Visualization of Archimedian and Platonic polyhedra using a web environment in Augmented Reality and Virtual Reality

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Abstract

This paper shows the development of a web environment for the construction of Archimedes and Plato polyhedra in Augmented Reality (AR) and Virtual Reality (VR). In this environment we used the geometric transformations of translation and rotation with the structure of hierarchies of HTML pages, without the use of the coordinates of each polyhedra vertex. The developed environment can be used in classroom to visualize the polyhedra in Augmented Reality, with the possibility of manipulations of the graphical representations by students in the environment created in Virtual Reality. Other studies that can be developed with the polyhedra modeled are areas, volumes and the relation of Euler. Another important content that can be developed is truncation, because seven Archimedes polyhedra are obtained by using truncation of Plato's polyhedrons. With this work, it becomes possible to develop didactic materials with a simple technology, free and with great contribution to improvement of the teaching of Geometry and other areas that use representation of 3D objects.

Keywords: Augmented Reality; Virtual Reality; Geometry; Polyhedra; Mathematics

1. Introduction

The three-dimensional concepts used in some disciplines may be better understood by students by using auxiliary resources. Nowadays concrete materials can be made with 3D prints for classes in Biology [1], Geometry [2] or disciplines with content involving 3D representations [3], [4]. The development of teaching environments or web applications also collaborates on visualizations and manipulations of concepts of Biology [5] and Geometry [6] and has been used as an attractive alternative to aid learning of the students.

The use of modeling with virtual technologies can also help in learning content that involves 3D concepts. Virtual Reality (VR) creates an immersive environment with manipulation of objects through controls and immersive glasses [7]. Environments developed in VR can help in the visualization of physical or biological phenomena, simulations of training situations, educational games, building simulations and other areas related to education.

Environments modeled with Augmented Reality (AR) technology are rendered on camera devices to place virtual objects together with the camera image environment, creating virtual layers of 3D objects and text over real-time camera image [8]. The applications of AR can help in the teaching of Geometry [9], Engineering [10], [11], Architecture [12] and Medicine [13]. The use of AR in the educational area demonstrates that it is a powerful tool, as it allows various forms of visual interactions in the learning of various subjects [14].

Learning content involving polyhedra in disciplines such as Euclidean Geometry, Descriptive Geometry and Technical Drawing almost always requires auxiliary materials such as polyhedra planned, assembled with alternative materials [15] or printed in 3D. Content that involves visualization of faces and edges, as well as area and volume calculations can be explored with manipulable materials or modeled in virtual environments.

The use of AR can complement traditional teaching materials in polyhedra teaching, as students can interact and visualize solids and their properties more effectively and meaningfully. VR can help students interact with the representations of the modeled polyhedra, facilitating the visualization and understanding of the objects.

In this work we show the web page structures that allow the construction of an environment with the technologies of VR and AR to represent the Archimedes and Plato polyhedra. The idea is to use an HTML encoded page for AR, with links to pages developed in VR. On the AR page, students view polyhedra from a variety of viewpoints and access VR sites to manipulate solids representations with mobile devices, computers, or even immerse themselves in the scene with VR goggles.

As can be seen in this paper, the commands used in building the proposed environment in AR and VR are quite intuitive, and require only a basic knowledge of HTML concepts. According to [16], some difficulties in the use of AR in the classroom can be overcome by using the environment proposed in this paper.

2. Polyhedra modeling

There are two characteristics for a convex polyhedron to be considered Platonic. The first feature is that all faces are formed by polygons with the same number of edges n. The second feature is that all vertices have the same amount of edges m [17]. An Archimedes convex polyhedron has regular polygonal faces and each vertex is the end of the same number of edges. There are 13 Archimedes polyhedra, where 7 are obtained through plane sections of the 5 Platonic Solids. The vertices of these polyhedra are combinations of two or more different regular polygons [18], [19].

This paper shows the elements of truncated octahedron and their tags for visualization in Virtual Reality environment. Constructions of the other solids of Archimedes and Plato can be made similarly.

The truncated octahedron is an Archimedes polyhedron formed by 14 faces: 8 hexagons and 6 squares. This polyhedron can be constructed using the symmetries of the faces, rotations, translations and the composition of its parts, without the need to obtain the coordinates of its vertices. In this kind of construction, the structural hierarchies of the web pages and geometric transformations are used, obtaining an optimized rendering of the model in VR and AR.

The first modeled face of the truncated octahedron is a square. Considering point O as the center of the sphere circumscribed, and the edge of polyhedron a = 3, the distance to the center of the truncated octahedron square is $dq = OO' = a\sqrt{2} \cong 4.243$ [18], where O′ is the center of the square. All edges of the squares translate dq with respect to the center of the circumscribed sphere.

