

Process of Appropriating Mathematical Concepts Mediated by Teaching Activity

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ABSTRACT

Given the importance of Teaching Orientation Activity (AOE) as an articulated set of teachers' intentionality with the possibility of promoting mathematical theoretical-scientific thinking, the purpose of this study is to analyze the process of appropriation of mathematical concepts of teaching elementary school teachers, inserted in a project of continuous formation in the perspective of Historical-Cultural Theory / Activity Theory. Regarding the process of apprehension of the investigated phenomenon, the historical logical movement of the concepts was based, based on the assumptions of Historical and Dialectical Materialism, advocated by Karl Marx. The evidence was being evidenced, especially through the preparation and development of AOE by teachers participating in continuing education, which allowed us to go beyond the appearance and fossilization of the process of appropriation of concepts, apprehending the historical mediations that comprised the phenomenon under investigation. On the techniques of data production, field observations, video recordings of the formative meetings and reflective session were worked on. It was adopted as an analytical procedure, the method of deployment in units of analysis proposed by Vigotski and complemented by the idea of "episodes" of learning (MOURA, 2004). It was observed that the teachers during the experience with the AOE, in a dialectical movement, little by little, were identifying different elements of the problem situations, commonly worked in the classroom, subsidized by formal logic. Thus, they acquire a level of awareness in the sense that as teachers, the need must be created, as well as the objective and subjective reasons and conditions for students to enter into learning activity.

Palavras-chave: Appropriation of Mathematical Concepts. Teaching Guidance Activity. Elementary school teachers.

Introduction

The present study proposes to analyze the process of appropriation of mathematical concepts of teachers of Elementary School, inserted in a project of continuous formation in the perspective of Historical-Cultural Theory / Activity Theory, which has as major exponents, the Russian theorists Vigotski and Leontiev.

We start from the assumption that empirical mathematical thinking, with a low level of abstraction, based basically on observation and perception, prioritizing the external and common properties of the concept, elaborated in everyday actions and experiences, for not providing students with training courses initial and / or continued teachers' mental actions through the solution of problems that can develop mathematical theoretical-scientific thinking, end up ignoring the peculiarities and regularities of the process of appropriation of mathematical concepts. In this regard, Sforni (s.d, p. 136) explains that

[...] seems to be supported by an easily observable hiatus, between the initial grades of elementary school and the sequence of schooling. While representation is prioritized in the first years of elementary school, in a transit between perception and synthesis itself, in adolescence it is considered that the student already has the capacity for generalizations. That is, it has the ability to deal with attributes that are not perceived by direct observation, the result of deductions, based on nexus and interlacing, and with the internal qualities of objects, using the deductive method.

We can infer from Sforni's reflections that teacher educators must pay attention to the need to work from the perspective of the concept appropriation process, as it is still quite absent from classrooms, which exhorts us to understand that one of the most important problems concerning mathematical learning is in teacher training courses.

It should be added that, for Ferreira (2009), teacher training courses do not seek to familiarize undergraduate students with the logical, psychological, historical and pedagogical foundations related to this process, which puts us in front of the paradox: it is the teacher's competence to propose the situations that lead students to conceptual elaboration, and, however, he is unaware of the peculiarities and regularities of this process, as well as the pedagogical procedures to be mobilized to carry them out.

In line with the reflections of Sforni and Ferreira, D`Ambrósio (1993, p. 35) clarifies that the view of Mathematics that prevails in school curricula reflects the perception of what society thinks of what this discipline is, seen and characterized by "[...] precise results and infallible procedures, whose fundamental elements are arithmetic operations, algebraic procedures and definition and geometric theorems".

Despite this problem, this was the meaning we had produced about mathematical concepts coming from academia, due to the absence of a "clear perception" (VIGOTSKI, 2004) of the relations of these concepts. As evidenced by Ferreira (2009), this is a question that remains emblematic in the teaching of Mathematics, as they ignore all the variables that allow, whether the student or the teacher, to know the genesis and nature of the concepts, limiting teaching in the perspective of empirical thinking.

The reflections hitherto made us want to find answers to the question that guided this research: What do the actions made possible by the continuing education of teachers reveal, in the perspective of the Historical-Cultural Theory / Activity Theory, above all, in the development of AOE, about the appropriation of mathematical concepts?

That said, in addition to the introduction, the text was structured with sections for theoretical reflections on AOE as a theoretical-methodological principle, for the explanation of the process of apprehending the

investigated phenomenon, for the analysis and discussion of the results on training actions mediated by the process of reflection on the AOE and the final considerations.

The AOE as an articulated set of the intention of Mathematics teachers

Leontiev (1977), when conducting a research whose objective was to investigate the principles of mental development and the problem of mental retardation in children who were not able to appropriately appropriate mathematical concepts, in a pre-established time and under conditions considered “normal”, observing them, in the object manipulation activity and in the establishment of relations with numerals, from a methodological point of view, brings us significant contributions. This theorist (1977, p. 115) found that:

The teaching of arithmetic should therefore not start with generalization, but with active training in the child of actions with external objects and, in parallel, with their movement and inventory. Subsequently, these external actions become language (count out loud), shorten and finally acquire the character of internal actions (mentally count), which are automated in the form of simple associative acts. However, behind them, complete actions on objects are now hidden, actions previously organized by us. That is why these actions can always be manifested externally again.

