An Optimal Eigenvalue Based Spectrum Sensing Algorithm for Cognitive Radio

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Abstract

Spectrum is a scarce resource, and licensed spectrum is intended to be used only by the spectrum owners. Various measurements of spectrum utilization have shown unused resources in frequency, time and space. Cognitive radio is a new concept of reusing licensed spectrum in an unlicensed manner. The unused resources are often referred to as spectrum holes or white spaces. These spectrum holes could be reused by cognitive radios, sometimes called secondary users. All man-made signals have some structure that can be potentially exploited to improve their detection performance. This structure is intentionally introduced for example by the channel coding, the modulation and by the use of space-time codes. This structure, or correlation, is inherent in the sample covariance matrix of the received signal. In particular the eigenvalues of the sample covariance matrix have some spread, or in some cases some known features that can be exploited for detection. This work aims to implement, evaluate, and eventually improve on algorithms for efficient computation of eigenvalue-based spectrum sensing methods. The computations will be based on power methods for computation of the dominant eigenvalue of the covariance matrix of signals received at the secondary users. The proposed method endeavors to overcome the noise uncertainty problem, and perform better than the ideal energy detection method. The method should be used for various signal detection applications without requiring the knowledge of the signal, channel and noise power.

Keywords—*Spectrum, sensing, detection, eigenvalues, cognitive radio.*

I. Introduction

The electromagnetic radio- frequency spectrum is a highly valuable natural resource, but its use is regulated by governments through licensing agreements. Careful studies on the current usage of the radio spectrum by several agencies reveal that some frequency bands are heavily used; other frequency bands are only partially occupied, while most frequency bands in the spectrum remain largely unoccupied. Spectrum utilization can be improved significantly by making it possible for a secondary user to access a spectrum hole unoccupied by the primary (licensed) user at the right location and time. Cognitive radio, inclusive of Software-Defined Radio (SDR), has been proposed as the means to promote the efficient use of the spectrum by exploiting the existence of spectrum holes.

Cognitive radio is a self-aware communication system that efficiently uses spectrum in an intelligent way. It autonomously coordinates the usage of spectrum by identifying unused radio spectrum on the basis of observing spectrum usage, monitoring parameters in its radio frequency environment and reconfiguring its transmission and reception parameters to enable
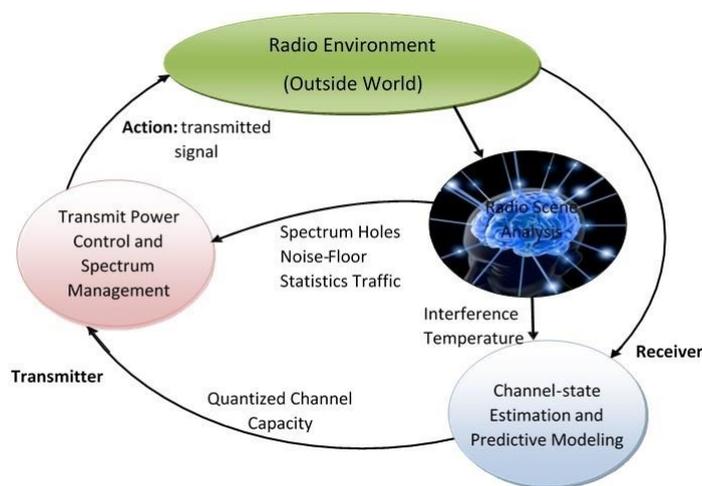
Fig.1. Basic Cognitive Cycle

Efficient communications with reduced interference. The reconfigurability of a cognitive radio is provided by the software defined radio platform, upon which a cognitive radio is built. The most advanced software-defined radio can additionally sense the environment and react appropriately according to the changes in the parameters it has sensed, in real time.

The cognition cycle by which a cognitive radio may interact with the environment is illustrated in Figure 1. Stimuli enter the cognitive radio as interrupts, dispatched to a cognition cycle for a response. Such a cognitive radio continually observes, orients itself, creates a plan, decides and then acts. In addition, learning may be pursued in the background [1].The cognitive process therefore, starts with the passive sensing of RF stimuli and culminates with action.

The introduction of cognitive radios will inevitably create increased interference and thus degrade the quality of service of the primary system. The impact on the primary system, for example in terms of increased interference, must be kept at a minimal level. To keep the impact at an acceptable level, secondary users must sense the spectrum to detect whether it is available or not. Secondary users must be able to detect very weak primary user signals [2][3][4]. Therefore, spectrum sensing is a fundamental component in cognitive radio.

