Special Needs Student Developing Automaticity With Multiplication Facts Six to Nine

Estella P. De Los Santos (Corresponding author)
Professor, School of Education, University of Houston-Victoria
Victoria, TX, United States of America
delossantose@uhv.edu

Abstract

In previous studies, the student used skip counting and her hands to recall [1] all the multiplication facts for the ones to fives and also gained automaticity [2] for the ones to fives facts. In the current study the ninth-grade student was able to memorize the multiplication facts (0 to 10) for the sixes to nines and gained automaticity with 80% accuracy. The overall goal of the current study was to improve the student's automaticity of multiplication facts (0-10) for the sixes through nines, such as 6 x 7 = 42. The student had been taught multiplication concepts in her elementary years and she has a basic understanding of the meaning of multiplication facts as shown in the previous studies. She understood that 6 x 7 is a representation of six sets of seven or 7 + 7 + 7 + 7 + 7 + 7 = 42. The student had previously used skip counting and flash cards with illustrations of the multiplication facts to memorize the facts. In the current study a multiple baseline design was used to measure the acquisition of fluency of multiplication facts over time. The student was able to develop automaticity with 80% accuracy by giving the solution within 3 seconds for all of the facts (0-10) for the sixes through nines.

Keywords: autism, learning disabilities, multiplication facts, automaticity

1. Introduction

The student was in the ninth grade and this investigation was conducted during the school year. The student was in a special education classroom at a secondary school. She received all of her academic instruction in the special education classroom and went to other classrooms for electives such as art and physical education. The student was diagnosed with mild learning disabilities and autism. She was performing at the fifth grade level in reading, writing and mathematics.

According to the student's classroom teacher, the student was having difficulty with several mathematics concepts including fractions. The researcher had previously been working with the student on renaming and simplifying fractions. The student was able to understand fraction concepts as long as explanations were given and concrete models were used. The student was not able to rename or simplify fractions abstractly as she did not know the basic multiplication facts by memory. She could not rename fractions such as thirds and fourths to twelfths nor simplify fractions such as 4/12 without specific instructions and the use of manipulatives. Therefore, the researcher changed the focus of the lessons from learning fraction concepts to memorizing multiplication facts. Once the multiplication facts were memorized, the researcher then worked on developing automaticity with the multiplication facts. The task of memorizing all of the facts seemed to overwhelm the student; so the task was divided into two parts: memorizing the ones to fives then memorizing the sixes to nines. A review of the ones to fives was maintained as the sixes through nines were memorized.

2. Literature Review

The National Council of Teachers of Mathematics (NCTM) has set the following as one of the standards for grades 3 through 5: "develop fluency with basic number combinations for multiplication and division" [3]. The NCTM recommends using multiple models such as skip-counting, area models, and relating known facts with those that are not known. According to the NCTM, if the basic facts are not known by the end of the fourth grade, the student "must either develop strategies so that they are fluent with these combinations or memorize the remaining harder combinations" [3]. The Common Core State Standards for Mathematics for Grade 3 states that, "By the end of Grade 3, know from memory all products of two one-digit numbers" [4].

2.1. Rationale

It is important that students be able to recall basic multiplication facts because they are necessary in mastering more complex concepts in mathematics [5], [6], [7], [8], [9], [10]. The multiplication facts are especially important in learning fraction concepts such as equivalent fractions, simplifying fractions, and finding common denominators of fractions. Students without learning difficulties usually master memorizing multiplication facts by grades 3 to 4, while it usually takes children with learning difficulties longer [5], [11], [12], [13].

2.2. Conceptual Understanding Stages

A conceptual understanding of basic addition and multiplication facts is essential before students are expected to progress to more complex mathematical topics [14], [15], [16]. Students may progress through three stages in learning the meaning of addition and multiplication: concrete, semi-concrete, and abstract stages. In the concrete stage of learning, students use different types of manipulatives to represent sets and counters in sets to understand addition and multiplication concepts. In the semi-concrete stage, drawings are used to represent sets and counters in sets for example circles for sets and dots in the circles as counters. In the abstract stage, students understand addition and multiplication concepts without using manipulatives or drawings [17], [18], [19], [20] [21].

2.3. Stages in Memorizing Basic Facts

Students should first have a conceptual understanding of multiplication facts such as "3x4=12" means "3 sets of 4 counters" and is equivalent to the sum, "4+4+4=12". Next, it is important that students memorize the basic multiplication facts such during their elementary school years [8]. Having used multiple models for multiplication such as groups, arrays, area models, jumps on a number line, etc, most student will memorize some of the basic facts such as 2x3=6 and the doubles such as 4x4=16.