The author therefore expresses his concern with the organization of mathematics teaching. Given this concern, and in line with this author, Moura (2010), adds by saying, so that the teacher becomes aware of the need to organize teaching in a way that can fulfill the school's primary role, that is, to enable students to students development of mathematical theoretical-scientific thinking, it is their responsibility, a priori, to make a self-assessment of their pedagogical activity. For this, it should reflect, for example, on the activities that involve the manipulation of materials as presented, often without the understanding of the teacher and, consequently, of the students, that the concepts, whether mathematical or not, are alive, fruits of human activity in movement.

On this issue, delimiting the national and local scenario, the school has found it difficult to socialize mathematical knowledge. When analyzing these issues regarding the organization of teaching in Mathematics, more specifically those concerning the teaching and learning process and teacher training, the activities proposed by the teacher must be problematized in order to provide students with triggering learning situations, which may lead to them to the development of theoretical-scientific thinking and, consequently, to the appropriation of mathematical concepts (MOURA, 1996, 2001; MOURA, 2010).

From the perspective of the Historical-Cultural Theory / Activity Theory, we understand activities that trigger learning to those activities that, in their structure, include, among other variables: the genesis of the concept / essence; the explanation of the needs that moved humanity to appropriate this concept; the appearance of problems; human needs in a given activity and the way men developed solutions or syntheses in their logical-historical movement (MOURA, 2010).

After making these considerations, we emphasize that Moura (1996, 2001), in the search for a didactic-methodological proposal, along the lines explained above, supported by the precepts of the Historical-Cultural Theory / Activity Theory and in the perspective of the logical-historical, aimed and appropriated AOE, understood as "[...] the articulated set of the educator's intentionality that will use instruments and strategies that will allow him to get closer between subjects and the object of knowledge" (MOURA, 1996, p. 19).

The AOE, in the development of this investigation, previously understood only as a unit between teaching and learning, also came to be seen as a unit between teaching, learning and research and, therefore, becoming a theoretical-methodological model that the researcher develops in the research course - the Research Guiding Activity (AOP) -, maintaining the same activity structure proposed by Leontiev (ARAÚJO, 2013), as already commented.

But what is AOE anyway? And what is your central objective? In response to this question, Moura (1996) tells us that it is all teaching activity that is structured in a way to provide the subjects involved in the activity, mediated by content, negotiating meanings and presenting a triggering situation, elaborated, for example, through a "virtual history of the concept" or games, involving the logic and history of the concept, from questions that guide the essence of the concept. In this sense, it is worth reinforcing that AOE has as its central objective "[...] to promote conceptual learning" (MOURA; SFORNI; ARAÚJO, 2011, p. 40), based on planned actions, "appropriate to the age to teach concepts theoretical" (KOZULIN, nd).

In the theoretical and methodological perspective of AOE, virtual stories of the concept "are problem situations posed by characters from children's stories, legends or from the history of mathematics as triggers of the child's thinking in order to involve him in the production of the solution of the problem which is part of the context of the story. " (MOURA, 1996, p. 20).

Given the importance of AOE as an articulated set of the intention of Mathematics teachers with the possibility of promoting mathematical theoretical-scientific thinking, the first step to be developed by these teachers is to carry out a study of the history and needs that led humanity to create the concept to be worked with the students, aiming to reach the genesis of this concept. This step is substantial to ensure greater efficiency in forwarding actions and subsequent steps, namely, the triggering situation of learning and collective synthesis (POZEBON; HUNDERTMARCK; FRAGA, 2012).

In a nutshell, the triggering situation of learning "constitutes the objectification of the teaching activity, which contemplates the elaboration, the collective solution and the genesis of the concept" (MORAES, 2008), thus bringing the triggering problem that, in the most of the time, it is presented, taking as a starting point a virtual story. From this perspective, in this stage, materials and games, for example, can be used as complementary didactic instruments.

Finally, the stage of collective synthesis, understood as the moment when, necessarily,

[...] the class should arrive at an answer that is common to all and "mathematically correct" to the problem. This answer must be related and coincide with the historical construction of the concept, so it will be mathematically certain. The educator's action becomes essential in this moment of sharing actions and ideas,

where everyone must reach a solution similar to the experience by man over time (POZEBON; HUNDERTMARCK; FRAGA, 2012, p. 5).

We could, therefore, synthesize that given the potential of AOE as a theoretical-methodological and research instrument, it is an instrument that, necessarily, must be part of the organization of teaching, as well as research, since it presents itself as a possibility of lead the student to reflect, to inquire, to raise hypotheses and to seek solutions to the triggering problems presented, without excessive dependence on the teacher.

On the process of apprehending the investigated phenomenon

Regarding the process of apprehending the phenomenon investigated in this study - mathematical learning mediated by the teaching guiding activity -, we show that the study in reference was anchored on the assumptions of Historical and Dialectical Materialism, advocated by Karl Marx.

