Investigating Elementary School Mathematics Teachers' Knowledge of Students about some Numbers

Zuya, Habila Elisha
Science Education Department,
School of Technology and Science Education, MAUTECH, Yola, Nigeria.
Email: ehzuya@yahoo.com; elishazuya2@gmail.com

Abstract

The study investigated the competence of elementary mathematics teachers' knowledge of students about some numbers. Studies have shown that there are common misconceptions that students exhibit in dealing with numbers. This study wanted to determine how competent elementary teachers are in identifying and dealing with such students' misconceptions. A sample of 30 teachers was randomly selected as participants in this study. The instrument for data collection was questionnaire designed for teachers to respond to through detail explanations in writing. Participants were presented with the problem situations and were expected to respond to the questions or tasks. The problem situations were a given problem and the hypothetical students' solutions of the given problem. The study revealed that while some teachers were competent in addressing students' misconceptions in the topic area, others had difficulties themselves in understanding the problem situations and the hypothetical students' solutions.

Key words: *Elementary school teachers, teachers' knowledge, students' misconceptions*

1. Introduction

A number of factors may affect the teaching of mathematics, but teachers play an important role in the teaching process. There is a common belief that if a mathematics teacher knows mathematics very well, then he/she is the best person to teach mathematics [1]. Teacher's knowledge has been the subject of intense research for long, and the range of knowledge that teachers draw upon everyday is vast [2]. [2] noted that this knowledge includes the knowledge of the content, students, curriculum, pedagogy and of psychology. According to [1], the components of mathematics teachers' knowledge are the knowledge of mathematics, the knowledge of mathematical representations, the knowledge of students' cognition and the knowledge of teaching and decision making. This study focused on teacher's knowledge of students' cognition.

Knowledge about students' misconceptions is one of the critical components of pedagogical content knowledge or knowledge of teaching mathematics. According to [3], an understanding of students' common misconceptions and effective methods to help students avoid them are important aspects of mathematical pedagogical content knowledge. Besides teaching in such a way as to help students avoid misconceptions, teachers must also have strategies for dealing with those that inevitably arise. Pedagogical content knowledge as defined by [4] is the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for teaching. This definition emphasizes three things- content, pedagogy and students – and the connections among them. [5] stressed knowledge of multiple ways of representing the content and student. This includes knowledge of students' typical preconceptions and misconceptions, and strategies for helping students recognize their understanding.

[6] said that to be a teacher requires extensive and highly organized bodies of knowledge. [7] shared the same view, as he said the single factor which seems to have the greatest power to carry forward our understanding of the role of the teacher is the phenomenon of teachers' knowledge. According to [6], pedagogical content knowledge is the knowledge of effective teaching, which includes knowledge of content, knowledge of curriculum and knowledge of teaching. They explained further, that knowledge of content consists of broad mathematics knowledge as well as specific mathematics knowledge at the grade level being taught. Knowledge of curriculum includes selecting and using suitable curriculum materials, fully understanding the goals and key ideas of textbooks and curricula.

1.1 Conceptual Framework

[6] presented a model for the network of pedagogical content knowledge, (see Figure 1). The network shows that knowledge of teaching can be enhanced by content and curriculum knowledge. In this network, three types of knowledge- content, curriculum, knowing students' thinking- interact with each other and are able to transform from one form to another around the central task of teaching. They maintained that teaching can be seen as either a divergent or a convergent process. A divergent process of teaching is one that is based on content and curriculum knowledge, but is without focus and ignores students' mathematical thinking [6]. On the other hand, a convergent process of teaching focuses on knowing students' thinking, which consists of four aspects: building on students' mathematical ideas, addressing students' misconceptions, engaging students in mathematics learning, and promoting students' thinking mathematically [6]. They pointed out that teachers are often satisfied with students' knowing or remembering facts and skills but are not aware of students' thinking or misconceptions about mathematics.

