Didactic Sequence for Teaching Exponential Function

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ABSTRACT

This paper presents a methodological proposal for the teaching of exponential function, resulting from the application of a didactic sequence involving exponential function, where evidence of learning and the consolidation and application of mathematical concepts in problem solving were identified and analyzed. The Didactic Engineering of Michèle Artigue (1988) was used as a research methodology. As theoretical contributions that guided and enabled the development of the research, we chose the use of Mathematical Investigation in the classroom; Didactic Sequence in the conception of Zabala (1999); the Articulated Units of Conceptual Reconstruction proposed by Cabral (2017) and assumptions of Vygotsky's theory. A didactic sequence composed of five UARC's was elaborated to work the exponential function, with a view to *minimizing the difficulties naturally imposed by the content to be explained. Microgenetic analysis of verbal interactions between teacher and students was used to analyze the results of the application. The results show that the students participating in the experiment showed evidence of learning, recorded during the process, and began to have a good understanding of the concepts and properties related to the topic, in addition to a good performance in carrying out the activities, facts that corroborate the potential of the didactic sequence proposed herein.*

Keywords: Math. Teaching. Didactic Sequence. Exponential Function.

1.Introduction

Mathematics is a science, a language, a set of knowledge built thanks to the collective effort that man has been building throughout his history. Since ancient times, human beings have used their capacity for analysis and creativity to modify the environment in which they live and invent the tools that make it possible to satisfy their needs. It also allowed him to reason about natural, social and thought phenomena, which, together with everyday practice, led him to structure various sciences.

In Brazil, the main documents that guide Basic Education are the Law of Guidelines and Bases for National Education, Law No. 9394/1996, the National Education Plan and the National Curriculum Parameters (PCN).

Andrade (2013) emphasizes that the reasons why mathematics are taught and the consequent need for learning it is due to the fact that it is extremely present in their daily lives. Furthermore, as part of an increasingly developed and competitive society, we must add that some outstanding professions generally require mathematical knowledge and reasoning. However, when it comes to teaching mathematics, especially in Basic Education, there are several recurrent problems.

The low indexes in the indicators of public education in Brazil, such as IDEB, SAEB and ENADE, the low performance in assessments such as Prova Brasil and ENEM, and a high number of failures in the subject of Mathematics is seen with dissatisfaction by the school community, which in leads us to reflect and investigate how mathematics teaching is being conducted and what to do to reverse this situation.

For a long time, more precisely in the 1970s, a teaching trend called “technicist” emerged, where contents were presented as a “cake recipe”, that is, as programmed instruction. It was also characterized by a process based on the "transmission of knowledge", where the teacher, considered the "owner of knowledge" transmitted the content, usually summed up in the memorization of formulas and repetition of manipulative exercises, and the student just limited himself to listening and execute the instructions given. But could it be that during this process there was real learning on the part of the students?

Silva (2005) considers that in this teaching model, the student being only a passive subject in the classroom, leaves aside his ability to critically analyze a given situation, giving the idea that what matters are only calculations and procedures. routine. This is not to say that “calculate” type exercises should be removed, as they are essential in learning resolution techniques and properties. However, promoting a teaching of mathematics in which the memorization and manipulation of meaningless formulas in the context of the student is overvalued, and it is only up to this subject to "accept" what is taught, without any possibility of questioning or reflection, of course it is not the best path to quality teaching and learning.

Among the mathematics contents studied in the classroom, the study of functions is considered one of the most important, as such relevance is given by the fact that the concept of function establishes relationships with several other mathematical concepts and can be applied in the study of phenomena in several knowledge areas. Caraça (1989) is more emphatic when he says that the concept of function is established as a mathematical tool that helps man to understand the processes of influence and interdependence that are intrinsic to things and beings in our universe.

These approaches, according to Abrantes et.al (apud REBELO, 2011), can create confusion and learning difficulties, as students are faced with several terms around a single concept - that of function. Another problem that is observed is the way functions are presented in textbooks. Emphasis is usually given to its algebraic form, its manipulations and the mathematical rigor regarding the exaggeratedly formal language.

It is necessary to continue looking for new methodologies in such a way that students are the main protagonists of their learning, that is, that privilege their ability to build new knowledge and that this knowledge can be applied in their daily lives. The didactic sequences configure a current trend for teaching in basic education, especially in Mathematics. This type of approach allows the construction of knowledge, enabling experimentation, generalization, abstraction and formation of meanings.

