assigned to be the neutral element, after which it must be proved that given the element x of the set, operated with the neutral element, is the same as element x. Finally, the inverse element assumes that the inverse element exists in the operation and then verifies whether given element x operated with x-1 results in the neutral element.

Another point to highlight from generalization is the fact that to demonstrate if given a set with a binary operation is Group, it is enough to verify the three Group axioms. The verification of associativity occurs in manipulating with the operation to verify the veracity of equality. In the neutral element, we search for the element.

During the didactic session, the teacher uses the Mediation foundation, where is the way that the teacher becomes a mediator of knowledge for the student, not the content player. The student can also provide a solution in which the teacher has not thought about their planning, and this is characterized as a foundation of Sequence Fedathi called adidatic situation.

4. Final Considerations

It is noted that it is possible to develop didactic sessions in the teaching of Algebra under the aegis of a Dynamic Geometry software, although the content limits the use of the tool in only examples unlike the Differential and Integral Calculus that can have the geometric character even in the concepts and not only boiled down to exemplification.

The use of *GeoGebra* software for the teaching of Algebra is a possibility even with these difficulties in inserting a geometric relationship to the content. And so, one can have a lesson plan oriented to teaching the Algebraic Structure Group, which links the *GeoGebra* software.

The research was directed to the use of the computational tool in the teaching of Algebra, but it is noteworthy that the course of the activity is idealized by the Sequence Fedathi teaching methodology, as it provides the teacher with a guide on how to proceed and what posture should take in the classroom.

Thus, there is a contribution to the use of *GeoGebra* in Algebra teaching as well as Mathematics Education in higher education. We use here only an Algebra clipping for the applicability of *GeoGebra*, which causes a limitation of the work. However, it is suggested to use this tool to teach the concept of other Algebraic Structures, such as Rings, Integrity Domain and Bodies.

7. References

[1] ALVES, Francisco Régis Vieira; ARAUJO, Ana Gleiceane Dias de. Ensino de Álgebra Abstratacom Auxíliodo Software Maple: grupos simétricos Sn. **Conexões**: Ciência e Tecnologia, Fortaleza, v. 7, n. 3, p.25-35, nov. 2013.

[2] BORGES NETO, Hermínio. Uma proposta lógico-construtiva-dedutiva para o ensino de Matemática. 2016. 28f. Tese (Ascenção a Professor Titular) – Faculdade de Educação, Universidade Federal do Ceará, Fortaleza, 2016.

[3] BORGES NETO, Hermínio; CAPELO BORGES, Suzana. As Tecnologias Digitais no Desenvolvimento do Raciocínio Lógico. **Linhas Críticas**, v. 13, n. 24, p. 77-87, 2007.

[4] BORGES NETO, Hermínio. **Sequência Fedathi:** fundamentos. Curitiba: Crv, 2018. 136 p.

[5] ELIAS, Henrique Rizek; SAVIOLI, Angela Marta Pereira das Dores. Dificuldades de graduandos em Matemática na compreensão de conceitos que envolvem o estudo da estrutura algébrica grupo. **Educação Matemática Pesquisa**, São Paulo, v. 15, n. 1, p.51-82, 2013. Disponível em: <https://revistas.pucsp.br/emp/article/view/10353/pdf>. Acesso em: 05 out. 2019.

[6] FRANCO, Hernando José Rocha; SOARES, Carlos Alberto Santana. Conflitos de Aprendizagem na Disciplina de Álgebra Abstrata. **Revista Paranaense de Educação Matemática**, Campo Mourão, v. 2, n. 2, p.160-178, jan./jun. 2013. Disponível em: <http://www.fecilcam.br/revista/index.php/rpem/article/view/888>. Acesso em: 05 out. 2019.

[7] GONÇALVES, Adilson. **Introdução à Álgebra.** 5. ed. Rio de Janeiro: Impa, 2013. 194 p.

[8] KLUTH, VerildaSperidião. O Movimento da Construção das Estruturas da Álgebra: uma visada fenomenológica. **Bolema**: Boletim de Educação Matemática, Rio Claro, v. 20, n. 28, p.95-112, 20 ago. 2008. Disponível em: <https://www.periodicos.rc.biblioteca.unesp.br/index.php/bolema/article/view/1533>. Acesso em: 06 out.

2019.

[9] NOGUEIRA, Clélia Maria Ignatius; PAVANELLO, Regina Maria. A Abstração Reflexionante e a Produção do Conhecimento Matemático. **Bolema**: Boletim de Educação Matemática, Rio Claro, v. 21, n. 30, p.111-130, 06 out. 2008. Disponível em: <http://www.periodicos.rc.biblioteca.unesp.br/index.php/bolema/article/view/1784>. Acesso em: 05 out. 2019.

[10] SANTANA, José Rogério. **Do Novo PC ao Velho PC:** a prova no ensino de matemática a partir do uso de recursos computacionais. 2002. 171 f. Dissertação (Mestrado) - Curso de Programa de Pós-graduação em Educação, Faculdade de Educação, Universidade Federal do Ceará, Fortaleza, 2002.