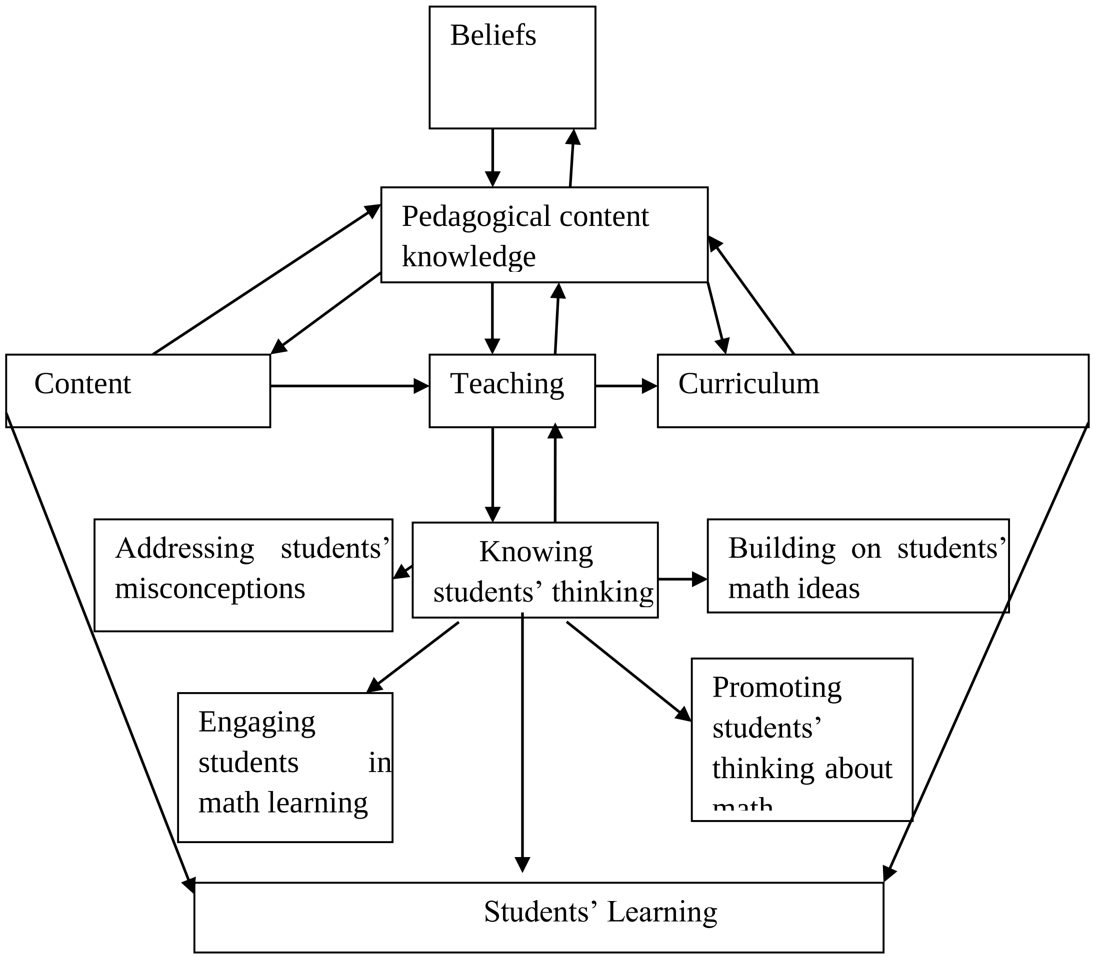

Figure 1: Network of PCK (Source: An, Kulm and Wu, 2004)

[6] maintained that in convergent process of teaching, the teacher does not only focus on conceptual understanding and procedural development, making sure that students comprehend and are able to apply the concepts and skills, but also consistently inquires about students' thinking. This study focused on one of the components of teacher's knowledge-knowledge of students' thinking. According to the conceptual network, knowing students' thinking has four aspects, and the study focused on one of the four aspects - addressing students' misconceptions. The mathematics teachers' competence in addressing students' misconceptions in some number concepts was focused upon. Specifically, the ability of teachers to predict, identify, discuss and suggest strategies in dealing with students' misconceptions associated with arithmetic operations (subtraction and division) and zero was investigated. Mathematics teachers are by their training expected to identify and discuss students' misconceptions and errors. Therefore, the study is of significance, as number concept is indispensible in learning mathematics.

1.2 Students' Misconceptions in Numbers

Students exhibit various misconceptions that are documented in the literature [8]. The addition operation, according to [8] is the one that students have least difficulty with. [8] pointed out that misconceptions that are associated with addition operation, indicate students' difficulty with the concept of place value.