Memorization techniques should be used to teach the students how to memorize the remaining facts [3]. Memorizing basic math facts will help students in learning other mathematical concepts such as fractions in the middle grades and algebra beyond the middle grades.

Three phases that children usually progress in memorizing basic multiplication facts are described in the research. Phase one involves using counting strategies such as counting blocks, tallies, fingers, etc. Phase two involves using reasoning strategies such as using patterns, logic, relationships in numbers, etc. Phase three is fast and accurate mastery of the multiplication facts [6], [7], [8], [9], [10]. In the third phase, students have memorized the multiplication facts and can recall the product with speed and accuracy. In the first two phases cognition is conscience, deliberate and slow, while in phase 3 cognition is non-conscience, automatic, and fast [6]. Automaticity is the ability to automatically recall the facts without using counting or reasoning strategies [8], [9], [10].

In learning the multiplication facts, students learn relationships between number facts such as multiplying with the numbers zero and one, and multiplying “twin numbers” together such as 2x2, 3x3, 4x4, etc. An example using the commutative property of multiplication is learning the relationship between “6x3=18” and “3x6=18” [5], [6], [8], [9]. Learning the commutative property allows students to realize that they only need to memorize half of the facts on a multiplication chart [6], [7].

In phase two, mental reasoning strategies are used to find relationships in the numbers. For example, when multiplying 7x8 the student may know that 7x7=49 so therefore 7x8 would be seven greater or 7x7+7=56. In reality the student is using the distributive property of multiplication over addition: 7x8 = 7(7+1) = (7x7) + (7x1) = 49 + 7 = 56 [5], [8], [9]. Connections between the arithmetic and algebra should be made by teachers teaching algebra.

2.4. Two Views on Memorizing Basic Facts

Rote memorization results in routine expertise where knowledge can be applied to familiar tasks but not with new tasks. Meaningful memorization results in adaptive expertise which can be applied to familiar as well as new tasks. There are two views as to how children learn basic facts. The passive storage view [6] or conventional wisdom view [7] describe how multiplication facts are memorized by rote through repeated practice and reinforcement. The active construction view [6] or number-sense view [7] describe how the facts are memorized meaningfully.

With the passive storage view or the conventional wisdom view, the association between problem and solution does not consider conceptual understanding nor developmental readiness of the child. The learning may be accomplished without counting or reasoning. Positive and negative reinforcements may be used to motivate the students to learn the facts. The fact recall part of the brain does not involve the conceptual and reasoning part of the brain [7]. Phase 1 and 2 can help; but are not necessary for phase 3. Memorizing the multiplication facts can be achieved through extensive practice such as using flash cards and timed drills alone [6].

With the active construction view or number-sense view, phase 1 and 2 are necessary for phase 3. Meaningful memorization of the multiplication facts is achieved by creating a rich network of factual, relational, and strategic knowledge. For example, multiplication by “zero” and “one” become rules that lead to automaticity. Multiplication of “doubles” leads to reasoning that helps the students memorize other multiplication facts [6]. Mastery of computational fluency happens by discovering the numerous patterns and relationships that interconnect the basic combinations [7]. Students learn better by learning patterns and relationships rather than by rote memorization. Knowledge that is connected to prior learning is stored in long term memory better than isolated facts [7].

With the passive storage view or the conventional wisdom view, phase 3 instruction is achieved by well-designed and extensive drill. The key to memorization of basic multiplication facts is practice. With the active construction view or number-sense view, instruction in all three phases should include discovery of patterns and relations connecting the basic multiplication facts. The emphasis is on relating new knowledge to previous knowledge. A focus on the structure, rather than memorizing individual facts by rote memory, is more likely to lead to retention of facts [6].

According to the passive storage view or conventional wisdom theory, learning difficulties are due to deficits in the learner [6], [7]. These children are labeled as "learning disabled" and often characterized as inattentive, forgetful, often confused and unable to apply knowledge. Children labeled as learning disabled rely heavily on counting strategies, may be taught reasoning strategies; but will not invent them on their own, difficulty in learning number facts especially those with numbers higher than 5, and have a high error rate in recalling basic number facts [7].

According to the active construction view or number-sense view, learning difficulties are due to defects in traditional instruction [6], [7]. Although children with learning disabilities do have cognitive impairments, they are capable of learning basic multiplication facts. One reason that learning disabled children have difficulties is because they lack the informal knowledge to develop successful problem solving and reasoning skills. For example, they may lack the informal knowledge to develop an understanding of composition and decomposition which are foundations for learning many mathematical concepts. A second reason is that if the students have been taught with a focus on memorization, they have not developed skills such as looking for patterns and relationships, reasoning skills, nor developed an appreciation for mathematics [7]. Students who have been taught with a focus on memorization may confuse rules such as multiplication by "zero" and "one". They may lose count if they are skip counting and give the wrong answer. Students taught by rote memorization do not think about the reasonableness of an answer and may give an answer that is unreasonable. These strategies make them prone to errors [7].

