A different formulation for the Fermat's last theorem

A. Mazaris
Phd in Mathematics, Number Theory
a.mazaris@yahoo.gr

Abstract

It is a big challenge, not only historical, to research Fermat's Last Theorem rising mathematics of Fermat's era. In the present work, based on a mathematical background known in 16th century, we will show that the wording of the theorem should be different.

The concept is based in works of L.Serbi[2] an A.Mazaris[3].

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Introduction

The most famous mathematical problem is probably Fermat's Last Theorem. After four centuries proved, first time, in 1995 by A.Wiles, but remained the challenge, because Wiles proof was based on an extensive mathematical background unknown in Fermat's era.

In this study, we attempt to prove, using mathematics known in 1637, that the wording of Fermat's Last Theorem should be different.

First of all we will prove Fermat's Last Theorem partially. With the assistance of this proof we will argue and explain why the formulation of the theorem had to be different.

A novel and targeted approach

Fermat's famous Last Theorem states that:

If a, b, c are positive integers, then there is no natural integer $n > 2$ such that:

$$a^n + b^n = c^n \tag{1}$$

We will show that: if a is bigger or equal to b

(this would necessarily happen for one of a, b)

and $n > a$ then Fermat's Last Theorem holds.

Because of this proof, we are going to present to you, the area who has meaning to search if FLT holds is only $2 \leq n \leq a, \quad a \geq b$

So the wording of FLT should be:

If a, b, c are positive integers then there is no natural integer n such that:

$$2 < n \leq a, \quad a \geq b$$

$$a^n + b^n = c^n$$

We assume that a, b, c, n are positive integers numbers that satisfy the theorem.
Dividing equation (1) by a^n (obviously positive integer) results:

$$\frac{a^n + b^n}{a^n} = \frac{c^n}{a^n}$$

Then:

$$1 + \left(\frac{b}{a}\right)^n = \left(\frac{c}{a}\right)^n \tag{2}$$

Because $c > a$, always there is $k \in Z_+^*$:

$$c = a + k, \quad c, a, k \in Z_+^* \tag{3}$$

We replace the eq.(3) to eq.(2):

$$1 + \left(\frac{b}{a}\right)^n = \left(\frac{a+k}{a}\right)^n$$

Then

$$1 + \left(\frac{b}{a}\right)^n = \left(1 + \frac{k}{a}\right)^n \tag{4}$$

Because there are preconditions for using Bernoulli inequality (see Appendix A):

$$\left(1 + \frac{k}{a}\right)^n \geq 1 + n\frac{k}{a} \quad , \quad \frac{k}{a} \in [-1, +\infty) \tag{5}$$

Also is very easy (see Appendix B) to prove that:

$$2 \geq 1 + \left(\frac{b}{a}\right)^n \tag{6}$$

Combining eq.(4), eq.(5) and eq.(6):

$$2 \geq 1 + \left(\frac{b}{a}\right)^n = \left(1 + \frac{k}{a}\right)^n \geq 1 + n\frac{k}{a}$$

So:

$$2 \geq 1 + n\frac{k}{a}$$

Then:

$$1 \geq n\frac{k}{a}$$

then

$$\frac{a}{k} \geq n$$

Finally:

$$a \geq n \tag{7}$$

Results

The relation $a^n + b^n = c^n$, , $a, b, c, n \in Z_+^*$ could hold only if $n \leq a$, $(a \geq b)$.

So it has no meaning to search $c \in Z_+^*$:

$$3^5 + 4^5 = c^5$$
$$3^6 + 4^6 = c^6$$
$$3^7 + 4^7 = c^7$$
$$\dots$$

Only in these cases we would search for a number c $(c \in Z_+^*)$

$$3^4 + 4^4 = c^4$$
$$3^3 + 4^3 = c^3$$
$$3^2 + 4^2 = c^2$$
$$3^1 + 4^1 = c^1$$

Finally, if we had 2 numbers $a, b \in Z_+^*$ and we would like to search all possible triads: $a^n + b^n = c^n$ (a, b known numbers,

n, c unknown numbers)

We will stop searching c, when we check the triads of numbers, when $n > a$ (in the position of n we will have replace all the positive integers up to a and checking if there is c who satisfy the equality).

Because of this proof we understand that:

Fermat's Last Theorem holds when $n > a$ $(a \geq b)$

So it has no meaning to try prove FLT when $n > a$ and we could wording FLT like that:

If a, b, c are positive integers then there is no natural integer n such that:

$$2 < n \leq a, \quad a \geq b$$
$$a^n + b^n = c^n$$

We have to remain that $2 < n \leq a$ because this is not obvious at all.

We want to remark that all terms (except a) of relation

$a^n + b^n = c^n$ have restrictions:

$b \leq a$, $c < a + b$, a is free but these restrictions are obvious for someone works maths.

This is not happens with restriction $n \leq a$

Discussion and conclusion

The result concerns also Pythagorean Theorem.

The reason that there is no $c \in Z_+^* : 1^2 + 1^2 = c^2$ is exactly that: $2 = n > a = 1$

For two given numbers $a, b \in Z_+^*$ there are infinite possible combinations that should be tested to find out if the relation holds $(a^n + b^n = c^n)$

With the restriction $n \leq a$ the combination are a.

We presented a partial proof of FLT because we limit the area of search in the field $n \leq a$.

The FLT always holds when $n > a$

APPENDIX A

Inequality Bernoulli:

$$(1+x)^n \geq 1+nx, \quad x \in [-1,+\infty), \quad n \text{ positive integer}$$

APPENDIX B

Proof:

$a,b \in \mathrm{N}^*: \quad a \geq b \;\rightarrow\; 1 \geq \frac{b}{a} \;\rightarrow\; 1^n \geq \left(\frac{b}{a}\right)^n \;\rightarrow\; 1+1 \geq 1+\left(\frac{b}{a}\right)^n \;\rightarrow\; 2 \geq 1+\left(\frac{b}{a}\right)^n$

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None

Conflict of interest

The author declares that there is no conflict of interest.

References

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