The apothem of the square measures half the edge, ie $ap' = a/2 = 1.5$. By defining the edge as a cylinder with height equal to edge $a = 3$, base radius $r_c = 0.03$, and the face vertex as a sphere of radius $r_e = 0.2$, the position of edge with vertex A is obtained by translating in the OO′ direction with distance dq, with the second translation in the orthogonal direction at OO′ and distance ap' (Figure 1). The other three sides of the square are obtained by rotating 90º around the line OO′.

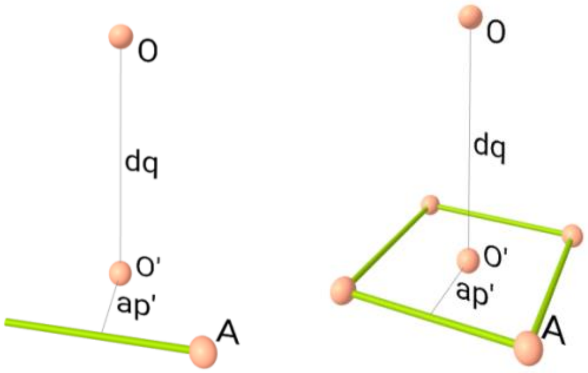


Figure 1: Construction of the edges of the truncated octahedron squares faces.

The distance from the center of the circumscribed sphere to the center of each hexagon of the truncated octahedron measures $dh = OO'' = a\sqrt{6}\ /\ 2 \cong 3.676$ and the apothem of the regular hexagon measures $ap'' = a\sqrt{3}\ /\ 2 \cong 2.598$. Another measure used to position the hexagons adjacent to each square is the dihedral angle of this polyhedron, which measures $\alpha = 125.16º$ [18], [19]. The position of the edge with vertex B is obtained by rotation with angle α and center O, translation in the OO″ direction with distance dh, with the second translation in the orthogonal direction to OO″ and distance equal to the apothem ap'' (Figure 2). The other sides of the hexagon are obtained by rotating 60º around OO″.

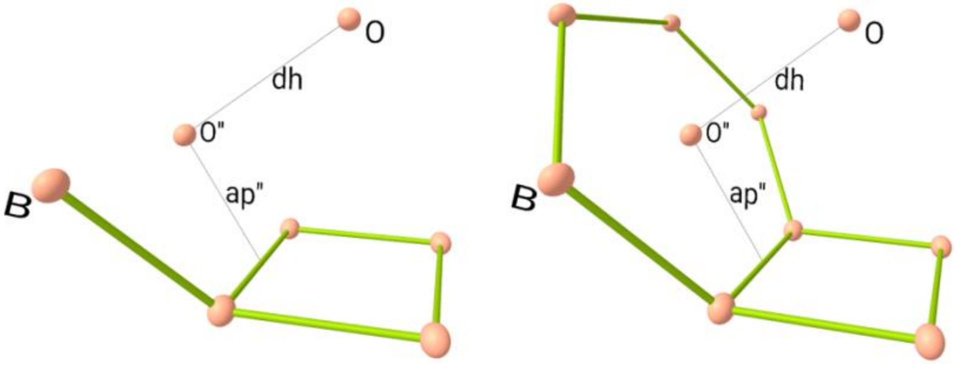


Figure 2: Construction of the hexagon edges of the truncated octahedron.

The other three hexagons are obtained by 90° rotations around the OO′ axis (Figure 3). This block represents one of the parts of the polyhedron that can be fitted into another symmetrical block. In this polyhedron, it is enough to fit the top through a 180º rotation around the OO′ axis and a translation with a measurement equal to $2dq$, obtaining the complete truncated octahedron (Figure 4).

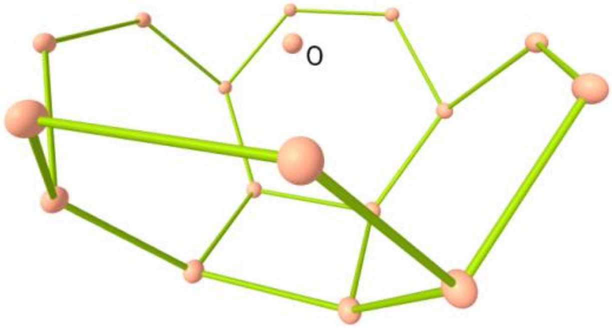


Figure 3: Truncated Octahedron Bottom Block.