Historical and Dialectical Materialism is based on the principles of dialectical logic guided by Hegel and proves to be an effective methodological instrument for understanding the real, as it is the very expression of the relationship between the laws of thought and the laws of objective reality, which gives it a status of method in permanent construction, in the same way as its object of study.

According to the precepts of this method, man is seen as a concrete, historical and singular being, which is constituted in the relationship with his reality, in the dynamics of the relations between objective and subjective conditions, which are revealed in the historical movement of social life in a way dynamic, created by men collectively and, at the same time, creator of these men. In other words, in this perspective, the world is a material reality to which man is present and can know and transform it.

In keeping with the assumptions of the method in evidence, we signal that the complexity that the mathematical concept encompasses requires that the issue of its historicity be placed in its correlation with the logical, understood as "[...] the element of mediation that allows thought theoretically recreate the history, constituting a means for its knowledge, and logic is the way this mediation takes place" (FERREIRA, 2007 apud FERREIRA, 2009, p. 56). In this view, we start from the assumption that as one of the categories of knowledge, the concept appears, develops and transforms. Therefore, discussing about the historical development of the mathematical concept was a substantial condition. There is no way to get to the essence of the concept without examining its formation and development process (KOPNIN, 1978).

It is worth pointing out that, thus, we look for signs of appropriation of the mathematical concept in order to apprehend, explain this process, based on the historical logical movement of the concepts. These evidences were being evidenced, mainly through the elaboration and development of AOE by the teachers participating in the continuing education, in the perspective of the Historical-Cultural Theory / Activity Theory, which allowed us to go beyond the appearance and the fossilization of the process of appropriation of concepts, apprehending the historical mediations that comprised the phenomenon under investigation. In addition to AOE as a data production technique, the following techniques were used: field observations and video recordings of the formative meetings and the reflective session. We also clarify that for the realization of the empirical study, we chose to delimit a sample that met the prerequisites: being a teacher

and / or coordinating teacher of Basic Education (Elementary Education) and being available to participate in the investigation, making a total of 10 (ten) participants. Thus, in order to guarantee anonymity, we use codenames: Teacher T1, T2, T3, T4, T5, T6, T7, T8, T9 and T10.

In addition, we have adopted the method of split into units of analysis as an analytical procedure. For Kosik (2011), the act of knowledge is a process of decomposition / separation of the parts of the elements that make up the whole, reality (the phenomenon investigated). Furthermore, "[...] the 'concept' and the 'abstraction', in a dialectical conception, have the meaning of a method that decomposes the whole in order to be able to reproduce the structure of the thing spiritually, and, therefore, to understand the thing". (KOSIK, 2011, p. 18, emphasis added).

Despite the words of this theorist, conceptual knowledge is the only mechanism through which we can know reality in its concreteness, in its concrete totality, since reality is not a combination of things that happen, but, rather, things that happen. are interconnected and that constitute the analyzed reality.

That said, the analysis of the process of appropriation of mathematical concepts by teachers participating in continuing education was complemented by the idea of "episodes" of learning. The definition of an episode was proposed by Moura (2004) because he understands that it is a set of actions that triggers the process of seeking the answer to the problem in question. Moretti and Moura (2011, p. 437), supported by Moura (2004), complement this understanding, clarifying that the process of clipping and analyzing episodes reveals actions "[...] in the process of training the participating subjects".

When gathering this information, Moura (2004, 2013) defines episodes as those reports that aim to narrate moments in which it becomes visible the awareness of a fact that restructures the thought of the investigated subject in the face of the question posed and that gives a new quality to the solution you work out.

In this way, we come to understand the episodes as moments of interaction in the formative encounters made possible by continuing education, in which mathematical concepts moved, producing meanings for the subjects, from the development of the training actions proposed by the mentioned project. In the case of this investigation, the episodes are the conversations and actions that reveal interdependence between the elements of the training actions. For corroborating the ideas of Moura (2004), we clarify that, when working on the logic of the episodes in this study, these were not defined based on a set of linear actions, which occurred in consecutive months or even years. Thus, in the excerpts of the conversations of the subjects of this research, present in one unit, episodes appear at different times.

Training actions mediated by the process of reflection on AOE

Given its potential, as previously discussed, it was possible to verify that the AOE enabled the teachers participating in the continuing education to reflect and become aware of the training actions, unlike the didactic-pedagogical orientations imbued with formal logic, in which the teacher prioritizes tasks (lists of exercises, dictations of loose words, homework and repetitive math problems, etc.) and teaching teaching resources concerned only with fulfilling the curriculum, often in an unplanned, unintended manner. In our understanding, formative action is any and all conscious action, that which, as Leontiev (1978, p. 303)

explains, "[...] is formed, therefore, within a sphere of relations already constituted, within such or such an activity that determines psychological particularities. ".