With the passive storage view or conventional wisdom theory, massive drill and practice is not recommended. It is recommended that students are introduced to one set of basic facts and that the students learn those facts before advancing to another set [7].

With the active construction view or number-sense view, the memorization of basic facts should be integrated with conceptual understanding, problem-solving and reasoning strategies. Students should be encouraged to develop number sense and invent informal strategies. Students should develop meaningful mastery of facts by being encouraged to focus on looking for patterns and relationships among numbers [7].

2.5. Studies on Developing Automaticity

Parmar et al. indicate that students with learning disabilities have different needs in the mathematics classroom [22]. Thorton and Toohey [23] describe basic guidelines when planning the learning of basic facts with learning disabled students. Factors to consider include prerequisite knowledge, monitoring and assessing, the sequence that facts are presented, teaching strategies prior to drill, learning styles of the student, pacing, verbal prompts, and self-monitoring skills.

Low achieving students and students with learning disabilities have difficulty in developing automaticity. Student with learning disabilities are more likely to use counting strategies rather than instant recall of multiplication facts [9], [10].

Students with learning disabilities may have difficulties expressing themselves verbally or in writing. They may have difficulties following a set of verbal or written instructions. They may have cognitive difficulties with memory, attention and language. Teachers can make adjustments so that the child with learning disabilities can master the concepts and knowledge. Teachers can use explicit and systematic, provide additional support and instruction, adjust the environment, re-explain and model and describe the outcomes desired [24].

Morin and Miller [21] taught three seventh grade students with mild intellectual disabilities to memorize multiplication facts. Students were taught using the concrete-representational-abstract teaching method and students did improve on a pretest-postest measure.

Wood et al. placed the 100 multiplication facts (0-9) into six sets [25]. Facts with zero (19 cards), facts with one (17 cards), two (15 cards), five (13 cards), nines (11 cards) and other facts (25 cards). The results showed that fourth and fifth grade students improved their fluency on multiplication facts after instruction and on to a follow up measure. The weakest effects occurred with the last set of cards or those labeled as facts other than zero, one, two, five or nine.

Bouck et al. investigated the use of Pentop computers to improve multiplication of three middle school students with mild intellectual disabilities [11]. Each student was being educated in a special education classroom with pull-out classes for art, music, and physical education. The three 12-year old students were performing at a fourth grade level in mathematics. The FLY™ Pen was used to improve the students' fluency with multiplication. Students were first taught multiplication facts for two weeks. Base line scores were acquired and pre-training on use of the FLY™ Pen was given to the students. The intervention took place for a period of three weeks. Assessment scores were taken during the intervention phase. One week after the intervention, students completed a maintenance assessment on the multiplication facts. The results showed that the students did improve on percentage of multiplication problems solved correctly from the base line to maintenance phases [11].

The constant time delay teaching strategy has been used in many studies. This strategy involves using multiplication fact flash cards. If the student replies incorrectly or takes more than a set number of seconds (usually 2s-4s) to respond, the teacher repeats the problem and correct answer. The card is placed 2-3 cards back in the stack of cards [8].

Giving the answer to the student after the designated time period is important so the student focusses on immediate recall instead of taking time to derive the answer [8]. If the student does not recall the answer within 3-4 seconds, he or she is immediately told the answer and asked to repeat it [8]. Small sets of multiplication facts learned to high levels of mastery before adding more facts is important to achieving automaticity [8].

Baker and Cuevas [10] investigated the use of strategies in recalling multiplication facts. Their subjects were 3rd, 5th, and 8th grade students. The findings corroborated previous research that shows students struggle with recall of single digit multiplication facts. This is alarming because the lack of fluency with multiplication facts puts their future success in the mathematics classroom in jeopardy.

Automaticity should be achievable for most students. There are different strategies that students use to recall multiplication facts such as skip counting, using their fingers to count up, and the hand trick for multiplication by 9. Baker and Cuevas [10] found that 13.1% of the participants in their study used a strategy and that strategies were used on the most difficult problems. The results showed the percentage of automaticity was 52.4% for 3rd grade students, 75.7% for 5th, and 65.7% for 8th grade students. The percentage of automaticity was higher for easier problems than for harder problems such as multiplication facts of 6, 7, 8, and 9.