In subtraction, [9] pointed out that the students' misconceptions can be summed up as 'Smaller-from-Larger' or inadequate knowledge of subtraction by borrowing method. [9] refers to 'Smaller-from-Larger' as subtracting the smaller digit from the larger not taking their place value into consideration.

[8] and also asserted that division operation is the most problematic among the four arithmetic operations. The first is the belief that the divisor should always be smaller than the dividend [10]. [10] found that 80% of 30 children presented the solution: 15÷5=3 when they were asked the question:

"*Five pounds of trail mix was shared equally by fifteen friends. How many pounds of trail mix did each friend get?"*

In fact, what is more disturbing was that 27 of a sample of sixty-five elementary school teacher trainees gave the answer also as15÷5.

The source of this misconception lies clearly in the students' early encounters with division [11]. Such encounters are almost always in situations where a whole number has to be divided by one of its factors. Further research [12] suggests that students encounter very few instances where the divisor is greater than the dividend.

1.3 Misconceptions associated with zero

Students have many misconceptions when they encounter zero in arithmetic operations [8]. Perhaps the most common is the problem that students have in 'borrowing' from zero in the process of subtraction. The use of zero in multiplication and division is also the source of a large number of mistakes and misconceptions among students of all ages [11]. One of the most common mistakes involving zero is the failure by many students to realize that multiplying any number by zero yields zero. [11] found that students had misconceptions associated with zero. For instance, $9 \times 0 \times 8 = 72$. This failure stems probably from the difficulty that many students have in interpreting a multiplication by zero. For many, zero represents nothing. As such, a multiplication by zero just leaves the number unchanged. Often students are confused when trying to decide whether to write or omit zero. They are often told that zero at the end of a decimal number has no value and therefore can be omitted without changing the number. Thus

36.40 is identical to 36.4

Similarly, when dividing 1632 by 8, students are taught not to write the ‘0’ that 3 divided by 8 would yield and divide 16 by 8 instead. As a result, many students become confused and are unable to determine exactly when should zero be written and when it should be omitted. Thus, students often make mistakes of the type:

$$\begin{array}{r|l} & 48 \\ \hline 4 & 1632 \end{array}$$

It is not difficult to see the rationale behind the result [8]. Since 4 does not divide into 3, students often move on to divide 4 into 32 to obtain 8. Thus the zero that should have been written between 4 and 8 is omitted, resulting in 48 instead of 408. In this study the competence of teachers’ was investigated in relation to predicting, identifying and discussing students’ misconceptions in hypothetical students’ solutions.

1.4 Statement of the Problem

The problem for this study was to find out whether elementary school mathematics teachers are competent enough to address students’ misconceptions associated with some arithmetic operations and zero.

1.5 Purpose of the Study

The main purpose of the study was to investigate elementary school mathematics teachers’ knowledge of students about some number concepts. Specifically, the study focused on elementary school mathematics teachers’ competence in addressing students’ misconceptions associated with arithmetic operations (subtraction and division) and zero (multiplication and division involving zero).

1.6 Research Questions

The following research questions guided the study:

1. How competent are elementary school mathematics teachers in predicting students’ misconceptions associated with subtraction?
2. How competent are elementary school mathematics teachers in identifying students’ misconceptions associated with division operation?
3. How competent are elementary school teachers in suggesting strategies that would help in addressing students’ misconceptions?
4. How competent are elementary school mathematics teachers in discussing students’ misconceptions associated with zero?

2. Methodology

2.1 Research Design

This study employed the qualitative approach in examining the respondents’ answers to the questions asked. [13] pointed out that qualitative research is mainly concerned with process and not product. And that it is better able to identify the problems experienced by respondents in answering questions. The data collected were qualitative in nature, as they were the explanations respondents gave, though in writing, in response to students’ misconceptions.

2.2 Participants

The participants to the study, who were 45 in number, were elementary school mathematics teachers teaching in both public and private schools. The participants were randomly selected through simple random sampling from 2 public and 3 private schools, and were holders of Nigeria Certificate in Education (N.C.E) with at least three years of teaching experience. Nigeria Certificate in Education (N.C.E.) is the minimum qualification for teaching profession in Nigeria. Holders of this certificate undergo three year training; taking courses in Mathematics, pedagogy, educational psychology and participating in practical teaching. Hence, these participants were suitable for the study.