Spectrum sensing is the ability to find available frequencies or timeslots to transmit in. However, there are several factors which make the sensing problem difficult to solve. First, the signal-to-noise ratio (SNR) of the primary users received at the secondary receivers may be very low. Secondly, fading and time dispersion of the wireless channel may complicate the sensing problem. In particular, fading will cause the received signal power to fluctuate dramatically while an unknown time dispersed channel will cause unreliable coherent detection [5][6]. Thirdly, the noise/interference level changes with time which results in the noise uncertainty [4][7]

In spectrum sensing, therefore, the main objective is that of designing an optimal algorithm for exchanging spectrum sensing data between nodes to reliably detect spectral holes for use by the cognitive radio and to reliably detect when the primary transmitter comes on. The problem is then that the algorithm needs to have as little delay as possible so that once channels are available one can transmit immediately and, of course, with as few false detections and false no-detections as possible.

The spectrum sensing functionality can be implemented in either a non- cooperative or a cooperative fashion. Matched filter detection, energy detection and cyclostationary feature detection are three classic non-cooperative spectrum sensing methods. Cooperative spectrum sensing can effectively improve the sensing performance in a fading environment. In cooperative spectrum sensing, local sensors individually sense the channels and then send information to the network center, and the network center makes the final decision according to a certain fusion rule.

Eigenvalue based spectrum sensing methods use the eigenvalues of the sample covariance matrix to detect the primary transmitter without requiring information of primary user signals or noise power level. Eigenvalue

based detection techniques studied in literature include maximum-minimum eigenvalue (MME) detection [18], energy with minimum eigenvalue (EME) detection [17], maximum eigenvalue detection (MED) [20] and maximum eigenvalue to trace (MET) detection [19]. In [13], the simulation and performance for MME detection and EME detection are presented for the Nakagami-m fading channel.

Moreover, in the eigenvalue based methods, the expression for the decision threshold has been derived based on the asymptotic or limiting distributions of extreme eigenvalues. The exact decision threshold is calculated for MME detector in [11]. By using the exact decision thresholds, the detection performance of MME detector achieves significant performance gains. An eigenvalue based spectrum sensing technique with finite number of samples and sensors is proposed in [16]. The authors express the distribution of the largest eigenvalue of finite sample covariance matrix in the form of sum of two gamma random variables.

These eigenvalue-based detectors have been shown to perform well when the signal to be detected is highly correlated. The proposed methods can be used for various signal detection applications without knowledge of the signal, the channel and the noise power. Furthermore, different from matched filtering, the proposed methods do not require accurate synchronization.

However, all of the above eigenvalue based spectrum sensing methods require multiple sensing nodes or receiving antennas. In some networks such as cognitive radio ad hoc networks [9] and cognitive radio cellular networks [12], CR users are mobile and they communicate with each other using small and flexible devices. It is impractical to equip a small mobile device with multiple antennas due to the required size of these antennas. More specifically, the space between two antennas must be at least of the order of $\lambda/2$, λ being the wavelength used for transmissions. For example, for the commonly used 2.4 GHz frequency band, the required distance is 6.125 cm. Even four antennas can be too big to be mounted on a laptop and the situation will get worse for small mobile device [9].

Therefore, this research work addresses the problem of spectrum sensing in a single receiver system. An optimal maximum-minimum eigenvalue detection method using a single antenna is proposed for cognitive radio networks. The temporal smoothing technique is used to form a virtual multiantenna structure for the implementation of proposed detection method based on single antenna. The proposed approach makes use of the power method to obtain the maximum and minimum eigenvalues to avoid the eigenvalue decomposition processing. The decision threshold is derived based on latest random matrix theories.