In this way, training actions are capable of causing qualitative changes in teaching activity. In the words of Moura (2001, p. 51), these qualitative changes "[...] can be added to or merged with those already acquired by teachers". Furthermore, it is the formal actions that lead the teacher to develop his conscience for the appropriation of theoretical and scientific knowledge as an object of pedagogical activity. From this perspective, the teacher's performance is charged with intentionality, since he aims in his activity the reason that drives him (RIGON, ASBAHR, MORETTI, 2010).

Thus, we infer that this may negatively affect the movement of appropriation of mathematical concepts and, of course, the development of theoretical and scientific thinking. Hence the need to organize teaching in a humanizing, against hegemonic perspective. We are referring to the perspective embraced in this study that conceives knowledge as a product of human activity in its multiple dimensions and, as a result, embodies the socio-historical process of its production (RIGON; ASBHAR; MORETTI, 2010).

In view of this reflection and observation, it is also enlightening in this context that this made us understand the central objective of AOE: the promotion of the learning of mathematical concepts, in a movement similar to that of its original production dynamics - the historical logic of the concept, as remembered by Moura, Sforni and Araújo (2011). In this perspective, it justifies the fact that it is one of the training actions that most enabled us to evidence the appropriation of the concept of measure by teachers in the learning space created by continuing education, during the collective development of teaching activities.

Thus, for a better explanation of the analysis and discussion of the data produced about the organization of the teaching of Mathematics in continuing education on screen, through the formative actions mediated by the process of reflection on the AOE, we chose to divide the unit of analysis on screen into three episodes: Episode 1 - qualitative leap in the teaching of Mathematics: teachers' meanings about the triggering situations of learning; Episode 2 - teachers' reflections revealing evidence of the appropriation of the concept of measure; Episode 3 - synthesis of the overcoming made possible by the continuing education of mathematics teachers.

Episode 1 - Qualitative leap in the teaching of Mathematics: teachers' meanings about the triggering situations of learning

We recognize and defend that, in the scenario of pedagogical activity, situations that trigger learning are proposed and developed as part of the AOE. Soaked by the assumptions of the Historical-Cultural Theory / Activity Theory, we observe that from the experience in the learning space created by the continuing education about this problematization format, culture appears as something to be appropriated by the subjects in learning activity and, as a result in addition, they internalize "[...] a set of knowledge that allows them to be among the subjects and exchange meanings with them, to be able to share knowledge so that together they can build new and better ways of living." (MOURA, 2007, p. 41).

In this perspective, we understand that the proposition of situations that trigger learning, according to the theoretical-methodological perspective assumed in this investigation, aims to make the subjects in teaching learning activity aware of the need for the mathematical concept to be taught (MORETTI, 2007).

That is why the problem situations are presented to the teacher and students as enabling the organization of teaching, unlike problem solving, "[...] in the sense of exercise or application to ensure the apprehension of a certain mathematical content." (RIBEIRO, 2011, p. 137). First, because the triggering situations of learning are applied and developed in the collective and they need the appropriation of concepts, as a social objective, either to the subject in learning activity, or to the subject in teaching activity. Second, because so understood, AOE becomes the theoretical-methodological principle seen as a training unit, whether for the teacher or for the students (MOURA, 1996, 2001, 2004, 2010; MOURA, SFORNI, ARAÚJO, 2011).

In this way, the analysis of the teachers' conversations, selected for this episode in the light of the reflections of the teachers participating in this study, show quite striking evidence of the transformation of the teaching organization to a new quality, by developing new meanings to problem situations, motivated by training continued from the perspective of Historical-Cultural Theory / Activity Theory.

As an illustrative example, we have the speech of Teacher T6, stating that, "[...] it was and is being a very important job in the case of 1st year children and when it comes to when we make the comparison as that the books present us as a model of how things work for us to follow [...] a work like that, which is much more meaningful and the student learns a lot. The teacher learns a lot more "(Videography / Formative Meeting). When criticizing the format of problem situations proposed in textbooks, devoid of meanings and senses for the student, this teacher becomes aware of the limits and possibilities of these problem situations. In fact, his speech provides foresight of the transformation of the teaching organization to a new quality in the context of its pedagogical activity.

When we focus on the learning movement of mathematical concepts in the scenario of continuing education, strictly speaking, Teacher T10's reflections meet the meanings developed by Teacher T6, complementing that, in the course of problem solving, the stages of formation must be highlighted concept and interdependence with its conceptual nexus. It is what emerges from the speech of this teacher: "[...] they reveal to us the stages of construction of the concept. [...] this is knowledge [...] Then suddenly we discover the nexuses. No the child is only required to give a mathematical name, a mathematical drawing but, mainly, we also discover that there are these stages in the construction [...]" (Video recording / Formative Meeting). The reflections of the present teachers are in line with Moretti's ideas (2007, p. 103, emphasis added) about this discussion. For this author, the problem situation, in the sense advocated in this study,

[...] it is not understood as an "application exercise" of concepts previously presented by the teacher. The problem-situation [...] presupposes a first approximation of the student with the concept involved in the proposed situation in such a way that, constituting the proposed situation as a problem for the learning subject, it is possible for him to appropriate the construction history of human thought.