2.3 Instrument for the Study

Four questionnaires were designed to explore how teachers address students' misconceptions associated with some number concepts. These items were the ones reported in the literature as the most common misconceptions of students. Each questionnaire showed work from a hypothetical student, and requested teachers to indicate their responses with respect to the question asked. The questionnaires relating to misconceptions associated with arithmetic operations (subtraction and division) had 2 items each and are shown in Table 1. In the subtraction questionnaire, the hypothetical student assumed that subtraction is commutative, and subtracted 3 from 7 because 3 was the smaller digit. The concept of smaller-from-larger was assumed not considering the positions of the digits. The participants were asked to predict the student's assumption or thinking. In the division questionnaire, the student had misconceptions that divisor should always be smaller than the dividend. The participants were requested to identify the source of the misconception. In both questionnaires (1 & 2) they were asked to suggest strategies that would help in removing the misconceptions.

Two questionnaires pertained to misconceptions associated with the concept of zero, and both had 1 item each, see Table 2. In the questionnaire relating to multiplication involving zero, the student ignored the zero. The participants were requested to discuss the student's thinking. For the division questionnaire, the hypothetical student omitted the zero that should have been written between 2 and 4, and wrote 24 instead of 204. The participants were requested to discuss the student's solution.

Table 1 Questionnaires on Misconceptions associated with arithmetic operations

Problem 1

The following is a student's solution:

$$\begin{array}{r} 543 \\ -237 \\ \hline 314 \end{array}$$

(a) Predict the student's thinking.
(b) How would you address this misconception?

Problem 2

Five kilograms of flour was shared equally among ten friends.
How many kilograms of flour did each friend get?
The student's solution: $10 \div 5 = 2$

(a) Identify the source of this misconception.
(b) Suggest strategies to deal with this misconception.

Table 2 Questionnaires on Misconceptions associated with zero

Problem 3

A student was asked to multiply $9 \times 0 \times 8$.

The student's solution: $9 \times 0 \times 8 = 72$.

(a) Discuss the student's thinking.

Problem 4

A student was given the problem $1632 \div 8$.

This is the student's working: $\begin{array}{r} 24 \\ 8\overline{)1632} \end{array}$

(a) Discuss the student's solution.

3. Results

3.1 Teachers' predictions of student's misconceptions about subtraction

Majority of the teachers predicted that the student thought smaller number is always subtracted from bigger number. This prediction was made by 20 (66%) out of the 30 teachers that responded to the question. Four of the teachers predicted that the student thought subtraction is commutative. These teachers showed competence in their prediction, which is ability to understand students' thinking process. However, the remaining 6 teachers had the following as their responses: "*The student thinks mathematically, but not attentive to teaching*"; "*The correct answer is 206*" and "*The student thinks you subtract highest from lowest, he subtracted 7 from 3 which is wrong*". This last group of teachers displayed not only incompetence, but lack of understanding the problem situation and/or the question they were asked to respond to.

3.2 Teachers' identification of student's misconception about division

More than half of the teachers (18 (60%) of them) identified the following as the source of the misconceptions:

Arrangement of numbers, divide bigger by smaller
Not told about decimal point in division, that is why he divided number of men by flour
The larger number is always the dividend and the smaller always the divisor
The answer must be whole, that is the dividend must be divided by one of its factors
Lack of understanding word problem

These responses revealed the teachers' competence in identifying the student's source of misconceptions about the division operation.

Eleven of the teachers had the following as their responses to the item 'Identify the source of the student's misconception': "*The student should have divided 10 from 5kg of flour (5000)*" and "*The student did not think very well before solving, he had confidence that the problem was a simple one*". This group of teachers did not only show incompetence in identifying the source of the student's misconception in the division problem, but their responses revealed having difficulty themselves with the problem situation. One of the teachers said," *The student supposed to divide 5kg by number of friends to get 5/10 = 1/2kg, but he divided number of friends by mass of flour to get 10/5 = 2*". This teacher avoided what he was asked to do- to identify source of misconception not to solve the problem or to say how the problem would be solved.