To improve the visualization of each polyhedron, the parallel perspective was used. Figure 5 shows a view of the truncated octahedron with the parallel perspective, combined with a caption showing the main polyhedron measurements and properties.

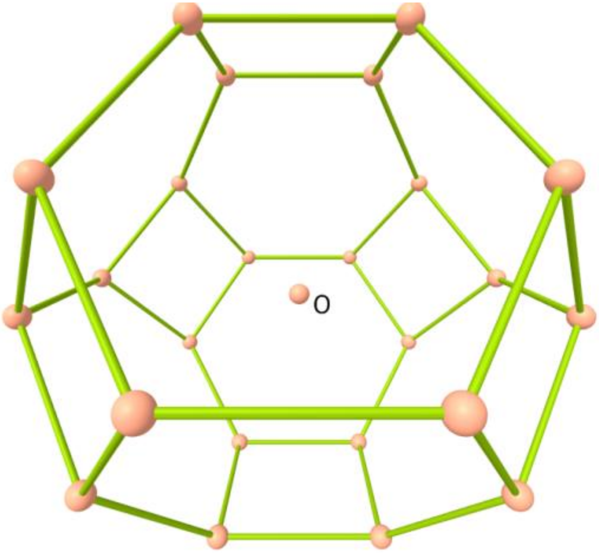


Figure 4: Complete truncated octahedron represented in VR.

Other more complex Archimedes polyhedra can be constructed with the same methodology shown, with more repetitions of the main block. For example, the icosidodecahedron main block is formed by a regular pentagon and five equilateral triangles (Figure 6). This module can be repeated on the sides, forming the bottom of the solid. The composition of these modules can be translated twice the distance from the center of the sphere to the pentagon of the first module and rotated 180° about the main axis, as shown in the truncated octahedron.

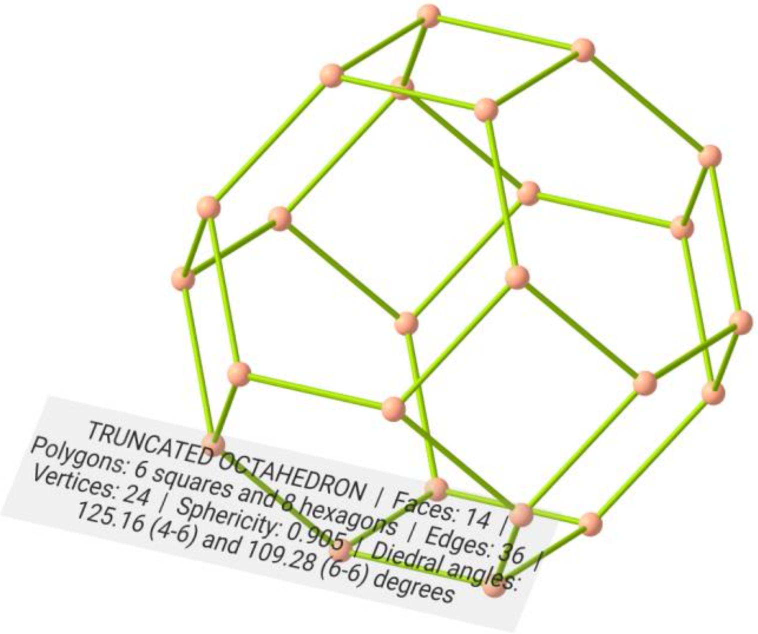


Figure 5: Truncated octahedron represented in VR with parallel perspective.

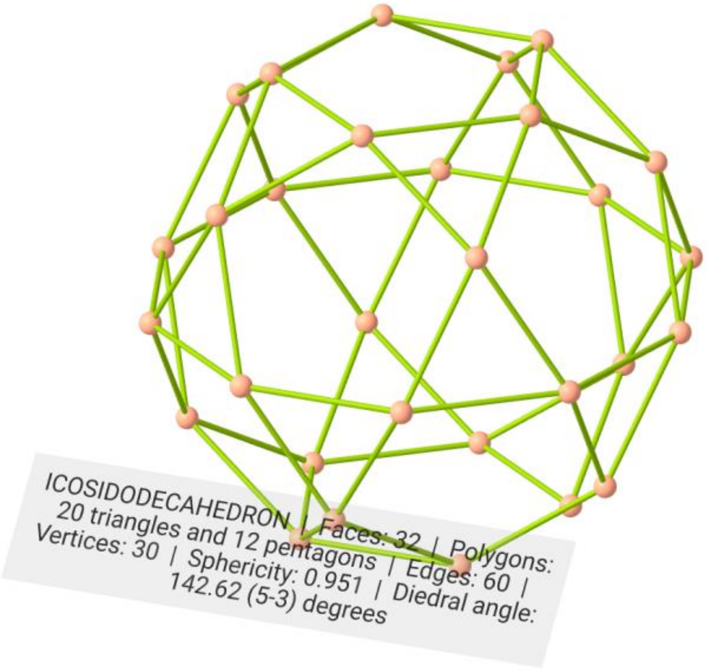


Figure 6: Icosidodecahedron represented in VR with parallel perspective.