Moretti is totally in tune with Leontiev's precepts when he explains that the problem-situation must be thought out, constituted in such a way as to enable the subject to the historical appropriation of human thought, that is, of scientific concepts. Under these conditions, this problem situation should have the status of "learning activity", so that, as Moura (1996) complements, it becomes a tool with potential for the

development of meanings and senses about the mathematical concepts worked, demanding, therefore, for the teacher and the student, the acquisition of skills that allow them to appropriate these concepts and, certainly, the teaching organization.

It is worth emphasizing, therefore, that among these skills, according to the analysis of Teacher T8's speech, based on his experience regarding the operationalization of training actions in the learning environment provided by continuing education, mediated by collective discussions, the game is presents as an enabler of the appropriation of mathematical concepts. In order to verify this result, we record the fragments of this teacher's speech: "[...] the game puts the child in front of a problem situation, the challenging game, in contrast to the extensive standard exercise lists in schools, provides children with possibility to know, act and modify the object of knowledge "[...] (Videography / Formative Meeting).

It is worth mentioning that, in the analytical process of the fragments of Teacher T8's speech, there is evidence that there has been a transformation of the teaching organization to a new quality about the teacher's understanding of the dynamics of problem solving involving the game. The awareness of this teacher that the game presents itself as a possibility of "knowing, acting and modifying the object of knowledge" can be exemplified as a major factor in the development of his mathematical theoretical-scientific thinking. Why do we say that? Because, certainly, for this teacher, the game as a methodological resource present in the triggering situation of learning, with objectives and intentionality, contributes to the organization of teaching, its transformation in the pedagogical activity and, of course, in the development of the mathematical theoretical-scientific thinking of the students. and teachers.

Given the importance of the game in the mathematics learning process, we understand that it is in the act of playing that where one can observe the movement of the appropriation of concepts. Liublinskaia (1973, p. 44) shares this understanding with us, stating that "[...] through action, language and feelings the child reflects reality. With its development, the relationships between the main processes of knowledge, your place and role in the game. "

In addition to these findings, the analysis of this episode also showed us the awareness of the training actions by the teachers, as understood in this study, which, in our opinion, contributed satisfactorily for them to enter into learning activity.

For example, Teacher T4 when socializing his experience about the "triggering situation of learning - length", elaborated by one of the subgroups of continuing education, under our supervision and guidance, in the dazzling of his reports it is possible to notice the awareness the formative action (or directed action) on the part of this teacher and collective, namely: [...] and in that very simple activity, we asked ourselves: 'My God, how is it? I've done it so many times and I didn't realize it (activity of measuring children's height with string) ... how did the children think about it? ' It was very remarkable for me, watching the children doing the activities. When we studied the measurement part, it was very striking for me, the children realized that their height is an attribute and can be extracted and represented on a string, it is not simple. That for me was quite an impact. If you don't look at the activity, you don't hear what the kids are saying, you don't notice these things. In early childhood education, their body is their measure, there is no separate measure (laughs) "(Reflective Session).

As we can see, Teacher T4, in addition to questioning and criticizing his pedagogical activity more closely before continuing education in focus, also produces meanings about his concrete students, as so

well explained in the report: "it was very striking for me, looking at children doing When we studied the measurement part, it was very striking for me, the children realized that their height is an attribute and can be extracted and represented on a string, it is not simple. That for me was a very big impact. does not look at the activity, does not listen to what the children are saying, does not perceive these things. "On the process evidenced in the analysis of Teacher T4's speech, with the purpose of presenting further clarifications, Sforni (2004, p. 132) brings us arguments that theoretically clarify this problem:

Action awareness is what allows the subject to dominate and move the activity. Domain because action, when conscious, moves to the level of conscious operations, allowing it to be automated and at the same time controlled by the subject. Mobility for being requested or modified according to the operational composition of a new action according to the conditions of its realization.

Thus, we can synthesize that the reflection process of becoming aware of the action (or directed / formative action) presents itself as one of the components of the functional structure of the activity with great potential for the development of consciousness and, consequently, of theoretical-scientific thinking mathematician (DAVIDOV, 1988a, 1988b).

Despite this synthesis, we affirm that the meanings produced, not only by Teacher T4, but by the other teacher participating in this study on the problem situation involving the concept of measurement, present themselves as revealing what Sforni (2004, p. 133) characterizes as the "[...] moment of 'taking possession' of the action itself, a fundamental and differentiating moment of learning scientific concepts in relation to the learning of spontaneous concepts.", positively affecting the transformation of the teaching organization in Mathematics to qualitative leap.

Episode 2 - Teachers' reflections revealing evidence of the appropriation of the concept of measure

In this episode, in particular, we have the purpose of identifying evidence of the appropriation of the concept of measure by the teachers, subjects of this research. To achieve this purpose, we consider the different relationships of these subjects with the object of knowledge and, thus, analyze and explain data produced from the transcripts of the filming of the formative meetings, without losing sight of the objective and the problem of this investigation.