The didactic sequence according to Perreti and Costa (2013) also allows for interdisciplinarity, as when dealing with a topic in the listed discipline, it may use specificities of others, allowing for the exploration of knowledge globally, reducing fragmentation. In addition to this positive point, during planning it is possible to determine the possibilities for interdisciplinary work during the desired time.

The Articulated Units of Conceptual Reconstruction - UARC correspond to a didactic sequence model proposed by Cabral (2017) to serve as a reference for the production of new didactic proposals with the objective of teaching curricular contents of the subject of mathematics in basic education. Even though it is a recent concept, many works have already been developed according to this idea, and presented positive results, with improvements in the learning process.

According to Cabral (2017), activities based on the Articulated Unit of Conceptual Reconstruction (UARC's) enable students to explore regularities and perceive, even if intuitively, the importance and usefulness of establishing generalizations, in addition to a more active participation in the teaching process. Due to the problems related to teaching and learning in mathematics, specifically with an exponential function, and considering methodologies that have shown positive results in teaching, it led us to the following question: What are the potentials of a didactic sequence created specifically for the teaching of exponential function a from problem solving and structured from the perspective of articulated units of conceptual reconstruction?

2. Material and Methods

2.1 Didactic Engineering

The concept of Didactic Engineering emerged in the area of Didactics of Mathematics (perspective of French didactics) in the early 1980s, first in 1982 by Yves Chevallard and Guy Brousseau, then in 1988 by Michèle Artigue.

It had as its foundation the concern with a certain "innovative ideology" present in the educational domain, which opens the way for experiences in the classroom. According to Artigue (1988), the expression "Didactic Engineering"' was created for didactic work compared to the work of an engineer who, to carry out a project, relies on scientific knowledge in his area, accepts to submit to a control of scientific nature, but at the same time, it is forced to work on objects more complex than the debugged objects of science. Therefore, it consists of facing, with all the means at its disposal, problems that science does not want or cannot take into account.

Similarly, Carneiro (2005) emphasizes that:

> [...] the term "Didactic Engineering" is inspired by the work of the engineer, whose production requires solid, basic and essential scientific knowledge, but also requires facing practical problems for which there is no preliminary theory, at which point it is I need to build solutions (CARNEIRO, 2005, p. 89).

Didactic Engineering, treated as a research methodology, was primarily characterized by an experimental scheme based on "didactic achievements" in the classroom, that is, on the construction, realization, observation and analysis of teaching sessions. It is also characterized, according to Almouloud (2007) as experimental research by the record in which it is located and by the validation modes that are associated with the comparison between a priori analysis and a posteriori analysis.

Furthermore, Didactic Engineering is characterized as a particular way of organizing methodological procedures in research in Mathematics Education. From a conception that contemplates both the theoretical and the experimental dimensions, Machado (apud PAIS, 2009) states that:

> [...] Didactic Engineering manages to link the theoretical plan of rationality to the experimentation of educational practice. Thus, Didactic Engineering, as a methodology, enables a methodological systematization for the practical realization of research, taking into account the dependency relationships between theory and practice (MACHADO, apud PAIS, 2009, p. 99)

Didactic Engineering is a research methodology that was created with the purpose of analyzing didactic situations, which are the object of study of Didactics of Mathematics. At the same time, it is related to the movement to value the teacher's practical knowledge, with the conviction that theories developed outside the classroom are insufficient to capture the complexity of the system and to somehow influence teaching traditions (CARNEIRO, 2005). In this approach, the question is aimed at the possibility of acting rationally, based on mathematical and didactic knowledge, highlighting the importance of "didactic realization" in the classroom as a practice of investigation.

Didactic Engineering can also be divided into four methodological phases: Preliminary Analysis; Priori design and analysis; Experimentation; Posterior Analysis and Validation:

In the Preliminary Analyzes, the objectives are to identify the teaching and learning problems of the object of study, to delineate in a reasoned way the question(s), the hypotheses, the theoretical and methodological foundations of the research, that is, this is where it occurs the mathematical organization, the analysis of the didactic organization of the chosen object and the research question is defined.

Almouloud (2007) describes that the Preliminary Analysis phase can include the following aspects: Study of the Mathematical Organization; Analysis of the Didactic Organization of the Chosen Mathematical Object.