II. System Model

Assume that the frequency band of interest has a central frequency f_c and bandwidth *W*. During a particular time interval, the frequency band may be occupied by only one primary user. Several secondary users are randomly distributed in the cognitive radio network. Each secondary user is equipped with a single antenna. In this research work, the non-cooperative spectrum sensing scheme is considered, that is, the sensing work is completed by only one secondary user (only one source, one receiver). For signal detection, two hypotheses can be formulated: (1) hypothesis H_0 : there exists no signal (only noise); (2) hypothesis H_1 : there exists both the signal and additive white noise. The binary hypothesis test can be replaced by:

$$H_0 : x(n) = w(n), \qquad n = 0, 1, \cdots$$
$$H_1 : x(n) = \sum_{k=0}^{N} h(k)s(n-k) + w(n) \qquad (1)$$

Where $x(n)$ denotes the discrete signal at the secondary receiver, $s(n)$ is the primary signal seen at the receiver, $h(k)$ is the channel response, *N* is the order of the channel, and $w(n)$ are the noise samples. Considering a sub-sample *M* of consecutive outputs and defining

$$\hat{x}(n) = [x(n), x(n-1), \cdots, x(n-M+1)]^T$$

$$\text{w}\hat{\ }\,(n) = [w(n), w(n-1), \cdots, w(n-M+1)]^T \quad (2)$$
$$\text{s}\hat{\ }(n) = [s(n), s(n-1), \cdots, s(n-N_1-M+1)]^T$$

This results in

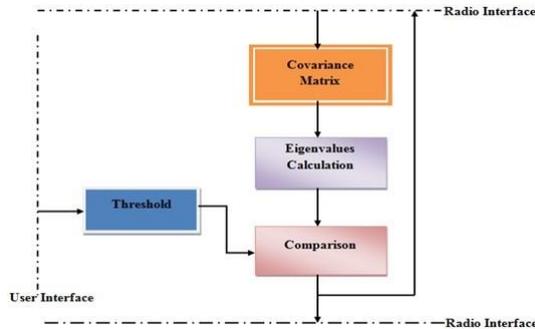$$\text{x}\hat{\ }(n) = \text{Hs}\hat{\ }(n) + \text{w}\hat{\ }\,(n) \qquad\qquad (3)$$

Fig. 2.Eigenvalue-Based Spectrum Sensing Algorithm Flow Chart

Where

H is an M x $(N+M)$ matrix, defined as

$$H = \begin{pmatrix} h(0) & \ldots & h(N) & \ldots & 0 \\ & \ddots & & \ddots & \\ 0 & \ldots & h(0) & \ldots & h(N) \end{pmatrix}$$
$$(4)$$

Considering the statistical properties of the transmitted signal and channel noise,assume that the noise is white and that the noise and the transmitted signal are correlated. Let R be the covariance matrix of the received signal, that is,

$$\text{R} = \frac{1}{N_s} \sum_{n=M}^{M-1+N_s} \hat{\mathbf{x}}(n)\hat{\mathbf{x}}^H(n) \qquad\qquad (5)$$

Where N_s is the number of collected samples. If N_s is large, based on the assumptions made earlier,it is verified that

$$\text{R} \approx \text{E}[\text{x}\hat{\ }(n)\text{x}\hat{\ }^H(n)] = \text{HR}_s\mathbf{H}^H + \sigma_w^2\mathbf{I}_M \qquad (6)$$

Where R_s is the statistical covariance matrix of the input signal, $\text{R}_s = \text{E}[\text{s}\hat{\ }(n)\text{s}\hat{\ }^H(n)]$, σ_w^2 is the variance of the noise, and I_M denotes an $M \times M$ identity matrix.

Let $\hat{\lambda}_{max}$ and $\hat{\lambda}_{min}$ be the maximum and the minimum eigenvalues of R , and $\hat{\rho}_{max}$ and $\hat{\rho}_{min}$ be the maximum and the minimum eigenvalues of HR_sH^H. Then

$$\hat{\lambda}_{max} = \hat{\rho}_{max} + \sigma_w^2 \quad and \quad \hat{\lambda}_{min} = \hat{\rho}_{min} + \sigma_w^2 \qquad (7)$$

Obviously, $\hat{\rho}_{max} = \hat{\rho}_{min}$ if and only if $\text{HR}_s\text{H}^H = \delta\text{I}_M$ where δ is a positive number.

Again, obviously, when the primary signal is present

$$\hat{\lambda}_{max} = \hat{\rho}_{max} + \sigma_w^2 \ , \ \hat{\lambda}_{min} = \sigma_w^2$$ and when the primary

Signal is absent $\hat{\lambda}_{max} = \hat{\lambda}_{min} = \sigma_w^2$

Hence, if there is no signal, $\hat{\lambda}_{max}/\hat{\lambda}_{min} = 1$; otherwise $\lambda\hat{\ }max/\lambda\hat{\ }min > 1$

The ratio of $\hat{\lambda}_{max}/\hat{\lambda}_{min}$ can be used to detect the presence of the primary signal. However, $\hat{\lambda}_{max}$ and $\hat{\lambda}_{min}$ are the estimated eigenvalues.