Archimedes' best known polyhedron is the truncated icosahedron, a solid used as a base for making soccer balls (Figure 7). In this polyhedron the main block is formed by a regular hexagon with adjacent regular pentagons and intercalates regular hexagons (a pentagon with 5 adjacent hexagons can also be considered). This block can be repeated 6 times at the sides and then at the top of the polyhedron.

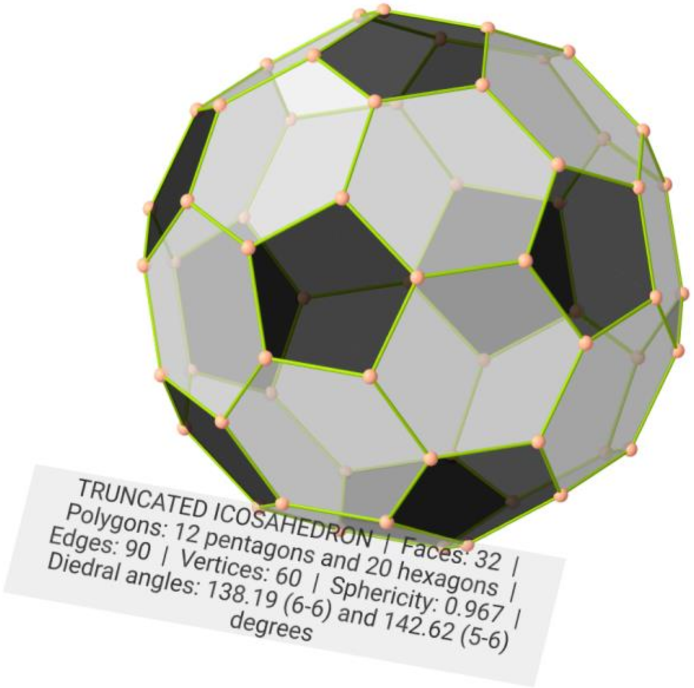


Figure 7: Truncated Icosahedron represented in VR with parallel perspective.

The construction of each Platonic polyhedron is relatively simple, as each solid has equal measurements of dihedral angles and distances from the center to faces. In this work the same methodology was used for the construction of the Archimedes and Plato polyhedra. Figures 8 and 9 show icosahedron and dodecahedron, respectively, in the programmed Virtual Reality environment.

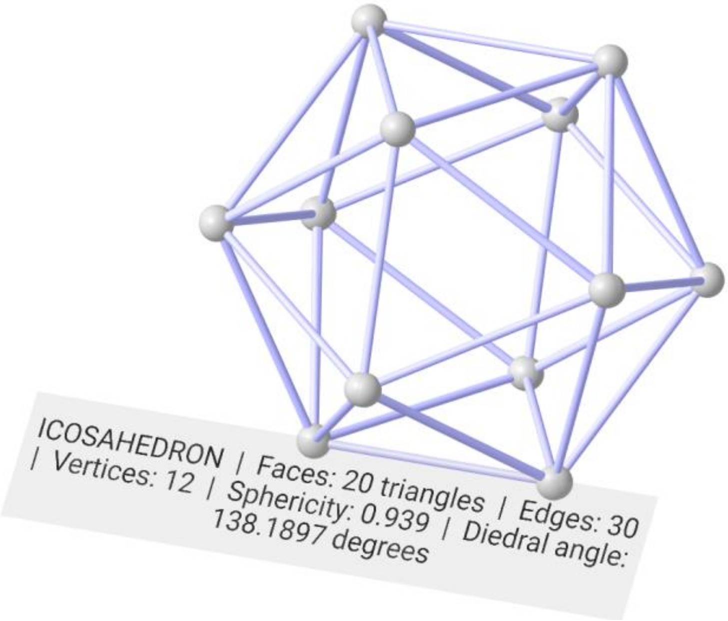


Figure 8: Icosahedron represented in VR with parallel perspective.