In this context, it is important to justify that we refer to the concept of measurement, driven by the factors: relevance given to this concept by the participants of continuing education due to the need for theoretical and methodological deepening of it; awareness of research participants that the concept of measure, in fact, is the general basis for the process of appropriating the concept of number (DAVIDOV, 1982).

It was possible to verify, for example, in the statements recorded below, that teachers understand that the concept of measurement was elaborated from the concrete needs of men in the process of evolution of humanity: Teacher T2: "in this matter of time [...] then when he settled in, maybe the need for time arose. He was going to plant ... When will he be born? [...] How long will it take to get to a place, the shortest distance [...] a certain space of an object, organize the furniture, in short, you organize this space in a rational way. Cut a fabric without wasting "(Videography / Formative Encounter); and Teacher T1: "we

measure out of curiosity, because of the need for control, to make sense of what we do, to plan, so as not to waste time, energy and materials, to perceive the evolution" (Videography / Formative Meeting).

In this same episode, another revealing aspect of evidence of appropriation of the concept of measure that deserves to be highlighted, is the understanding on the part of some teachers that, although still in the condition of nomad, the measure was already presented as a necessity imposed on men. To prove our observation, as an example, we present the statements: Teacher T3: "[...] the measure of time, it exists even in nomadic peoples" (Videography / Formative Meeting); and Teacher T1: "exactly, even though I am a nomad, I think there is a need for guidance" (Videography / Formative Meeting). In this regard, Teacher T1 understands that the measure "[...] consists of a human need, so we thought about it, you will talk about the generic man, the history of humanity in this need for measurement. So, when you put it today you will reverse history "(Videography / Formative Meeting). However, we still observe that Teacher T2 is partially antagonistic about the understandings of teachers T1 and T3. In his view, the need for the measure arose only when man stopped being a nomad.

Regarding the meanings presented by teachers T1, T2 and T3 about the human need to measure, opportunely, we emphasize that, according to the study developed by Maldaner (2011) about the history of civilizations, since ancient times, men felt the need to discover ways of measuring objects, mainly due to the abandonment of nomadic life and the need to form communities, cities and kingdoms. In the study undertaken by Maldaner, the author presents the case of the Egyptians, who developed a complex system of measures as a result of the annual floods of the Nile River.

For corroborating these statements, the contributions of Silva (2004, p. 38, emphasis added) are valuable:

Primitive man did not need a very elaborate measurement system. Their metrological needs were certainly only for some rustic indications of positions, approximate distances and ratios of quantities such as 'greater than', and 'heavier than' or 'less than' and more 'lighter than'. However, from the moment it was necessary to cultivate the land or transfer the animals to more fertile pastures, there was also the need to communicate more conveniently in metrological terms, and it may have been at that moment that the first units of measurement appeared. And for ease, they were based on dimensions of the human body. Which took itself as the standard of measurement.

Regarding the theoretical contributions of Silva (2004), it is pertinent to emphasize that the analysis we made of Teacher T3's speech, according to the record, "what I had thought was that some things are dimensions of the body itself, dimensions of an object [. ..]"(Videography / Formative Encounter), is in line with what was explained by the aforementioned author, that is, that in the action of measuring, initially, men used their own body parts as units of measurement.

Thus, still regarding the analysis of the data selected for this episode that deal with the reflections undertaken by the teachers on the appropriation of the concept of measure, we find evidence that, in this movement of appropriation, the study of the history of the concept is one of the guiding elements of the

work educational organization. Understood in this way, the concept's history enables the process of appropriating mathematical concepts. In this regard, as remembered by Lanner de Moura (2007, p. 73), in the theoretical perspective adopted in this investigation about the study of the history of the concept, it is understood "[...] not in the factual, chronological, fortuitous sense, but in its fundamental meaning: of man creating himself through the development of his conceptual rationality".

Still on this issue, we consider it pertinent to highlight that the history of the concept alone is not a satisfactory pedagogical instrument, since we also need to define as a methodological premise the unity between the logical and the historical, assuming that the logical is the abstract interpretation of history (KOPNIN, 1978).

Continuing the analytical process of the data selected for this episode, there were also evidences of the teacher's awareness that in the process of appropriating the concept of measure, initially, he needs to appropriate the culture brought by the students, or better, of the knowledge empirical results for them hitherto appropriate. To illustrate, we highlight Teacher T3's reflections: "[...] then she will bring with her the culture she has to situate that particular measure [...]" (Videography / Formative Meeting). These reflections corroborate the ideas of Vigotski (2008, 2009). For this theorist, the thought does not pass directly to the subjects, much less the empirical thought is automatically subsumed by the theoretician-scientific, since it is only "[...] in the dialogical-pedagogical walk that the two orders meet of concepts: everyday concepts are incorporated and surpassed by scientists." (GASPARIN, 2011, p. 115).

The analysis of the data also revealed that, in the movement of teaching learning provided by continuing education about the appropriation of the concept of measure, teachers faced the need to appropriate other concepts related to the measure - the conceptual nexus, defined by Lanner de Moura (2007, p. 73) as those concepts that constitute "[...] the concept in its complexity, and which are part of its creation dynamics [...]". Among others, we identified in the fragments of the speeches of the participating teachers: comparison (Teacher T1), quantity (Teacher T1), quality (Teacher T1, Teacher T4), greatness (Teacher T1, Teacher T2), measurement standard / measurement units (Teachers T2, T1 and T3) and tools / instruments (Teachers T1, T2 and T3).