Instructional strategies that can be used for students with different academic levels include explicit timing and cover-copy-compare strategies [26]. Codding et.al. found that students who at are at a frustration level with math fact fluency did better with a cover-copy-compare strategy and students at a normal range of math fact fluency did better with explicit timing. The cover-copy-compare (CCC) strategy involves 5 steps: 1) look at problem and answer, 2) cover the problem and answer, 3) record the answer, 4) uncover the problem and answer, and 5) compare answers [26] [27]. A recent study [28] found no significant difference between Math Cover, Copy, Compare intervention procedures for children with Autism Spectrum Disorder (ASD).

Burns investigated the use of Incremental Rehearsal (IR) to increase fluency of single-digit multiplication facts [29]. A gradual increase of known to unknown facts was used with the final stage being 90% to 10%. Three third-grade elementary students with learning disabilities and low fluency with multiplication facts were the subjects. Base line data was collected using single worksheets with 35 randomly selected single-digit multiplication facts. The experimental treatment was administered twice per week for 10-15 minute periods. The 100 (0 to 9) single-digits facts were written on index cards and identified as “known”, if the child correctly stated the answer within 2 seconds and identified as “unknown” if the child was not successful. The treatment was an IR sequence where 1 card presented was “unknown” and 9 cards presented were “known”. The Incremental Rehearsal method was successful in increasing fluency of single-digit multiplication facts [29].

Lessons presented with direct instruction methods are presented systematically. Direct Instruction (DI) was used with flash cards and model-lead-test correction procedures with elementary students with learning disabilities [30], [31], [32].

Cravalho et al. investigated the use of DI flashcards to teach elementary students with learning disabilities their addition facts [30]. A baseline score was obtained followed by direct instruction and correction models of teaching. The student was shown a flash card with an addition fact and the problem was read by the researcher. Following the student’s response, the researcher would respond with “Yes”, or “No” followed by the correct answer. If the student responded with an incorrect response, the researcher used a model, lead, test, and retest correction procedure. The missed card was then placed two cards back in the stack. The results showed that all students made progress using DI flashcards in learning addition facts [30].

Lund et al. investigated the use of DI flashcards in teaching multiplication facts to sixth grade students with learning disabilities [31]. The researchers in this study required the students to state the problem themselves rather than the researcher. A correct response was considered a correct answer within 5 seconds. Mattingly and Bott [32] also used a 5 second constant time delay procedure in teaching multiplication facts to fifth and sixth grade students. The DI lesson was followed by a math racetrack game. The racetrack was a board game with 28 spaces for multiplication facts. The student was timed for speed and accuracy. The problem was read by the student. The researcher responded with either "Yes" or "No" followed by the problems with the correct answer. If the student's response was correct, they were allowed to go on to the next square. If the student's response was incorrect, the student had to repeat the problem after the researcher's correction. The DI and math racetrack intervention was effective in increasing the students' accuracy and fluency on basic multiplication facts [31].

Glover et al. also investigated the use of DI flashcards to teach multiplication facts to adolescents with learning disabilities [33]. The researchers were successful in teaching basic multiplication facts to 11 and 12 year old students using DI flashcards with back three cards for errors strategy [33].

3. Methodology

The single-subject study served to (a) identify the unique needs of the student, (b) guide the selection of instructional content and materials, (c) create an individualized education plan (IEP), (d) monitor student progress, and (e) evaluate learning [34]. Adequate representation of content, appropriate scope and sequence of the content and developmentally appropriate content were addressed to insure accurate measures.

A single-subject multiple baseline design [35], [36], [37] across sets of multiplication facts for sixes to nines with Direct Instruction (DI) flashcards [29], [30], [31], [32] were used. Multiple baseline designs are useful for measuring the acquisition of fluency of skills over time [21], [29], [30], [31], [32], [33].

The researcher had worked with the student in five previous studies [1], [2], [39], [40], [41]. The student's previous knowledge and history were used to develop an individualized education plan to help her memorize multiplication facts (0-10) for the sixes through nines [1], [2]. The previous studies showed that the student had a conceptual understanding of basic multiplication facts; but did not have them memorized. The researcher began by engaging the student in skip counting exercises. The student's hands were used and each finger represented the basic facts from 1 to 10. The interventions were using flash cards with illustrations of the multiplication facts and skip counting. The student was able to skip count using her fingers with accuracy but not with speed. The student was able to memorizing multiplication facts (0-10) for the ones to fives over a two month period [1]. The student was also able to develop automaticity for the ones to fives [2]. The purpose of the current study was for the student to memorize the multiplication facts and to develop automaticity for the (0-10) facts for the sixes to nines.

In the current study, the student first recited the multiplication facts for the twos to sixes using her fingers. After these were memorized, the student proceeded to recite the twos to sevens by memory, then the twos to eights and finally the twos to nines. The memorization process took an entire year because of COVID-19 restrictions and the student not attending school.