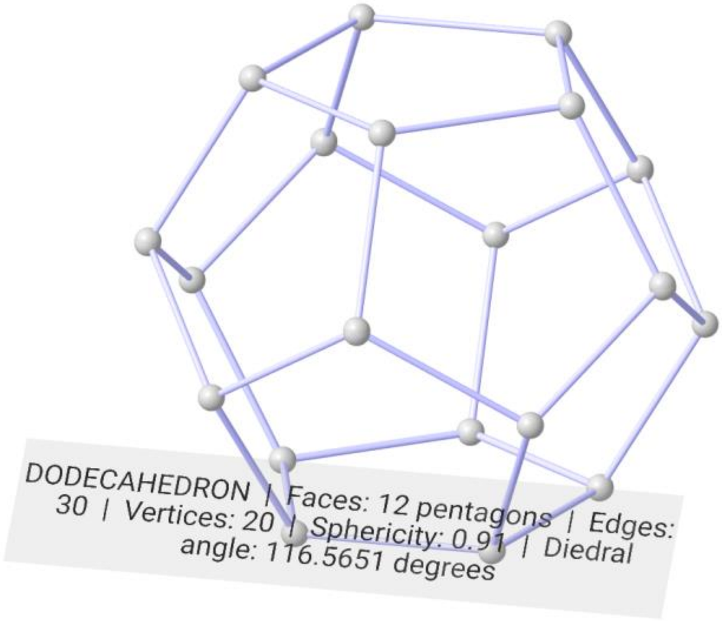


Figure 9: Dodecahedron represented in VR with parallel perspective.

With the elements defined, Archimedean and Platonic polyhedra can be modeled in AR and VR for classroom use. The same methodology can be used on polyhedra that have symmetries of the faces positions.