3.3 Teachers' strategies in dealing with students' misconceptions in subtraction and division

Besides the request to predict and identify students' misconception in subtraction and division operations, the teachers were also requested to suggest strategies that would help in avoiding or removing the misconceptions. Regarding the subtraction problem, majority of the teachers suggested that the concept of '*place value'* and '*subtraction by borrowing'* be addressed. For those who predicted that the student thought subtraction is commutative, suggested the use of number line to illustrate that subtraction is not commutative. These are indicative of the teachers' competence in suggesting strategies that would address the misconceptions. The rest of the teachers could not make useful suggestions that would deal with the misconceptions. For instance, '*the student should be given closer attention'; 'the student should be corrected to do the proper way of subtraction'; 'the student should be taught to borrow 1 from* 4' were their suggestions.

On the division problem, 15 teachers out of the 18 that were competent in identifying the sources of the student's misconception were also able to suggest useful strategies that would help in removing or avoiding the misconception. For instance, they suggested the following;

Problem situations should be created such that not only whole numbers are obtained as answers from division problems, but fractions as well.

Students should be told and shown that answers to division problems are not always whole numbers

In word problems, students should be properly guided to be able to identify the dividend and the divisor

However, 14 teachers suggested strategies that are either not helpful to the student or even irrelevant in addressing the problem. For example, they made the following suggestions:

I will tell him that the answer is not correct and will write down the correct answer for him

I will ask him to use long division method

I will ask the student to divide 5kg by 10

The student should pay more attention in class

One teacher said the student can be helped by re-teaching the topic.

3.4 Teachers' responses to student's misconceptions associated with zero

The teachers were requested to discuss the student's thinking with respect to the student's solution of the problem involving zero. The following teachers' responses indicated that they understood the student's thinking and were competent in discussing the student's thinking. Nineteen of the teachers responded thus:

The student thinks that since it is zero, then he will only multiply 9 by 8 to get his answer, ignoring zero

The student thinks that since zero is nothing, so you multiply 9 by 8 to get the answer

Zero multiply by any number is zero. He is not aware of this.

The concept of addition in which zero does not affect the outcome was used by the student thinking that zero will not affect the result

The student assumed that multiplication by zero leaves the answer unchanged as multiplication by 1.

The responses of the remaining 11 teachers revealed not only their incompetence in discussing the student's thinking, but their lack of understanding of the problem. For instance, their responses were: "*The student thought that the multiplicative property of a number greater than zero is stronger than that of zero"* and *"He does not think critically before solving mathematics problems*". These statements are symptomatic of the serious difficulties the teachers have themselves in understanding the problem.

3.5 Teachers' responses to the student's misconception involving zero in division

The teachers were expected to discuss the student's solution. Of the 30 teachers that responded, only 2 (6.6%) teachers showed some understanding of the student's solution. They responded by saying, "*Since students are told that zero at the beginning of a number is nothing, the student assumed that the zero resulting from 3 divided by 8 can be ignored or omitted*".

Seventeen of the teachers had the following responses:

The division was to start from the right hand side instead of the left hand side

The student should have started from units, hundred, etc

The student used short division method to solve the problem, he should have used the long division method

The student did not understand the concept of long division

The student did not respect the rule of division

These responses revealed clearly that the teachers themselves did not understand the problem situation and the question they were asked to respond to. The problem was solved using long division method, but the teachers said the student used short division method.

Eleven of the teachers responses were: *"The student divided 16 by 8 to get 2, and 32 by 8 to get 4"*, *"The student divided 16 by 8 and 32 by 8 giving him 2 and 4, which he brings together"* and *"The student thought the division of 4 digits number is in twos"*.

These responses did not point out or identify any error or misconception in the student's solution. This kind of responses revealed not only teachers' incompetence in discussing the student's solution, but also their inability to see the student's misconception in the solution.

