Proof through an equilateral hyperbola to find the infinite Pythagorean triples and a direct proof to find the sum of all odd numbers through a telescopic series.

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Abstract

This short article aims to demonstrate an infinite family of Pythagorean triples through a direct proof related to an equilateral hyperbola and the proof of the sum of odd numbers through a telescopic series discovered by this present author.

DEMONSTRATION OF THE RELATIONSHIP OF THE INFINITE TERNAS PYTHAGORICS WITH HYPERBOLE

Theorem 1. Every perfect square number can be represented by the equality 1 since $n \in \mathbb{Z}$.

1. $$n^2 = (2n-1) + (n-1)^2$$

Direct proof of theorem (I)

$$n^2 = (2n-1) + (n-1)^2$$
$$n^2 = 2n - 1 + n^2 - 2n + 1$$

Removing the equivalent terms on both sides of the equation, we have:

$$n^2 = n^2$$

Then,

$$n^2 = (2n-1) + (n-1)^2 \text{ QED.}$$

Theorem 2. Every perfect square number like x^2 can be represented by a Pythagorean triples, since x is odd or since $x = k + 1, k \in \mathbb{Z}$.

2.1

$$\left(\frac{x^2+1}{2}\right)^2 = x^2 + \left(\frac{x^2-1}{2}\right)^2$$

Theorem 3. Every perfect square number like x^2 can be represented by a Pythagorean triples, since $x \in \mathbb{Z}$.

3.1 $$(x^2+1)^2 = 4x^2 + (x^2-1)^2$$

Proof of theorem (3)

If we have a perfect square, number like that,

$$n^2 = (2n-1) + (n-1)^2$$

It follows that,

$$n = \frac{x^2+1}{2}, 2n-1 = x^2 \ \ and \ \ n-1 = \left(\frac{x^2-1}{2}\right)^2$$

Implies that,

$$n^2 = (2n-1) + (n-1)^2$$

So, It can be rewritten, as

$$\left(\frac{x^2+1}{2}\right)^2 = x^2 + \left(\frac{x^2-1}{2}\right)^2$$

Corollary 3.1.1 Every perfect square number as x^2 can be represented by a Pythagorean tender, though the equation 3.2 since x ∈ ℤ.

3.2 $$(x^2-1)^2 = (2x)^2 + (x^2-1)^2$$

Proof of the Corollary 3.1.

If $x \in \mathbb{Z}$, and

$$\left(\frac{x^2+1}{2}\right)^2 = x^2 + \left(\frac{x^2-1}{2}\right)^2$$

So, by multiplying each term by 4 in the expression above, we have to,

$$(x^2-1)^2 = (2x)^2 + (x^2-1)^2$$

RELATION BETWEEN PYTHAGORIC TRIPLES AND AN EQUILATERAL HYPERBOLE

Theorem 4. Every real number can be represented by a hyperbole equilateral.

4.1 $$y^2 - z^2 = 1$$

Proof of theorem 4

Let an equantion consider true like expression. 2.1

$$\left(\frac{x^2+1}{2}\right)^2 = x^2 + \left(\frac{x^2-1}{2}\right)^2$$

We can rearrange the terms of the above expression in such a way that

$$x^2 = \left(\frac{x^2+1}{2}\right)^2 - \left(\frac{x^2-1}{2}\right)^2$$

Dividing all the above equality by x^2

4.1 $$1 = \left(\frac{x^2+1}{2x}\right)^2 - \left(\frac{x^2-1}{2x}\right)^2$$

If we replace the terms of the expression, 4.1 like that,

$$y = \left(\frac{x^2+1}{2x}\right)^2 \ and\ z = \left(\frac{x^2-1}{2x}\right)^2$$

We have,

$$1 = \left(\frac{x^2+1}{2x}\right)^2 - \left(\frac{x^2-1}{2x}\right)^2$$

It can be rewritten, as

$$y^2 - z^2 = 1 \text{ QED.}$$

Moreover, the equation $y^2 - z^2 = 1$ is an equilateral hyperbole

A NEW RELATIONSHIP TO OBTAIN A NUMBER ODD NATURAL ANY THROUGH DIFFERENCE OF TWO PERFECT SQUARES

Theorem 5. Let a and b be two natural and consecutive numbers, such that $a > b$. Then,

$$a^2 - b^2 = a + b$$

Proof of the theorem 5

Suppose, a anb b, such that, $a = b + 1$, if it is so, then,

$$a - b = a - (b + 1)$$

And,

$$a - (b + 1) = 1$$

Then,

$$a - b = 1$$

Now, we can multiply by $(a + b)$ both side of the expression below, such that,

$$(a - b) \cdot (a + b) = (a + b)$$

Or, this way as it follows,

$$a^2 - b^2 = a + b \text{ QED.}$$

A FUNÇÃO QUE REPRESENTA A SOMA DOS NÚMEROS ÍMPARES DADOS DEMONSTRADO ATRAVÉS DE UMA SÉRIE TELESCÓPICA

Theorem 6. The sum of all odd numbers can be represented by a telescopic series, the sum of which is equal to the number of terms squared.

6. $$\sum_{k=1}^{n} (k)^2 - (k-1)^2 = n^2$$

Proof of the theorem 6.

As a result of **theorem 5** it has been said that:

$$\sum_{k=1}^{n}(k)^2-(k-1)^2=1-0+2^2-1+3^2-2^2+\cdots+(n-1)^2-(n-2)^2+n^2-(n-1)^2$$

Canceling all repeating members, we have to

$$\sum_{k=1}^{n}(k)^2-(k-1)^2=n^2QED.$$

Therefore, the sum of every consecutive number is an odd number, the sum of all two consecutive numbers up to a certain term n must be n^2.

Just remembering that the sum of two consecutive numbers is na odd number, Always.

THE FUNCTION THAT REPRESENTS THE SUM OF THE EVEN NUMBERS

Theorem 7. The sum of even numbers up to a given term n is equal to the product of his last term by his successor

$$\sum_{k=1}^{n}(2k)=n(n+1)$$

Proof of the theorem 7.

As the sum of all terms, even and odd, is given by the function,

$$f(n)=\frac{n(n+1)}{2}$$

For any natural n, you have to,

$$\frac{n(n+1)}{2}-n^2=\sum_{k=1}^{n}(2k)$$

Then,

$$\sum_{k=1}^{n}(2k)=n^2+n\ or\ \sum_{k=1}^{n}(2k)=n(n+1)QED.$$

FINAL CONSIDERATIONS

I hope that through this article I can contribute to the theory of numbers, as in this case, the direct proof of the sum of all odd numbers, which, as far as I know, was first proved, through mathematical induction, by Francesco Maurolycus in 1575.

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