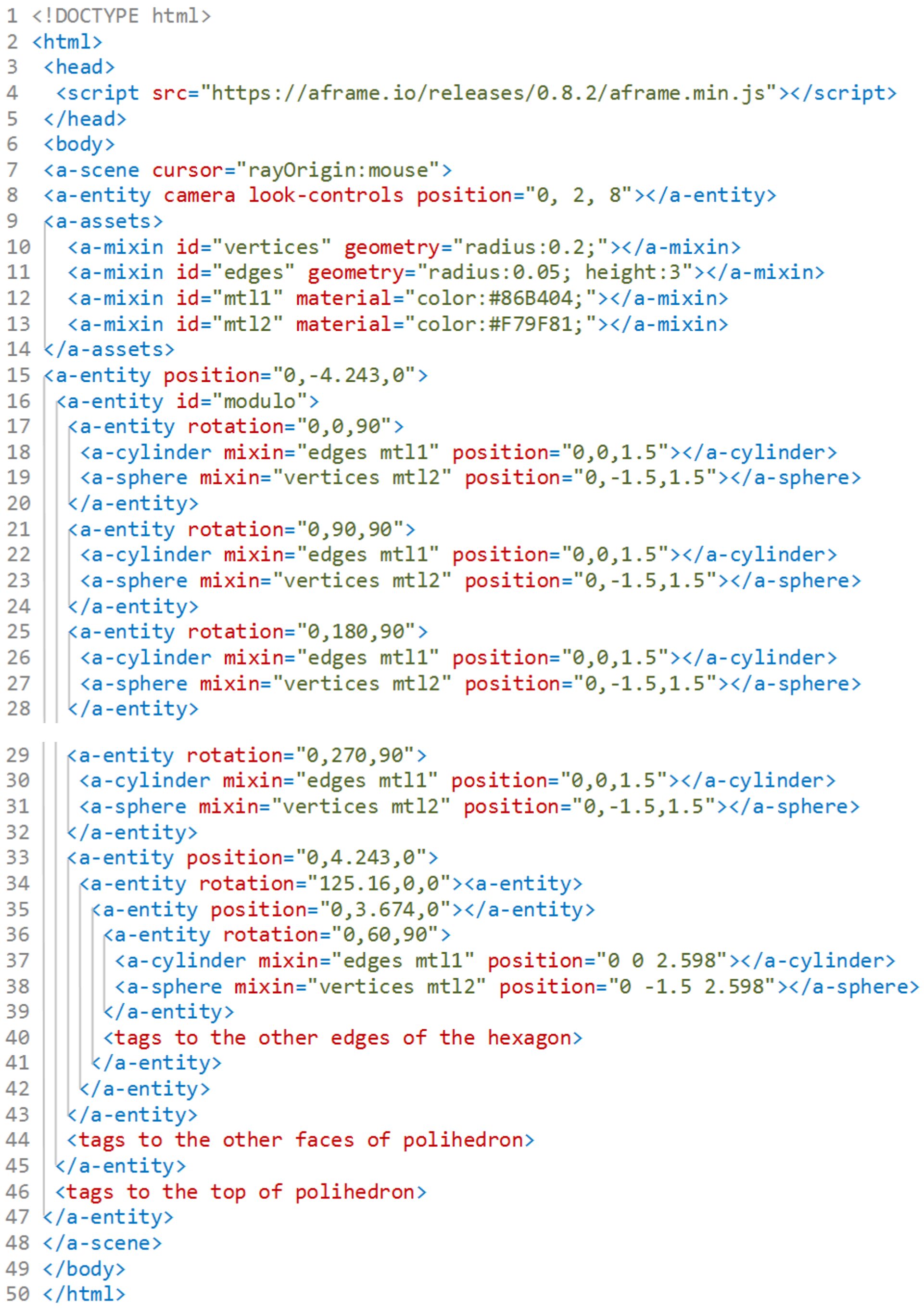
3. Virtual Reality

In a Virtual Reality programmed environment, a 3D object simulation occurs, giving the visitor a sense that the programmed objects are real [7]. The programmed environment in Augmented Reality, on the other hand, has VR objects integrated with the actual camera images of the device, creating the feeling that virtual objects are part of the real world.

The measurements shown in section 2 of the truncated octahedron were programmed in both VR and AR. In both cases, we used the A-frame libraries, an environment developed by the Mozilla VR team [20]. A-frame utilizes Java's Three.js library functions with pure HTML tags, allowing all VR or AR programming to be done on a webpage, which is made up of tags with inheritance and hierarchy principles [21].

The main tags for modeling a part of the truncated octahedron are shown in Figure 10. The information placed in this figure includes the square edge tags and one of the hexagon edges of the polyhedron main block. Other tags have similar structure and will be omitted. In the header tag of the HTML page is inserted the reference to the A-frame main library between lines 3 and 5. All library references can be inserted inside this header tag.

Polyhedron modeling is defined in the body tags of the HTML page. Lines 7 and 8 of Figure 10 are the definitions for user interaction with the mouse or VR control and the camera of the scene with starting position at coordinates x (right/left), y (height) and z (depth). Initial values are: x = 0 which centers the camera on the screen; y = 2 meters, representing the height of the observer; and z = 8 meters to distance the observer from the origin of the system, where the center of the truncated octahedron is represented.



```
1 <!DOCTYPE html>
2 <html>
3  <head>
4   <script src="https://aframe.io/releases/0.8.2/aframe.min.js"></script>
5  </head>
6  <body>
7  <a-scene cursor="rayOrigin:mouse">
8  <a-entity camera look-controls position="0, 2, 8"></a-entity>
9  <a-assets>
10    <a-mixin id="vertices" geometry="radius:0.2;"></a-mixin>
11    <a-mixin id="edges" geometry="radius:0.05; height:3"></a-mixin>
12    <a-mixin id="mtl1" material="color:#86B404;"></a-mixin>
13    <a-mixin id="mtl2" material="color:#F79F81;"></a-mixin>
14  </a-assets>
15  <a-entity position="0,-4.243,0">
16   <a-entity id="modulo">
17    <a-entity rotation="0,0,90">
18     <a-cylinder mixin="edges mtl1" position="0,0,1.5"></a-cylinder>
19     <a-sphere mixin="vertices mtl2" position="0,-1.5,1.5"></a-sphere>
20    </a-entity>
21    <a-entity rotation="0,90,90">
22     <a-cylinder mixin="edges mtl1" position="0,0,1.5"></a-cylinder>
23     <a-sphere mixin="vertices mtl2" position="0,-1.5,1.5"></a-sphere>
24    </a-entity>
25    <a-entity rotation="0,180,90">
26     <a-cylinder mixin="edges mtl1" position="0,0,1.5"></a-cylinder>
27     <a-sphere mixin="vertices mtl2" position="0,-1.5,1.5"></a-sphere>
28    </a-entity>

29    <a-entity rotation="0,270,90">
30     <a-cylinder mixin="edges mtl1" position="0,0,1.5"></a-cylinder>
31     <a-sphere mixin="vertices mtl2" position="0,-1.5,1.5"></a-sphere>
32    </a-entity>
33    <a-entity position="0,4.243,0">
34     <a-entity rotation="125.16,0,0"><a-entity>
35      <a-entity position="0,3.674,0"></a-entity>
36       <a-entity rotation="0,60,90">
37        <a-cylinder mixin="edges mtl1" position="0 0 2.598"></a-cylinder>
38        <a-sphere mixin="vertices mtl2" position="0 -1.5 2.598"></a-sphere>
39       </a-entity>
40       <tags to the other edges of the hexagon>
41      </a-entity>
42     </a-entity>
43    </a-entity>
44    <tags to the other faces of polihedron>
45   </a-entity>
46   <tags to the top of polihedron>
47  </a-entity>
48  </a-scene>
49  </body>
50  </html>
```

Figure 10: A-frame VR page tags for truncated octahedron modeling.