After the multiplication facts were all memorized, the researcher continued by testing for automaticity for the sixes to nines. There were 44 flash cards randomly divided into 4 sets of 11 cards. The goal was to go through each set of cards, by the researcher stating the multiplication fact on the card and allowing the student 3-seconds to respond with the correct answer. The researcher recorded a "+" or "-" for each of the 11 cards to record mastery or non-mastery respectively. One session was conducted per week and consisted of three attempts to finish the set of 11 cards with at least 80% accuracy. A record of at least 9 out of 11 correct was a successful attempt. Once the student was able to achieve at least 80% accuracy on three consecutive attempts, the student proceeded to the next set of 11 cards. Whenever a new set was introduced, the researcher used two instead of three attempts to familiarize the student with the new set of cards and gave the student 5-seconds to respond instead of 3-seconds. The first attempt did not count as a success since 5-seconds was allowed.

3.1. Subject

The student was in the ninth and tenth grade when this investigation was conducted. The student is in a special education classroom at a secondary school for grades 9-12. She goes to regular classes for physical education and art. The student was diagnosed with mild learning disabilities and was performing at the fifth grade level in math, reading and writing. The student was also diagnosed with mild learning disabilities and autism.

3.2. Materials

The student had been taught to skip count for the multiplication facts, twos to nines, using the fingers. At the beginning of each session, the student used her fingers to review the two to nine multiplication facts. Flash cards for the sixes to nines were used in this study. There were eleven multiplication flash cards for the six to nine facts. For example, for the multiplication by six facts the following cards were used: 6 x 0, 6 x 1, 6 x 2, 6 x 3, 6 x 4, 6 x 5, 6 x 6, 6 x 7, 6 x 8, 6 x 9, 6 x 10. The facts were written in vertical form on the card rather than horizontal form. In all there were 55 cards covering the multiplication facts for the sixes to nines.

4. Results

A single-subject multiple baseline design [35], [36], [37] across sets of multiplication facts for sixes to nines with Direct Instruction (DI) flashcards [29], [30], [31], [32] were used. Multiple baseline designs are useful for measuring the acquisition of fluency of skills over time [21], [29], [30], [31], [32], [33].

The first set of 11 cards took three weeks to master. At the first session or first week, the student answered 8 in the first attempt then 8 in the second attempt of the 11 flash cards correct by responding within 5-seconds. At the week two session, the student had 9 of the 11 cards correct in 3-seconds, then 8 and 9 of the 11 cards correct during the second and third attempts. At week three, the student had 9, 11 and then 10 cards correct out of the 11 cards in the first, second, and third attempts respectively. Since student answered 9 or more cards correct, mastery was at least 80% on all three attempts; therefore, the student progressed to the second set of cards at week 4. Table 1 contains the results for the first set of 11 cards (See Table 1) and Figure 1 is a graph of the data (See Figure 1).

Table 1. Developing automaticity set 1 results

Session	Attempt 1	Attempt 2	Attempt 3
Week 1—A	8	8	
Week 2—B	9	8	9
Week 3—C	9	11	10

The student achieved automaticity on set 1 after three weeks.