A. Detection Algorithm Flow Chart

Figure 2 illustrates the main parts of the proposed method. The sampled signal comes from the radio system interface, from which the covariance matrix is built. The eigenvalues of the matrix are the calculated with a

specific algorithm to form a maximum-minimum ratio; with the users threshold settings defined and signal presence detection done through comparison with the eigenvalues ratio.

B. Eigen-analysis of the Autocovariance Matrix

To better explain the detection algorithm, the eigenvalues of the autocovariance matrix is necessary.

It is assumed that the random process $x(n)$ is, in a wide-sense, stationary and its linear combinations of m basic components $S_i(n)$ is given by

$$x(n) = \sum_{i=1}^{m} a_i S_i(n)$$
(8)

Since the equation observed is $y(n) = x(n) + w(n)$, where $w(n)$ is a complex additive white Gaussian noise sequence with spectral density σ^2, the $M \times M$ autocovariance matrix for $y(n)$ can be expressed as

$$C_{yy} = C_{xx} + \sigma_w^2 I$$
(9)

Where C_{xx} is the autocovariance matrix for the signal $x(n)$,

$\sigma_w^2 I$ is the autocovariance matrix for the noise and M is the length of the covariance matrix. Note that if $M > m$, then C_{xx} which is of dimension $M \times M$ is not of full rank.

Now, an eigen-decomposition of the matrix C_{yy} is performed. Let the eigenvalues be ordered in decreasing value with $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$ and let the corresponding eigenvectors be denoted as $v_i, i = 1, \cdots, M$. It is assumed that the eigenvectors are normalized so that $\mathbf{v}_i^H \mathbf{v}_j = \delta_{ij}$ (H

Denotes the conjugate transpose). In the absence of noise, the eigenvalues $\lambda_i, i = 1, 2, \cdots, m$ are nonzero while $\lambda_{m+1} = \lambda_{m+2} = \cdots = \lambda_M = 0$. Thus, the eigenvectors $v_i, i =$

$1, 2, \cdots, m$ span the signal subspace. These vectors are called *principal eigenvectors* and the corresponding eigenvalues are called *principal eigenvalues*.

In the presence of noise, the eigen-decomposition separates the eigenvectors in two sets. The set $v_i, i = 1, 2, \cdots, m$, which are the principal eigenvectors, span the signal subspace, while the set $v_i, i = m + 1, \cdots, M$, which are orthogonal to the principal eigenvectors, are said to belong to the noise subspace. It follows that the signal $x(n)$ is simply linear combinations of the principal eigenvectors. Finally, the variances of the projections of the signal on the principal eigenvectors are equal to the corresponding eigenvalues of the covariance matrix. So, the principal eigenvalues are the power factors in the new signal space.

In the next subsection, the real maximum eigenvalue $\hat{\lambda}_{max}$ and minimum eigenvalue $\hat{\lambda}_{min}$ of the covariance matrix of the received signal will be obtained.

C. Power Method

In this section, the power method is exploited in order to calculate λ_{max} and λ_{min} for the detection of the primary signal. This way, the eigenvalues can be obtained by simple algebraic operations. This method reduces computational complexity since the eigenvalue decomposition processing is avoided. It is well known that the power method is an effective method to compute the maximum eigenvalue and the corresponding eigenvector (commonly referred to as maximum eigenvector) for a real-valued matrix B. It is important to note that this method is still very effective even if B is a complex valued matrix. For a complex-valued matrix, we have the following theorem.