The tags defining the colors, radius of the spheres representing the vertices ($r_e = 0.2$), the radius ($r_c = 0.05$) and height ($a = 3$) of the cylinders representing the edges of the polyhedron are placed between lines 9 and 14 of Figure 10. The code uses the a-mixin function to speed up page loading, as each property is set only once and used on the rest of the page with its reference by identifiers (id).

The tag set between lines 15 and 47 represents the construction of part of the truncated octahedron shown in section 2. The line 15 tag translates the polyhedron with the distance from the center of the circumscribed sphere to the center of the square ($dq \cong 4.243$). The tags between lines 17 and 20 show the construction of an edge of the square, with the translation equal to the apothem ($ap' = 1.5$). Between lines 21 and 32 are the rotations to construct the other edges of the square.

In lines 33, 34 and 35 are the commands to make a translation to the center of the sphere with distance *dq* (distance from the center of the square), the construction of the dihedral angle $\alpha = 125.16^{\circ}$ and the translation with distance $dh \cong 3.674$ to position the hexagon. These three commands are required not to lose the hierarchy reference of the HTML page. Within this tag can be made all constructions of hexagons adjacent to the first square constructed. It is noteworthy that the approximations made in the measurements of distances and apothems indicated did not cause inaccuracies in the construction of polyhedra.

Between lines 36 and 39 are the tags for building a hexagon edge. These tags rotate 60º with respect to the OO'' axis and translate with a distance equal to the apothem of the hexagon $ap'' \cong 2.598$. The other edges can be constructed with similar 120°, 180°, 240°, and 300° rotational tags from line 40 of the code shown in Figure 10. The tags on other faces can be inserted from line 44, and repeating to the top of the polyhedron can be set from line 46 with the constructions mentioned in section 2.

One way of user interaction with the programmed elements in the scene uses the orbit function [22], which allows the camera to move around the objects in the scene. When using VR goggles, the camera's movement with orbit function is automatic. On computers, tablets and smartphones, the camera can be moved around objects using the mouse, keyboard or touch.

4. Augmented Reality

Virtual Reality modeled elements can be mixed with physical objects shown through the camera of a device through Augmented Reality programming. The same structural tags shown in Section 3 in VR can be used in programming a page in AR, including the reference tag for the AR view developed by Etienne [23], which should be inserted into the page header with the A-frame referential tag.

Archimedes' two-polyhedron tags are illustrated in Figure 11. The scene tag in AR has the inclusion of webcam image embedding properties and mouse interaction capture or control radii on linked objects (lines 7 and 8). The scene in AR has barcode markers, which work with bit codes 0 and 1 in matrix form of images that are recognized through the webcam [24]. These markers act as reference point where specific positions can be set so that virtual objects appear in the actual webcam image.

When a barcode image is recognized in the AR scene, VR-modeled elements are activated. There are over 80 programmed marker options in the library developed by Etienne [23], which are represented by tags that includes the programmed VR elements that are activated. The most common markers are hiro, kanji and barcodes, illustrated in Figure 12.

Students can access their web page from their devices, target the printed programmed barcode markers, and their programmed VR objects appear on the AR device screens. The hiro marker was used for the AR visualization of the truncated octahedron. The structure of your tags is between lines 9 and 14 of Figure 11. In these tags, a-entity is used to group the elements of the polyhedron, position it over the marker and use scales. The link tag (line 10) creates the interaction for accessing VR programmed pages through blue circles that appear over the bookmarks. The kanji marker was used to show the truncated icosahedron in AR, with the tags between lines 15 and 20 of Figure 11. Figure 13 shows the two Archimedes polyhedra in AR.

```
<head>
 <script src="https://aframe.io/releases/0.8.2/aframe.min.js"></script>
 <script src=" https://jeromeetienne.github.io/AR.js/aframe/build/
    aframe-ar.min.js"></script>
</head>
<body>
<a-scene embedded cursor="rayOrigin:mouse" raycaster="objects:[link];"
   arjs='sourceType:webcam; detectionMode:mono_and_matrix; matrixCodeType:3x3;'>
<a-marker preset="hiro">
  <a-link href="octat.html" title="VR"></a-link>
  <a-entity position="1.5 0.8 0" scale="0.25 0.25 0.25">
   <tags of truncated octahedron>
  </a-entity>
</a-marker>
<a-marker preset="kanji">
  <a-link href="icosi.html" title="VR"></a-link>
  <a-entity position="1.5 0.8 0" scale="0.15 0.15 0.15">
   <tags of icosidodecahedron>
  </a-entity>
</a-marker>
```

Figure 11: A-frame AR page tags for Archimedes polyhedra modeling.