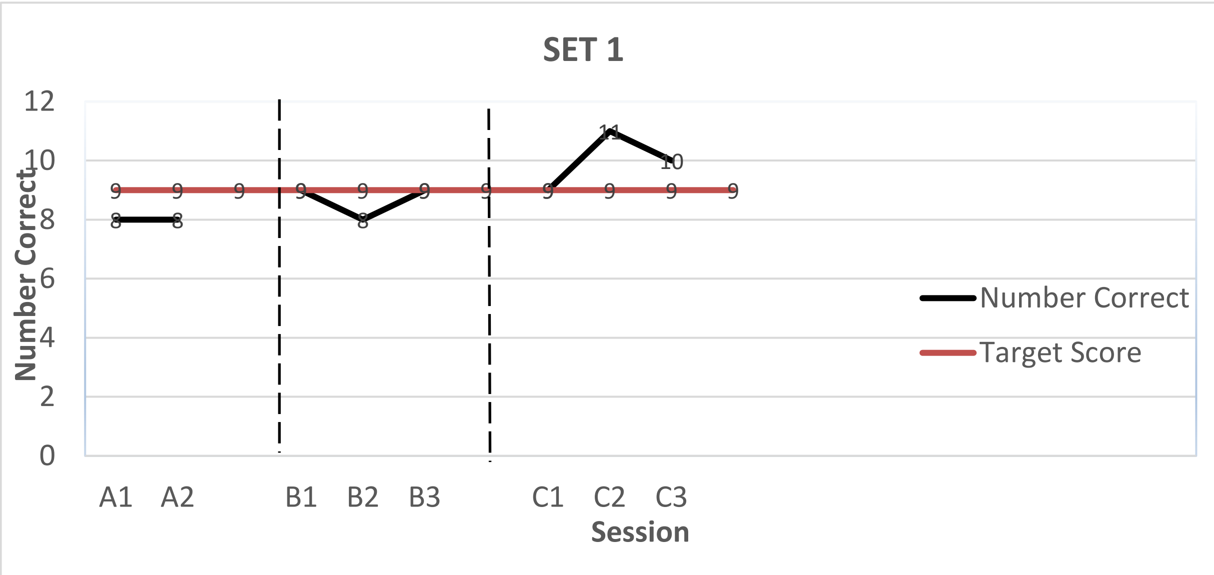

Figure 1. Developing automaticity set 1 results.
The student achieved at least 80% automaticity on three attempts after three weeks.

The second set of 11 cards took four weeks to master. At the fourth session or fourth week, the student answered 7 in the first attempt then 8 in the second attempt of the 11 flash cards correct by responding within 5-seconds. At the week five session, the student had 8, 9 and then 10 flash cards correct by responding within 3-seconds in the first, second, and third attempts respectively. At session six or week six, the student had 8, 10, and 10 cards correct in the three attempts. During session seven or week seven, the student mastered the set of cards by getting 9, 10, and 10 cards correct in the three attempts. Since the student answered 9 or more cards correct, mastery was at least 80% on all three attempts; therefore, the student progressed to the third set of cards at week 8. Table 2 contains the results for the second set of 11 cards (See Table 2) and Figure 2 is a base-line graph of the data (See Figure 2).

Table 2. Developing automaticity set 2 results

Session	Attempt 1	Attempt 2	Attempt 3
Week 4—A	7	8	
Week 5—B	8	9	10
Week 6—C	8	10	10
Week 7—D	9	10	10

The student achieved automaticity on set 2 after four weeks.

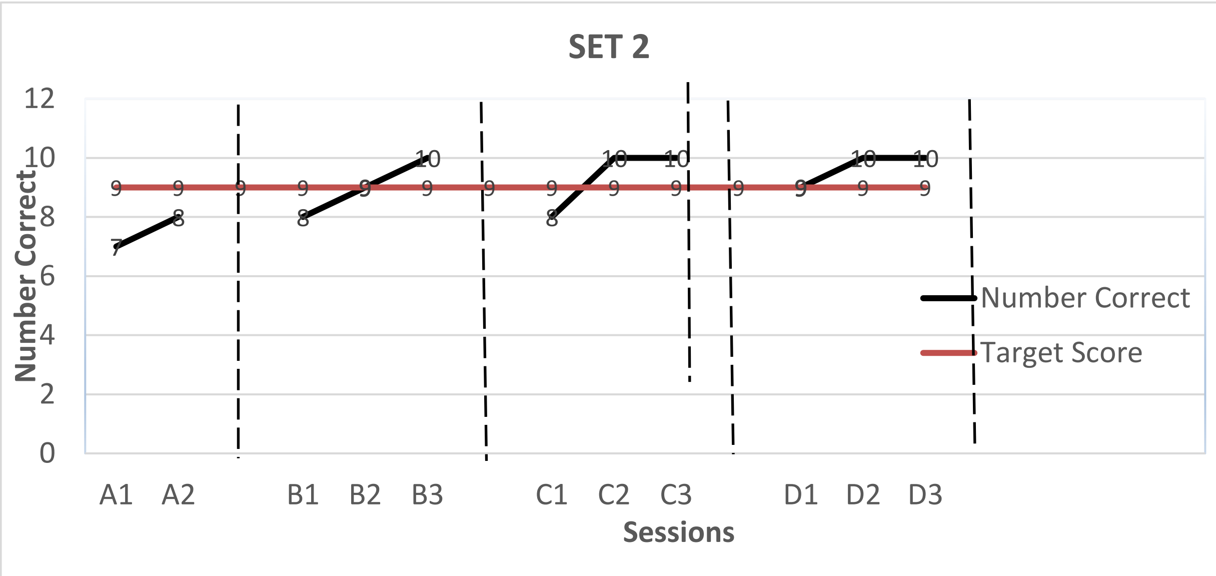

Figure 2. Developing automaticity set 2 results.

The student achieved at least 80% automaticity on three attempts after four weeks.