In the phase referring to Conception and Priori Analysis, the researcher must elaborate and analyze a sequence of problem-situations, in order to answer the question(s) and the hypotheses raised in the previous phase. According to Almouloud (2007), a problem situation is understood as:

> the choice of open and/or closed questions in a more or less mathematized situation, involving a field of problems posed in one or several domains of knowledge and knowledge. Its main function is the implicit use, and later explicit, of new mathematical objects, through questions posed by the students, at the time of problem solving (ALMOULOUD, 2007, p. 174).

Experimentation is the time to put into operation all the elaborated instruments, correcting them, if necessary, if the local analyzes of the experimental development identify this need, resulting in a return to the a priori analysis, for a process of complementation.

The last phase of Didactic Engineering, Analysis a Posteriori and Validation, is characterized by the set of data collected during the experimentation phase, that is, the observations made about the teaching sessions and the students' productions in the classroom or outside it. This information, according to Rozanski (2015), "contributes to the improvement of didactic knowledge regarding the conditions for the transmission of knowledge at stake". It is done in the light of a priori and preliminary analysis.

2.2 Following Teaching

The expression "Didactic Sequence" is frequently used in the field of Education to teach the most varied sciences. It corresponds to an ordered and sequential set of didactic activities designed with the purpose of teaching some knowledge of a field of knowledge. The main references on the study of didactic sequences are in the works of Zabala (1998), who more precisely defines a didactic sequence as being "a set of ordered, structured and articulated activities for the achievement of certain educational objectives, which have a principle and an end known to both teacher and student".

The use of didactic sequences in the teaching of Mathematics has gained prominence within Mathematics Education, as this resource, according to Batista, Oliveira and Rodrigues (2013) allows the teacher to make sense of the contents worked on in the classroom. Furthermore, with its use it is possible to achieve an investigative teaching, with problematization, organization of contents and application of knowledge.

2.3 Articulated Unit of Conceptual Reconstruction - UARC

Teaching math in a way different from what we were taught during basic education is undoubtedly a big challenge. Create an alternative that simultaneously distances students from the "traditional" class model (definition - example - exercise) and brings them closer to a dialogic discursive practice, promoting reflective verbal interactions, which somehow perceives the need, even if by intuition, and usefulness of establishing generalizations would be a great contribution to the teaching-learning process.

This conception, which serves as a structuring model for the elaboration of a didactic sequence, is called the Articulated Unit of Conceptual Reconstruction (UARC), proposed by Cabral (2017), who defines UARC as being “a set of empirical-intuitive arguments built through of structuring categories”, aiming to stimulate the reconstruction of a concept of mathematical knowledge. These categories will be explored later.

To reconstruct a given mathematical concept, it is necessary to elect a first UARC, which Cabral (2017) calls the Articulated Unit of Conceptual Reconstruction of the first generation (UARC-1). This is considered the starting point, which need not necessarily be a problem as is generally recommended. It is possible to start with a variety of positions within the concept that one wants to reconstruct, however, Cabral (2017) argues that:

> the choice of UARC-1 depends on a number of variables. The available time, the teacher's didactic and conceptual experience, the knowledge he has about what the students have mastered about certain prior knowledge, the learning objectives, etc. (CABRAL, 2017, p. 39)

In other words, no matter the amount of UARC's used, if they are correctly articulated, the student will reach the objective intended by the teacher. However, it is important to understand which elements or tools make up a UARC.

2.4 Microgenetic Analysis

Vygotsky was born in Belarus, in the city of Orsha, in 1986. He graduated in Law and studied Philosophy and History, but he dedicated a good part of his life to Psychology studies, even though he had no training in the area. Her studies were aimed at the psychological development of human beings, within the perspective of a dialectical materialist psychology, valuing the socio-historical factor in her theoretical-methodological constructions.

Among her main theories, the Historical-Cultural Theory, Vygotsky showed that the historical dimension, culture and social interaction are the main elements that influence mental development.

Vygotsky argues that culture becomes part of each person's nature, where psychological functions are a product of brain activity. About this, Coelho and Pisoni (2012) reinforce that:

> The child is born with only elementary functions and from the learning of culture, these functions are transformed into higher psychological functions, these being the conscious control of behavior, intentional action and the freedom of the individual in relation to the characteristics of the moment and the present space. (COELHO and PISONI, 2012)

In addition to the cultural historical process, the role of language in the mental development of human beings was also taken into account by Vygotsky, as according to Rabello and Passos (2010):

> its central issue is the acquisition of knowledge through the interaction of the subject with the environment. For the theorist, the subject is interactive, as it acquires knowledge from intra and interpersonal relationships and exchange with the environment [...] (RABELLO and PASSOS, 2010).