Theorem 1: For a Hermitian matrix $B \in C^{n \times n}$, if it has n linearly independent eigenvectors $b_1, \cdots, b_n$(k b_i k2= $1, for\ \forall_i \in [1, \cdots, n]$) and its eigenvalues satisfy the following relation $| \lambda_1 |>| \lambda_2 |\geq \cdots \geq| \lambda_n |$. Let v $_0 = \sum_{i=1}^{n} \alpha_i \mathbf{b}_i (\alpha_1 \neq 0)$. Take the vector $\mathbf{v}_0$ as the initial vector, and form a vector sequence according to the power of matrix B as follows:

$$v_k = Bv_k-1$$
$$m_k = max(v_k) = v_{ki}$$
(10)
$$v_k = v_k/m_k \quad (k = 1, 2, \cdots)$$

Where $v_k = [v_{k1}, \cdots, v_{kn}]^T$. Then, any one of the following statements are true:

$$b_1 \qquad max(v_k)$$

$$\lim_{k \to \infty} v_k = \underline{\hspace{1cm}}, or \ \lim \underline{\hspace{1.5cm}} = \lambda_1 \ (11) \ max(b_1) \ k \to \infty \ max(v_{k-1})$$

Proof: Under the above assumptions, the iteration vector v_k can be written as follows:

$$v_k = \mathbf{B}v_{k-1} = \mathbf{B}^k v_0 = \alpha_1 \lambda_1^k \mathbf{b}_1 + \alpha_2 \lambda_2^k \mathbf{b}_2 + \cdots + \alpha_n \lambda_n^k \mathbf{b}_n$$

$$= \alpha_1 \lambda_1^k \left(\mathbf{b}_1 + \sum_{i=2}^{n} \frac{\alpha_i}{\alpha_1} \left(\frac{\lambda_i}{\lambda_1} \right)^k \mathbf{b}_i \right) \qquad (12)$$

Since B is a Hermitian matrix, it easy to know that

$$\lambda_i \geq 0 \quad \forall_i \in [1, \cdots, n]$$

that is,

$$\lambda_1 > \lambda_2 \geq \cdots \geq \lambda_n \geq 0$$

Using the above analysis, $\lim_{k \to \infty} \left(\frac{\lambda_i}{\lambda_1} \right)^k = 0$ Therefore, for all sufficiently large k, it is clear that

$$v_k = \mathbf{B}^k v_0 \approx \alpha_1 \lambda_1^k \mathbf{b}_1$$

From (10), v_k can be expressed as $v^k = \frac{\mathbf{B}^k v_0}{max(\mathbf{B}^k \underline{\hspace{0.5cm}} v_0)}$. Notice that $\alpha_1 \neq 0$ and $v_{ki} = 1$ (it is the maximal component defined in (10) for the vector v_k). It can be easily verified that

$$v_k \xrightarrow{} \frac{\mathbf{b}_1}{max(\mathbf{b}_1)}$$

$$\frac{max(\mathbf{v}_k)}{max(\mathbf{v}_{k-1})} = \frac{\lambda_1^k max \left(\alpha_1 \mathbf{b}_1 + \sum_{i=2}^{n} \alpha_i \left(\frac{\lambda_i}{\lambda_1} \right)^k \mathbf{b}_i \right)}{\lambda_1^{k-1} max \left(\alpha_1 \mathbf{b}_1 + \sum_{i=2}^{n} \alpha_i \left(\frac{\lambda_i}{\lambda_1} \right)^{k-1} \mathbf{b}_i \right)} \to \lambda_1 \qquad (13)$$

This concludes the proof.

According to theorem 1, the maximum eigenvalue λ_{max} of the covariance matrix R can be solved by the power method. Since only one primary signal is concerned, R has only one maximum eigenvalue, the other $M - 1$ eigenvalues are all small eigenvalues. To get a more precise result, the miminum eigenvalue λ_{min} of R is computed as follows

$$\lambda_{min} = \frac{tr(\mathbf{R}) - \lambda_{max}}{M - 1}$$

Where $tr(R)$ represents the trace of R. Finally, the test statistic of the optimal detector is obtained: $= \lambda_{max}/\lambda_{min}$

D. Computational Complexity

Here, the computational complexity of the power method and eigenvalue decomposition when computing eigenvalues is briefly investigated. $O(n^3)$ is used to represent the order of n^3 multiplications. The eigenvalue decomposition processing solves for the complete set of eigenvalues and eigenvectors of the matrix even if the problem requires only a small subset of them to be computed. For the $n \times n$ matrix B , eigenvalue decomposition calls for $2n^3(t + 1)$ real multiplications where t is the maximum number of iterations required to reduce a super-diagonal element as to be considered zero by the convergence criterion [14]. Thus the computational complexity of eigenvalue decomposition is $O(n^3)$. The idea of the power method is only to compute the principal eigenvalues and eigenvectors. The method only consists of two main computational steps:

(a)   Obtaining the iteration vector v_k by computing $v_k = Bv_{k-1}$ *(b)* Computing $v_k = v_k/m_k$ in (10)

Since v_k is an $n \times 1$ vector, the computation of these two steps calls for $4n^2$ and $4n$ real multiplications, respectively. Suppose the number of iterations is S, then the total number of real multiplications is $4S(n^2 + n)$, that is, the computational complexity of the power method has a lower computational complexity than the eigenvalue decomposition processing when computing eigenvalues.