Thus, we verified the meanings and importance given by the investigated teachers of the conceptual nexus with a view to learning the mathematical concepts. It is interesting to note that in this discussion, in the opinion of Teacher T1, only "[...] through measurement does man manage to numerically express the quality of a phenomenon or an object" (Videography / Formative Encounter) and that greatness is a number that expresses quality. In the same way, in Teacher T2's reflections, we observe a reflection that meets Teacher T1's reasoning: "[...] I can only talk about the measure when I also make the relationship with quality [...] I measure that that will satisfy my need in the relationship that I have established." (Video recording / Formative Meeting).

In this way, the teachers mentioned share, among other meanings, that quantity can only be measured when compared to quality and quality as the variation of greatness. That is why when we think about measurement we are thinking about its qualities with the object to be measured.

Understood in this way, it is pertinent to emphasize that the qualities of an object depend on the environment in which it is inserted, on the functions of the section of reality arbitrarily cut out (CARAÇA, 2010). As an illustrative example, if we compare the quantities of 1000 butterflies and 1 donkey just for

the discrete aspect, the number of butterflies is greater than that of a donkey, however, under the continuous aspect, when comparing the mass of the donkey with the mass of the 1000 butterflies, the mass of the donkey will be much greater.

Continuing the analysis, in excerpts from the statements of teachers T3: "[...] some people may have thought like 'we measure?', That is, what standard of measures we use and not what tools we use" (Videography / Formative Meeting); T1: "I think there is a distinction between a measurement unit and an instrument" (Videography / Formative Meeting); and T2: "with which units and with what instruments? Unit of measurement not necessarily" (Videography / Formative Encounter), it is observed the importance that these teachers give to the conceptual nexuses: standard of measures, units of measurement and tools / instruments, for example understand that there is a difference between these attributes.

Regarding this manifestation, Caraça (2010) explains that as a result of becoming aware of the need not to limit ourselves only to knowing whether a given object is greater or less than another, through perception, without instituting a standard measure, there is a need to we also know how much this object measures. That is why we carry out procedures, among others, the establishment of a measurement standard in the process of comparing quantities of the same species - unit of measurement so that we can answer the question: how many times? Thus, in the movement of appropriation of the concept of measurement, there is a need for a number that expresses the product of the comparison with the unit - the measure of greatness in relation to the established unit.

Episode 3 - Synthesis of the overcomes made possible by the continuing education of mathematics teachers

According to Bernardes (2012), teacher training with the structure of "activity", in the perspective advocated by Leontiev, combined with the methodological principle of AOE, in which the subjects experience the transformation of the way of organizing teaching, consequently, enables the these subjects involved overcoming their pedagogical activity. In the case of this research, the analysis of the data produced in the teaching learning space created by continuing education, at different moments of the formative meetings and the reflective session, meets the proposal of Bernardes (2012). Why do we score this? Because the experience in this project of continuing education has positively affected teachers both in the form and content of thought, as well as in professional and personal changes.

The observation and analysis of the records of the teachers' statements T4: "[...] the theoretical-methodological perspective used here contributed to me learning what I had not learned in my entire training process. It is not just learning, but how I had no idea how to teach a 1 year old child the combination 1 to 1, for example [...] My view of this teaching today is completely different [...] The process of appropriating the theory allows a better reading of the school routine, of the many contradictions that we experience [...] Furthermore, the constant incentive to study enables our motives and leads us to build precious meanings, those that make us wake up and remember how much our role as a trainer and teacher is important "(Reflective Session); and T3: "[...] in the construction of the collective, everyone learns, builds himself as a person who mediates knowledge in the classroom [...] So ... I feel like this ... I feel built in group [...] The group has the characteristic of humanizing [...] It helped us to be better in all subjects and

to look at the student in a different way, with all his needs, in all his discoveries "(Reflective Session), explicitly confront the proposition of Bernardes (2012) already announced.

In this perspective, they are made explicit in the reflections of Teacher T4 and T3, who, in addition to continuing education, modify the form and content of the thought of their thoughts have changed themselves. In this episode, we also observed that Teacher T8 corroborates the ideas of this teacher, stating that: "I had a lot of difficulty in Mathematics as a student and then as a teacher too, especially when I worked with 3rd and 4th years ... 3rd. and 4th grade. I had to study a lot to work on that content with the students. It didn't come out that easily, fluently. I had to study a lot. And I was very concerned that my students would really learn. Today that is much clearer in this question of the concept, to appropriate the mathematical concepts and throughout my practice, sometimes I taught some little secrets like that [...] It is the way we learned Mathematics, that difficulty that I felt as a student, I had this concern that my students would not feel [...]. " (Reflective Session).