Figure 12: Barcode markers used in A-frame: hiro, kanji e barcode #20.

The AR developed page has all the links to VR of Platonic and Arquimedean solids visualizations shown in this work and is available at:

https://paulohscwb.github.io/polyhedra/

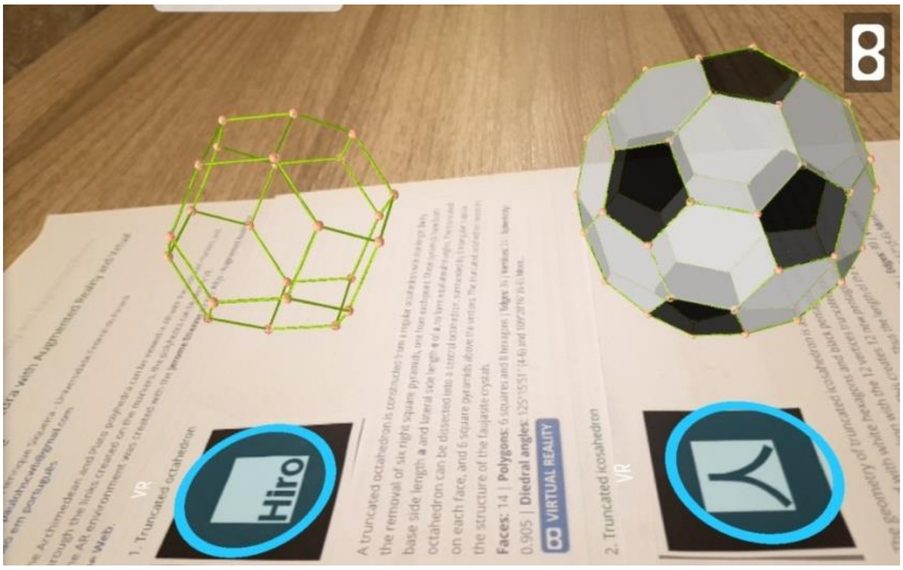


Figure 13: Visualization of truncated octahedron and truncated icosahedron in AR environment.

5. Conclusions

This paper shows a web-based system for viewing Archimedes polyhedra in Virtual Reality and Augmented

Reality. Through the visualization of printed markers, students can view in AR solids with any webcam and internet access device, and can use links to access the VR visualizations.

The methodology used in this work proposes the use of symmetries, translations and rotations for the correct positioning of each polyhedron face. This way, the rendering of each polyhedron is optimized by using web page programming hierarchies, making it easier to program multiple polyhedra on the same page.

The result shows that it is a useful tool for use in the classroom as it allows students to view and manipulate the polyhedra graphs on their devices or to use Virtual Reality glasses for complete immersion in the scene.

The programmed environment can be explored in geometry classes, assisting in the understanding of polyhedron elements or in topics such as area and volume calculations, Euler relationship, plane sections or simply visualizing each modeled solid. Plato polyhedra and dual polyhedra can also be explored in an environment similar to that proposed in this paper.

All polyhedron elements can be viewed in AR and VR, and students can pan the scene camera to find the best views of VR solids with A-frame tools to orbit the camera around objects.

The webpage programming tools shown in this paper are simple and intuitive, and can be used in classrooms with printed materials. Students access the programmed AR site, view solids with their printed markers, and can interact with programmed VR polyhedra. In this way, students can explore the geometric concepts involved more efficiently and dynamically.

Some advantages of creating AR and VR environments as web pages for classroom use are convenience, low cost, great performance, simplicity of programming, and operation on all types of smartphones and tablets. Another advantage of this tool is the almost immediate loading of the site as it is built in HTML with references from VR libraries developed in Java. Students do not need to download applications and multiple markers can be used on the same HTML page, which allows the creation of courseware with various topics programmed in AR and VR. This tool can be used in other disciplines such as Differential and Integral Calculus, Statistics, Biology, Chemistry, Physics, Engineering and other areas that use 3D graphical representations.

6. References

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