4. Discussion

4.1 Teachers' responses to students' misconception associated with arithmetic operations

One important finding of the study was the competence of the teachers in predicting student's thinking process in the subtraction operation. Twenty four (80%) of the teachers were able to predict some of the most common students' misconceptions reported in the literature. Twenty (66%) of the teachers predicted among others the following as the student's assumption: *the student thinks the smaller number is always subtracted from the bigger number*. [9] reported that students have the tendency of taking away the smaller number from the bigger number, irrespective of the position of the digits. The assumption of commutability was predicted by four (13%) of the teachers; and this had been reported by [8]. This implies that this category of teachers can engage students in meaningful learning of mathematics and also build students' mathematics concepts [6]

However, the study also revealed that six 6 (20%) of the sampled teachers were incompetent in predicting the student's thinking. In fact, their responses not only revealed their lack of competence, but difficulty in understanding the student's solution. This has negative implication in teaching and education in general. It implies that this category of teachers cannot teach, not even to talk of being effective in teaching. Hence, the issue of seeing teaching as a convergent process explained in the network of pedagogical content knowledge by [6] is far from being a reality. In fact, even the divergent process of teaching which is said to be based on just knowing and not understanding cannot be achieved. It is worrisome to note responses such as, "*The correct answer is 206*" or "*The student thinks mathematically, but not attentive to teaching*" as responses to the item '*Predict the student's thinking'*. Even if the teachers were requested to write the correct answer, 206 is not the correct.

On the identification of the sources of misconception associated with the division operation, it was revealed that18 (60%) of the teachers were competent in identifying some of the sources of the misconception. These teachers identified sources that included the common sources in the literature. For instance, "*The larger number is always divided by the smaller*" or "*The answer must be whole, the dividend is always divided by one of its factors*". The ability of teachers to identify students' sources of misconception is necessary in addressing students' misconceptions. The sources identified by the teachers were reported by [10][11]. They also pointed out that students encounter very few instances where the divisor is bigger than the dividend, hence the tendency to have such misconception.

However, another important finding of the study is the inability of 11 (37%) of the teachers to identify any source of the student's misconception. These teachers were not only incompetent, but also had difficulties themselves understanding the problem. This has serious consequences for the teaching and learning of mathematics. This is indicative of deficiency not only in the teacher's knowledge of the student, but also in the content and curriculum knowledge. [6] pointed out in their model that the knowledge of content, curriculum and knowing student's thinking interact among themselves to enhance teaching. These teachers could not exhibit any of this knowledge. [10] reported that 27 (42%) of a sample of 65 elementary teachers were unable to solve similar problem. What is very disturbing is the response: *The student should have divided 10 from 5kg of flour (5000).* A number is divided by another, and not from another. And the 5000 in parenthesis is assumed to be the result of the division. Needless to say that such teachers are not supposed to be in the classroom.

4.2 Teachers' responses to misconceptions with zero

With regard to the multiplication problem involving zero, 19 (63%) teachers were competent in knowing the student's thinking. Most of these teachers' responses were among the common ones reported in previous studies. For instance, responses such as: *The student thinks since it is zero, he will multiply 9 by 8 only to get his answer*; *The student assumes that multiplication by zero leaves the answer unchanged*, are said to be most common misconceptions associated with zero [11][8]. In fact, [8] reported that one of the most common misconceptions involving zero is the failure by many students to realize that multiplying any number by zero yields zero. These teachers could read the students' mind, indicative of the understanding of students' thinking. [3] said teacher's understanding of common student's misconception is an important aspect of mathematical pedagogical content knowledge. The remaining 11 (39%) of the teachers could not discuss the student's thinking, as their responses revealed lack of understanding of the problem by the teachers themselves. This again has negative implication on the teaching and learning of mathematics.

On the division problem involving zero, only 2 (7%) of the teachers made statements that revealed their competence in knowing student's thinking. The statement: *Since students are told that zero at the beginning of a number can be ignored, the student assumed that the zero resulting from 3 divided by 8 can be ignored or omitted*, indicated the teachers' knowledge of students' thinking. And the teacher's knowledge of student is necessary in dealing with students' misconceptions. Unfortunately, the rest were incompetent in discussing the student's solution. It was earlier noted in the literature that of the four arithmetic operations, division is the one that is most challenging. The inability of the majority of the teachers in this study to discuss the student's solution indicates that teachers could be one of the sources of students' misconceptions. Some of the responses of the teachers that are quite disturbing included: *The division was to start from the right hand side instead of the left hand side; the student should have started from units, hundred, etc; the student used short division method to solve the problem, he should have used the long division method*. These statements are suggestive of the teachers' inadequate mathematics pedagogical content knowledge. These teachers could not differentiate the long division method from the other methods of division at this level.