The third set of 11 cards took four weeks to master. At the eighth session or eighth week, the student answered 6 then 8 of the 11 flash cards correct by responding within 5-seconds during the first and second attempts, respectively. At the week nine session, the student had 9 of the 11 cards correct in 3-seconds, then 8 and 9 of the 11 cards correct during the second and third attempts. At week ten, the student had 9, 8 and then 10 cards correct out of the 11 cards in the first, second, and third attempts respectively. During week eleven, the student had 10, 11 and then 11 cards correct out of the 11 cards in the first, second, and third attempts respectively. Since the student answered 9 or more cards correct, mastery was at least 80% on all three attempts; therefore, the student progressed to the fourth set of cards at week twelve. Table 3 contains the results for the third sets of 11 cards (See Table 3) and Figure 3 is a base-line graph of the data (See Figure 3).

Table 3. Developing automaticity set 3 results

Session	Attempt 1	Attempt 2	Attempt 3
Week 8—A	6	8	
Week 9—B	9	8	9
Week 10—C	9	8	10
Week 11—D	10	11	11

The student achieved automaticity on set 3 after four weeks.

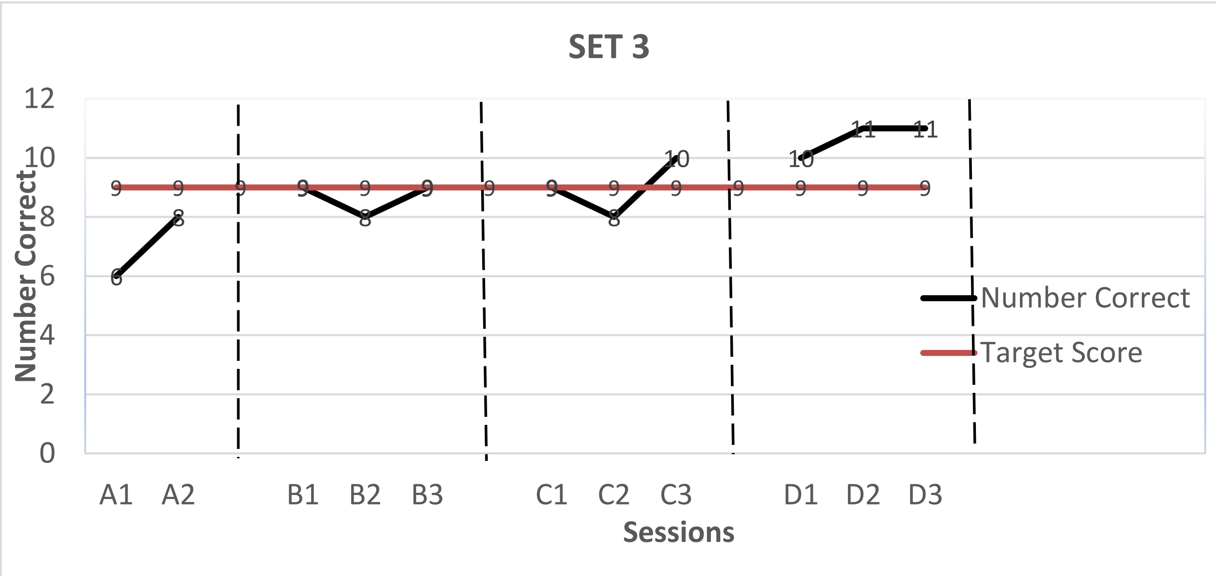

Figure 3. Developing automaticity set 3 results.

The student achieved at least 80% automaticity on three attempts after four weeks.

The fourth set of 11 cards took three weeks to master. At the twelfth session or twelfth week, the student answered 9 in the first attempt then 8 in the second attempt of the 11 flash cards correct by responding within 5-seconds. At the week thirteen session, the student had 7 of the 11 cards correct in 3-seconds, then 8 and 11 of the 11 cards correct during the second and third attempts. During week fourteen, the student had 9, 11 and then 11 cards correct out of the 11 cards in the first, second, and third attempts respectively. Since the student answered 9 or more cards correct, mastery was at least 80% on all three attempts; therefore, the student had completed all four sets of cards in fourteen weeks. Table 4 contains the results for the fourth set of 11 cards (See Table 4) and Figure 4 is a graph of the data (See Figure 4).

Table 4. Developing automaticity set 4 results

Session	Attempt 1	Attempt 2	Attempt 3
Week 12—A	9	8	
Week 13—B	7	8	10
Week 14—C	9	11	11

The student achieved automaticity on set 4 after three weeks.