Another point to be highlighted is that, when reflecting on continuing education, teachers T4, T3 and T8 share the same ideas regarding the possible professional and personal changes impacted by this training. In the analysis undertaken of the conversations of these teachers, it is evident that the training actions proposed by continuing education, mediated by the reflection process among the participants, positively affected the organization of the teaching of Mathematics, when they became aware of the importance of theory in the learning activity.

In this regard, we understand that it is the theory (or theoretical-scientific knowledge) that underlies the actions and operations of teachers in relation to the teaching organization, by mediating the process of raising awareness about the possibilities of learning, as well as the possibility of transforming praxis. in pedagogical activity (BERNARDES, 2012). Thus, as observed in the dynamics of the training actions proposed by continuing education, particularly in the AOE, the appropriation of a theory aimed at the development of consciousness, impacting on the formation of theoretical thinking, allows the subjects involved in the learning activity to develop meanings to his pedagogical activity, like Teacher T7, as recorded in the speech: "I learned a lot about mathematics, about the fundamentals, the concepts, the essence of the concept. Searching for this essence, in fact, I learned here and not during graduation. " (Reflective Session).

Regarding the reflection of Teacher T7 and the other collaborating teachers in this discussion, it is interesting to highlight that the collective organization of the training actions of the continuing education enabled these teachers to produce new meanings about the pedagogical activity, as a result of the actions being motivated. It is worth remembering that, due to the Historical-Cultural Theory / Activity Theory, the reasons depend on the development of actions (LEONTIEV, 1978, 2010). Referring to this problem, Moretti (2007, p. 120) brings us relevant considerations:

In this process, the new proposed actions aimed to objectify the new motive and, in this way, there is a coincidence between this and the objectified product. Thus, the organization of teaching, when reason and object coincide, becomes a teaching activity for the teacher. When producing new reasons in the course of the activity itself, the teacher also begins to attribute new meanings to it, which includes new

meanings to his actions, the instruments he chooses and the collective work process that created conditions for the perceived changes.

Thus, we share the thought of Moretti (2007), especially when stating that "[...] when producing new reasons in the course of his own activity, the teacher also starts to attribute new meanings to it, which includes new meanings to his actions, the instruments you choose and the collective work process ". Here we emphasize that, in the first months of research, next to the teaching learning space provided by continuing education, an aspect that called our attention was the fact that when we discussed the key concepts of the guiding theory of this study, with an emphasis on movement historical logical of the mathematical concepts, from the development of activities that trigger learning, according to the perspective of the AOE, the participation of the subjects of this research, became more and more questioning about the formation proposed by the project in focus. This brought overcoming the organization of mathematics teaching, due to the development of teachers' awareness, in particular, in the dimension of mathematical theoretical-scientific thinking, as seen in the analysis of previous episodes.

In this perspective, from the analysis of this last episode, we conclude, therefore, pointing out that the continued formation in the perspective of the Historical-Cultural Theory / Activity Theory positively affected the teachers, subjects of this research, by triggering professional and personal changes and new possibilities in the organization of mathematics teaching.

Final considerations

In view of the revealing evidence of the movement of the appropriation of mathematical concepts from the three episodes presented, we interpret that the continuing education in the perspective of the Historical-Cultural Theory / Activity Theory, had an impact on the organization of teaching in Mathematics of these teachers, as well as they produced and shared meanings.

First, because we observed that teachers during their experience with AOE, with emphasis on situations that trigger learning and from the perspective of logical-historical, gradually, in a dialectical movement, were identifying different elements of the problem situations, commonly worked in the classroom classroom, subsidized by formal logic. Thus, they acquire a level of awareness in the sense that as teachers we must create the need, as well as the objective and subjective reasons and conditions for students to enter into learning activity.

Second, because the referred continuing education consisted of several training actions, among others, the development of AOE, considering the historical logic of the concept and the triggering situations of learning, which through rules, conduct and responsibilities, including time, schedules, signs and instruments, collaborative forms of behavior and other variables. Under this meaning, the actions impacted the development of higher psychological functions of teachers, in particular, of reflection, awareness, theoretical-scientific thinking and teaching planning.

In view of these considerations, it is worth noting that continuing education in evidence has also provided us with qualitative leaps not only as a researcher in this formative space, but also as a mathematics teacher. As we developed meanings for our pedagogical activity, this mobilized us to act intentionally

towards the organization of teaching. On the other hand, when appropriating the assumptions of Historical-Cultural Theory / Activity Theory and the methodological principle of AOE, we devised a theoretical-methodological framework to analyze and explain the data produced for this research. In this movement, we were trained as researchers.

Thus, we believe that, among the contributions of this investigation, is the expansion of the theoretical field related to the pedagogical activity of the teacher who teaches Mathematics, especially in Basic Education about the appropriation of mathematical concepts. In other words, a pedagogical activity based on "[...] commitment to the opportunity for the teacher to be part of a movement for the appropriation and mobilization of theoretical and practical knowledge about teaching action" (MOURA, 2000, p. 173). That said, Although this research is restricted to a differentiated teacher training model - we hope that the issues addressed here may have an impact on the mobilization of other reflections on the concept appropriation process.

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