Such interactions between subject and the environment and their implications on the mental development of human beings led Vygotsky to elaborate the concept of mediation, which in his words, cited by Oliveira (2002), states that it corresponds to an intervention process of an intermediate element in a relationship, where such a relationship is no longer direct, and becomes mediated by this element.

Martins and Moser (2012) state that when the human brain learns a concept, it uses the mediation of words or language itself, that is, it is not possible to think without using words or images. Thus, the term language can be replaced by the term semiotic mediation. In this sense, considering the learning process, this is done through semiotic mediation or through interaction with the other, as words are used as a form of communication or interaction.

Vygotsky considers that a supposed new knowledge to be worked with a child must have as a starting point what he already knows, or what he believes has to do with the new that is being presented. In this case, learning must be matched with the child's developmental level.

An important methodological approach that has been dealt with in research in the areas of Education and Psychology, offered by Goés (2000), is called Microgenetic Analysis. According to the author:

> In general, it is a form of data construction that requires attention to detail and the clipping of interactive episodes, and the examination is oriented towards the functioning of the focal subjects, the intersubjective relationships and the social conditions of the situation, resulting in a detailed account of events. (GÓES, 2000, p. 9)

Generally, this type of approach is used to investigate dialogic interactions within the classroom environment, associated with the use of video recording, strategies for filming and transcription of interactive speeches (audio recordings), in order to identify genetic transcriptions as well. how to understand the steps of the actions of the subjects involved and explain their cognitive constructions, solutions and transformations.

Finally, Cabral (2004) reinforces that microgenetic analysis corresponds to a powerful methodological instrument for investigating the construction of knowledge when considering the encounter of subjects in teaching situations in the school environment, as the classroom is a stage for dialogic interactions that provide the teacher with an environment for pedagogical investigation. Thus, to carry out the didactic sequence, this tool was used to identify evidence of learning.

In this sense, the analysis of interactions in the construction of the Exponential Function concept has through the identification of interactive patterns provided by Microgenetic Analysis, enabling arguments to answer the central question of this research: What are the potentials of a didactic sequence created specifically for teaching of exponential function based on problem solving and structured from the perspective of articulated units of conceptual reconstruction?

2.5. Exponential Function

In the perspective of IEZZI (2004), let us consider the element $a \in \mathbb{R}+$, with $a \neq 1$. Then the function $f:\mathbb{R} \longrightarrow \mathbb{R}+$ defined by $f(x) = ax$ is called an exponential function of base a. It is important to emphasize that the restrictions $a > 0$ and $a \neq 1$ are necessary, because:

- For $a = 0$ and negative x, we would not have a function defined in $\mathbb{R}$, but rather an indeterminacy;
- For $a < 0$ and $x = ½$, for example, we would not have a function defined in $\mathbb{R}$, but in $\mathbb{C}$;
- For $a = 1$ and x any real number, we will have a constant function.

An exponential function is a type of function where the independent variable works as the exponent of a positive base. This function must be defined to have the following properties, for any $x, y \in \mathbb{R}$:

(i) $f(x + y) = f(x) \cdot f(y)$ or $ax \cdot ay = ax+y$

(ii) $f(1) = a^1 = a$

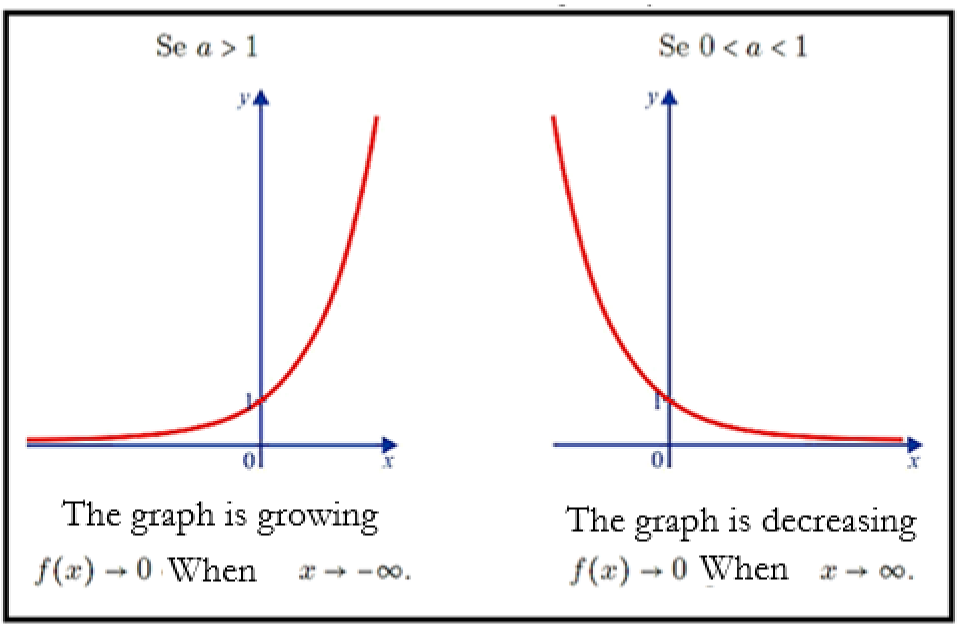
(iii) $x < y \Longrightarrow ax < ay$ $quando$ $a > 1$ e $x < y \Longrightarrow ay < ax$ $quando$ $0 < a < 1$.

The common characteristics of graphs of exponential functions in the form $f(x) = ax$ and $a \neq 1$ are:

- The graph is continuous.
- The domain is $(-\infty,+\infty)$ and the image-set is $(0,+\infty)$;
- It is not symmetric: it is neither an even function nor an odd function • Constrained inferiorly, but not superiorly.
- It has no local extremes;
- The graph intersects the ordinate axis at point (0.1) and does not intersect the abscissa axis at any point.

Below is figure 01:

FIGURE 01: Exponential Function Graph



Source: Stewart (2006)

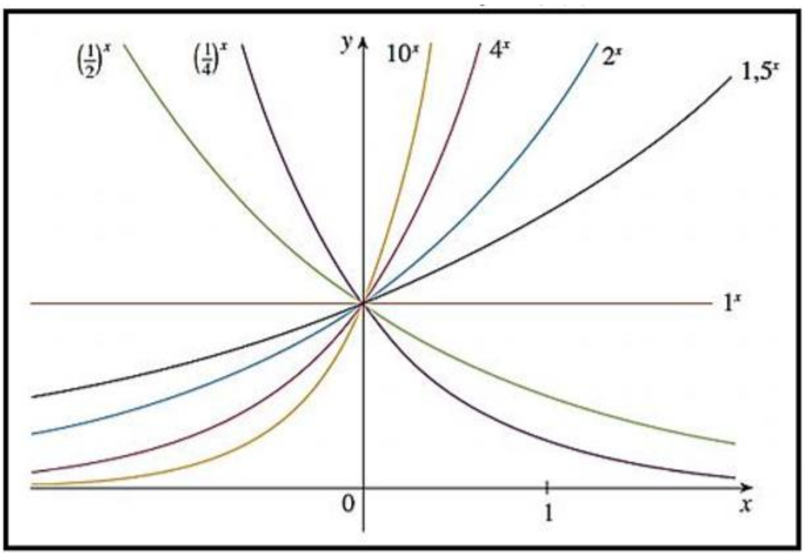
When $a > 1$, it is noted that, when da varies from left to right, the exponential curve $y = ax$ presents a slow growth while x is negative. As x grows, the growth of y becomes more and more accelerated. This is reflected in the slope of the tangent to the graph; for exceptionally large positive values of x, the tangent is almost vertical.

Likewise, when 0 < a < 1 , it is noted that when x varies from left to right, the exponential curve y = ax presents an accelerated decay while x is negative. As x grows, the decrease of y becomes slower and slower. This is reflected in the slope of the tangent to the graph; for very large positive values of x, the tangent is almost horizontal.

One of the most important characteristics of the exponential function is the fact that its graph approaches the abscissa axis (x axis) so that no intersection occurs. For a > 1, the function tends to zero when x decreases (or $x \longrightarrow \infty$). As for 0 < a < 1, the approximation with the x-axis occurs as x grows (that is, $x \longrightarrow \infty$). In this case, we can say that the x-axis (that is, the line y = 0) is a horizontal asymptote of the graph of the exponential function.