4.3 Teachers' differences in pedagogical content knowledge

The teachers were requested to suggest strategies that could deal with the misconceptions associated with the subtraction and division problems in questionnaires 1 and 2. Half of the teachers offered some useful suggestions with respect to the subtraction problem. They suggested adequate handling of the concept of *place value* and *subtraction by borrowing* as strategies for removing the misconceptions. They also suggested the use of number line to illustrate the non-commutability of subtraction. These teachers displayed some evidence of pedagogical content knowledge, and may see teaching as a convergent process [6]. On the other hand, the remaining half was not able to suggest strategies that would be helpful to the student in removing the misconception. They were not competent in predicting the student's thinking in the subtraction problem, hence their failure to suggest strategies aimed at dealing with the misconception. Their problem was not only the knowledge of students' misconception, but also the content knowledge as well.

Similarly, about half of the teachers who were able to identify various sources of students' misconceptions in division made suggestions that are useful in addressing students' misconceptions involving division operations. Their suggestions indicate that such teachers can engage students in mathematics learning, an aspect of knowing students' thinking. However, the remaining half had difficulties in suggesting useful strategies as they were unable to identify the sources of the misconception. The responses of the teachers to some extent revealed differences in their pedagogical content knowledge. While pedagogical content knowledge difference between those who were competent and those who were incompetent was obvious from their responses, still among the competent teachers there were some differences in their pedagogical content knowledge. For instance, while some suggested that problem situations be given where answers to division problems involve both whole and fractions, others simply suggested re-teaching the student.

5. Conclusion

The study revealed that while some elementary mathematics teachers are competent in addressing their students' misconceptions, others are not only incompetent but lack the subject matter knowledge. This situation has serious implications for the teaching and learning of mathematics, especially at this level of education. One such implication is that students would not be guided or engaged in meaningful learning of mathematics. Students learning under such teachers are likely to develop negative attitude mathematics instead of the required positive attitude. Institutions responsible for the training of this category of teachers should look inward with a view to having teachers that are competent or adequate in terms of their pedagogical content knowledge.

6. Reference

[1]Turnuklu, B. E. & Yelsidere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service Primary mathematics teachers' perspectives in Turkey. *IUPST: The Journal*, vol 1. Retrieved from www.k-12prep.math.ttu.edu

[2] Chick, H. L.,& Baker, M. K. (2005). Investigating teachers' responses to students' misconceptions, In H. L. Chick & J. L. Vincent (Eds.) *Proceedings of the 29th Conference of the International Group for the Psychology of mathematics Education, vol. 2, pp. 249*-256. Melbourne: PME.

[3] Graeber, A. O. (1999). Forms of knowing mathematics: What pre-service teachers should learn. *Educational Studies in Mathematics*, *38*, 189-208.

[4] Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57 (1), 1-22

[5] Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.
[6] An, S., Kulm, G. & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teacher in China and the U. S. *Journal of Mathematics Teacher Education*, 7, 145-172.
[7] Elbaz, F. (1983). *Teacher thinking: A study of practical knowledge*. London Croom Helm. Retrieved from www.getcited.org/pub/102217366.
[8] Sadi, A. (2007). Misconceptions in numbers. *UGRU Journal*, vol. 5, 1-7
[9] Resnick, L.B. (1982). *Syntax and semantics in learning to subtract*. In T.P. Carpenter (ed) Addition and Subtraction: A cognitive perspective, New Jersey: Laurence Erlhaum.
[10] Graeber, A. O.& Baker, K. M.(1992). Little into Big is the Way it always is, *Arithmetic Teacher*.
[11] Graeber, A.O. & Campbell, P.F. (1993). Misconceptions about Multiplication and Division, Arithmetic Teacher,408-411.
[12] Graeber, A. O.& Baker, K. M.(1988). Curriculum Materials and misconceptions concerning multiplication and division with decimals. *Sixth Congress of Mathematics Education,Budapest, Hungary*.
[13] Woods, P. (2006). *Qualitative research*. Faculty of Education, University of Plymouth.