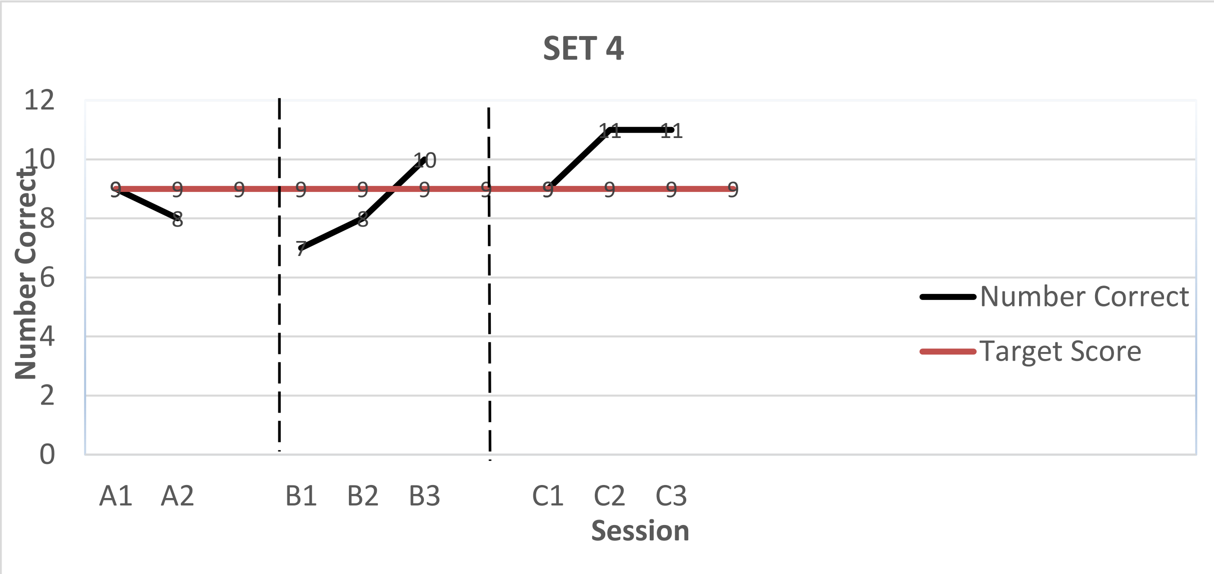

Figure 4. Developing automaticity set 4 results.

The student achieved at least 80% automaticity on three attempts after three weeks.

The student mastered all four sets of 11 cards for the sixes and nines facts (0-10) by responding correctly within 3-seconds for each multiplication fact. The student successfully developed automaticity in fourteen weeks with all the multiplication facts for the sixes to nines. The student had previously achieved automaticity for the ones to fives [2]. Therefore, the student now has all of her multiplication facts memorized with automaticity for the ones to nines.

5. Conclusions

According to the Individuals with Disabilities Education Act of 2004, “specific learning disability means a disorder in 1 or more of the basic psychological processes involved in understanding or in using language, spoken or written, which disorder may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations” [38]. The student in this study was diagnosed with mild disabilities. She was performing at about the fifth grade level in reading, writing and mathematics. The student was also enrolled in speech services at the public school which she attended.

Students continue to struggle with multiplication facts in the elementary grades. Students who do not develop automaticity with the multiplication facts in the elementary grades, struggle with mathematics in the middle school and are even further behind in high school mathematics. Basic multiplication facts are a building block for understanding higher mathematical concepts; so it is very important that elementary students achieve automaticity [8], [9], [10]. Low achieving students and students with learning disabilities have difficulty in developing automaticity. Student with learning disabilities are more likely to use counting strategies rather than instant recall of multiplication facts [9], [10].

It took several months for the subject in this study to memorize the multiplication facts for the sixes to nines. The student was quite successful at reciting the multiplication facts for the sixes to nines, using skip counting on her fingers as she did for the ones to fives [1].

Automaticity is developed by practicing a small set of multiplication facts and mastering that set before proceeding to another set of facts. The recall time should be immediately, within a couple of seconds, rather than taking time to derive the answer [8]. The student in this study was able to develop automaticity for the sixes to nines multiplication facts. She was able to respond with the correct answer within 3-seconds of hearing a problem stated by the researcher. The student was able to respond to all of the facts for the sixes to nines divided into four sets of 11 cards with at least 80% accuracy.

The use of calculators and computers may be a reason that students are not motivated to memorize the basic multiplication facts [10]. Summer or other vacation time from school is a perfect time for older students or adults to help elementary aged students memorize basic addition or multiplication facts. Counters and drawings may be used initially, followed by flash cards with pictures. Next, flash cards with the problem only may be used. Finally, the student is expected to give a verbal response from memory. Some students may need more time and practice to memorize all the multiplication facts. Memorization is the most basic level of learning and can be achieved by most students.

The researcher will continue to work with this student on other math concepts. The student is especially proud to know the multiplication facts by memory. She is able to answer successfully when asked to give the solution of multiplication problems by family and friends. She also takes great pride that she is the only one in her class, who has the multiplication facts memorized. The student states that she would like to work at a fast food restaurant; so the next lessons will be on counting money and giving change.

6. References

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