The graphs of the members of the family of functions $f(x)$ = ax are shown in Figure 02, below for various values of the base a. All these graphs pass through the same point (0.1), as mentioned above, because $a0$ = 1 for $a \neq 0$. We notice that the exponential function grows faster as a gets larger (for x > 0) and decreases faster as a gets smaller (for 0 < a < 1). As for a = 1, any real number x, $f(x)$ = 1 será will always be constant.

FIGURE 02: Family of Functions $f(x)$ = ax



Source: Stewart (2006)

3. Results and discussion

This didactic sequence in this investigation was composed of five UARC's, where each one deals with one or more topics related to the theme in question. UARC 1 and UARC 2 deal with the initial idea of the relationship between variables in exponential form, UARC 3 deals with the restriction of the base "a", UARC 4 deals with the graph and asymptote, and UARC 5 deals with some properties of this function such as as domain and image, growth and degrowth.

After each UARC, a formalizing intervention is carried out in order to materialize the ideas built in each moment. We chose to use the term "relationship" during the execution of the UARC's, as every relationship is a function, and the term "function" was addressed after the completion of the five UARC's, in the final formalizing intervention. Each one was developed in an attempt to overcome the difficulties pointed out in the preliminary analyses.

UARC 1: LEGO
TITLE: The Lego
OBJECTIVE: Discover a relationship between variables
MATERIALS: Building blocks, Pencil, pen, A4 paper
PROCEDURES: Read the text carefully and answer the proposed items.

The first meeting took place on April 16, 2018 (Monday), from 12:15 pm to 1:30 pm, where we explained to the participating students the procedures at that time, as well as the guidelines for the execution of the UARC's. The 12 students chosen were divided into three groups (Group A - 4 students; Group B - 4 students and Group C - 4 students).

Figure 03: Lego

Source: infoschool website

During the realization of UARC - 1, the students were provoked by the researcher teacher, through oral interventions, to respond to each of the structuring interventions, that is, the teacher read an item, the students from each team discussed among themselves, responded in a way verbal and then asked to write the answer on the activity sheet. These oral interventions are fundamental, as the student is involved in a kind of "discursive ping-pong", caused on the one hand by the structuring interventions that were materialized in written form in the didactic sequence and on the other hand, by these oral interventions, which help the teacher to modulate the students' approaches and distances in relation to the learning objectives (CABRAL, 2017).

This UARC was designed with the intention that students would observe, through interaction with the Lego toy, the dependence relationship between the number of floors and the number of formed towers. In some items, tables will be filled in so that the regularities of the data obtained can be perceived.

The UARC was also designed to overcome an obstacle observed in textbooks, which corresponds to not exploring the exponential relationship between the variables. The idea is for the student to notice the exponential growth in the number of towers as a function of the number of floors (blocks) used.

UARC 2 - MODELING MASS
TITLE: Benefits of Modeling Clay
OBJECTIVE: Discover a relationship between variables
MATERIALS: Pencil, pen, eraser and A4 paper
PROCEDURES: Read the text carefully and answer the proposed items.

This activity was also elaborated with the intention that the students would observe, through the interaction with the modeling clay, the dependence relation between the length of the dough and the number of removals.

The second meeting took place on April 17, 2018 (Tuesday), from 12:15 pm to 1:00 pm. The students organized themselves into the same groups and places that were established on the first day. Each student received the set of UARC activities - 2 and then a motivational reading on the benefits of modeling clay was done.

As in UARC 1, tables will be filled in some items in order to understand the regularities of the data obtained, in addition, it was also created to overcome the problem of working the relationship between the quantities observed in textbooks. The idea is for the student to notice the exponential decrease in the length of the modeling clay and the number of removals.

The idea was for them to note that since after each take-off, the length of the mass is reduced to half, so the given length could be written as the previous length multiplied by ½ , and that the initial length could be written as the value itself multiplied by 1. With this, students filled in a second box in intervention 15. Later, the key point of UARC 2 occurred when we were asked how we could write the initial length 160 as multiplication by ½ . The students analyzed and stated that from $160 \cdot 1$, the number one could be written as a power of zero exponent, leaving $160 \cdot (1/2)0$. In addition, the students were able to see that the other lengths could also be written as 160 multiplied by a base power ½

Finally, when asked if it was possible to write an expression that related the number of withdrawals n with the length of the mass C, following the regularity for n withdrawals, they concluded that the expression was of the form $C = 160 \cdot (1/2\ )n$.

At this UARC, we found that students did not have difficulties in interpreting the initial intervention. The first important point we observed was that the students were able to perceive that the removal of the parts influenced the length of the dough and that after each removal, the length of the dough was reduced to half.

UARC 3 - FILLING THE TABLE
TITLE: Filling in the Table
OBJECTIVE: Check existing and non-existent values for base "a"
MATERIALS: Pencil, pen, eraser, calculator
PROCEDURE: read the instructions below

The third meeting took place on April 18, 2018 (Wednesday) from 12:15 to 13:00. With the same groups formed, each student received the set of UARC 3 activities and were asked to use the calculator on their cell phones.

This UARC was created with the intention of working the restrictions of the base of the exponential function ($a > 0$ and $a \neq 1$). The table presents, in the first row, powers of rational, integer and natural basis, with exponent x, and in the first column, predefined values for the variable x. With the help of the calculator (if necessary), students should perform the calculations using the values of x for each power and fill in the entire table.

UARC 3 was created to overcome the difficulty of correctly justifying the reasons for these restrictions, because from the preliminary analysis of the analyzed textbooks, it was found that some do not justify such restrictions and, when it occurs, there is some misunderstanding, as observed in a of the books, where it is mentioned that for the base $a = 0$, there is a constant function for all $x \in \mathbb{R}$, with $x \neq 1$.

The central idea is that students can compare the data obtained and realize that for the negative base and zero base powers cannot be used for any value of x, and for base 1, any x used will always give a result of 1, not characterizing thus an exponential model, serving then only expressions greater than zero and different from 1, since for these, any value of x used will give a corresponding value.

UARC 4 - GRAPHIC CONSTRUCTION AND RECOGNITION

TITLE: Exponential relationship graph construction and recognition

OBJECTIVE: Explore the elements of the exponential relationship graph

MATERIALS: Pencil, pen, eraser, calculator, A4 paper, curve board

The fourth meeting took place on April 19, 2018 (Thursday) from 12:15 pm to 1:30 pm. Each student received the set of activities from UARC 4, which then explained the procedures to be done. We then started with the construction of graphs of $y = 2x$ and $y = (1/2)x$, with pre-established values for x. We leave the use of the calculator free to obtain y values, as well as guide the use of a decimal value of 0.5 for base ½ .

The students had no difficulty in calculating the y values for the established x values and when drawing the graph, some students used straight lines, as expected, others managed to draw a curve very close to the exponential curve.

This UARC was created with the intention of working the graphic construction of the exponential function and recognition of its sketch. First, we will work on the constructions of the graphs of the models $y = 2x$ and $y = (1/2\ )$, where for each example, tables with pre-established values for serão will be presented, starting with three numbers, -3, 0 and 3 , then with five numbers, -3, -2, 0, 2 and 3, and finally, with seven numbers, -3, -2, -1, 0, 1, 2 and 3. Later, the constructions will be worked. of the graphs of the models $y = 3x$ and $y = (1/3)x$.

UARC 4 was created to overcome the difficulty in understanding the graphic construction of the exponential relationship, because according to Oliveira (2006), when graphic representations are addressed in most textbooks, they usually appear without scale and without representation on appropriate paper , millimeter or grid.

It is also not proposed that the student build charts using these types of papers, and distortions may occur in the construction of scales, especially if they are not alerted by the teacher, thus constituting a didactic obstacle. The central idea of this activity is that students realize that the graph of the exponential relationship is a smooth curve, which does not intersect the abscissa axis.

UARC 5 - CHARACTERISTICS OF THE EXPONENTIAL RELATIONSHIP
TITLE: Characteristics of the Exponential Ratio
OBJECTIVE: To explore the elements of the exponential relationship $y = y$
MATERIALS: Pencil, eraser, A4 paper, calculator (if necessary)

This UARC was prepared with the intention of working with some characteristics of the exponential relationship, based on the constructions that were made in UARC 4. First, the domain will be worked, where students should check which values of x can be used. Then, the image will be worked, where, based on the items on the domain and on the constructed graphics, what values can obter be obtained.

The fifth and last meeting took place on April 20, 2018 from 12:15 to 13:30. Each group received the set of activities, and this UARC was carried out much more quickly, as the questions aimed to explore some elements of the exponential relationship based on the constructions of the graphs.

Asked about the values of x, the students realized that any value could be used. About the value of y, the students realized that it would always give a positive value, with no possibility of y being zero or negative.

Regarding the growth and decrease of the exponential relationship, the subjects were able to perceive that for the relationships $y = 2x$ and $y = 3x$, the values of y increase and for $y = (1/2)x$ and $y = (1/3)x$, the values of y decrease when we increase the values of x in both cases. In addition, they were able to relate growth and decay to the "a" base of the exponential relationship.

UARC 5 was created to overcome the difficulty of how these characteristics are approached, pointed out by Braz (2007) in the preliminary analyses, where the exponential function contents are treated without connection with other mathematical objects, thus, we think for this activity to work, at the same time, all these characteristics.

4. Final Considerations

The collected data and the survey of studies on the theme, exponential function, point out that the teaching process mostly occurs in a classic habitual way, that is, it essentially comprises object definition, examples and resolution of activities, most likely direct influence from the textbook adopted as the only methodological support reference. Such facts point to the need to present other didactic resources for the teaching of mathematics beyond the textbook, in our case, a didactic sequence.

This classic way of presenting mathematical contents, a fact that, according to the students, makes classes boring and meaningless when they are required to apply these contents in the resolution of contextualized activities. In addition, the survey points out that there is inadequate use of mathematical language, with an excess of symbols and without the proper concern to clarify meanings and terminologies, which contributes to the increase in difficulties related to learning the elements involved.

It was possible to observe that students have little or no basis to deal with mathematical content related to the exponential function and point their elementary difficulties to the process triggered by the teachers, that is, how they work the content in the classroom.

The elaboration of the didactic sequence was a unique process, considering the volume of theoretical and practical elements that was appropriated, in addition, it allowed a new view on the teaching and learning process of mathematics, modifying the pedagogical practice, thus showing the real importance of continuing the search for teaching methodologies that favor the teaching and learning processes.

During the application of the didactic sequence, we noticed a certain impact on the part of the students, as they were faced with an unusual situation, since they were used to the classic form of class - teacher, whiteboard and dialogued expository class, however, the each activity performed, it was observed that the students' enthusiasm and curiosity in relation to the procedures used grew, as well as the interest and acceptance of the teaching proposal that was being presented.

These interactions provided discussions that corroborated so that the objectives established in each UARC's were achieved and, consequently, the characterization of the learning signs. It was observed that students understood the proposed activities, promoted group discussions, and constantly interacted and contributed with colleagues who had greater difficulties until they also reached the goal established for each stage.

One of the objectives incorporated into the sequences of activities was the perception of constant regularities in the processes, a fact widely noticeable by all, as well as the prompt response to the teacher's interventions, responses supported by consistent arguments that led us to conclude that the students reached the expected level and, in some cases, far beyond what has been established.

We highlight the moment where it was observed that the students realized that one variable depends on the other, stating that towers cannot be formed without using the floors. This evidence characterized their perception of the existence of a dependency relationship between the variables involved.

In addition, there was a time when students use the term "function" in the answers to the questions made, although during most of the procedures the term "relationship" was used. This fact shows us that, most likely, the students understood the idea of function ahead of schedule.

With regard to graphical representations, it was evident that students were able to perceive the regularities associated with graphs of exponential functions. The situations mentioned demarcate strong evidence of learning, facts that lead us to conclude that the proposed didactic sequence has pedagogical potential for teaching exponential function and that it can contribute to the teaching and learning process of mathematical contents related to exponential function, that is, , improve the way of teaching and contribute to the improvement of student learning.

About the use of this didactic sequence of activities in the teaching of exponential function, it was found that with each UARC performed, the time spent by students in the development of activities decreased. In addition, the use of this resource proved to be favorable, as it enabled students to perceive regularities that culminated in the discoveries and constructions of concepts and properties. For this reason, we consider that our hypothesis about the potentiality of using didactic sequences for teaching this topic was confirmed and, therefore, the research question was answered.

Finally, as future developments of this work, the application of the didactic sequence now presented in other educational environments is recommended, with a view to producing new assessments that may corroborate those described here, expand the field of applications, and contribute to the improvement of this proposal.

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