E. Threshold Definition

In the general model of the spectrum sensing algorithm, a threshold must be determined to compare with a test statistic of the sensing metric in order to sense the presence of a primary user. Consequently, to find the threshold for the statistical test, it is important to study the statistical distribution of the covariance matrix.
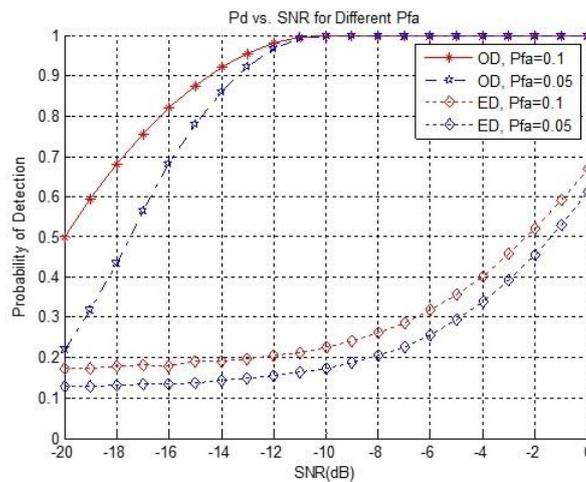
The eigenvalue distribution of R is very complicated [15]. Moreover, there is little or no information about the signal. In fact, it is difficult to know whether the signal is present or not. This in turn makes the choice of the thresholds very difficult. Therefore, in this subsection, random matrix theory is used to approximate the distribution of this random variable and derive the decision threshold based on the pre-defined probability of false alarm, P_{FA}.

When the primary signal is absent,R turns to R_w,the covariance matrix of the noise defined as

$$\mathbf{R}_w = \frac{1}{N_s} \sum_{n=M}^{M-1+N_s} \mathbf{w}^{\hat{}}(n)\mathbf{w}^{\hat{}H}(n) \qquad (14)$$

R_w is nearly a Wishart random matrix [15].

In recent years, the study of the eigenvalue distributions of a random matrix has become a very hot topic in the fields of mathematics, communication and even physics. The joint probability density function (PDF) of ordered eigenvalues of a Wishart random matrix has been known for many years [15]. However, since the

expression for the PDF is very complicated, no closed form expression has been found for the marginal PDF of ordered eigenvalues. Recently though, researchers have found the distribution of the largest eigenvalue [10] and smallest eigenvalue [8] for real and complex matrices.

Fig. 3.Probability of Detection versus SNR for Different Probability of False Alarm

III. Simulations

In the following section, some simulation results are given using randomly generated signals to illustrate the performance of the proposed optimal detection method.

Consider a licensed frequency band in the cognitive radio network with only one active primary user. The primary signal employs Binary Phase Shift Keying (BPSK) modulation and the center frequency is 8 MHz. the sampling rate is set at 32 MHz. N_s is the number of samples and P is the temporal smoothing factor. The results are averaged over 1000 tests using Monte-Carlo realizations (for each realization, random channel, random noise and random BPSK inputs are generated) written in Matlab.

The SNR of a CR is defined as the ratio of the average received signal power to the average noise power over the licensed frequency band.

$$SNR = \frac{E(\|\mathbf{x}(n) - \mathbf{w}(n)\|^2)}{E(\|\mathbf{w}(n)\|^2)} \qquad (15)$$

The probability of false alarm is required to be $P_{FA} \leq 0.1$, then the threshold is found. For Comparison, energy detection is also simulated with noise uncertainty for the same system. The threshold for energy detection is given in [3]. At noise uncertainty, the threshold is always set based on assumed/estimated noise power.

Figure 3 shows the probability of detection curves for optimal-detection and Energy Detection (ED). The results are taken for N_s = 100000 and SNRs varying from −20dB to 0dB. In the optimal detector, the temporal smoothing factor is 8. As shown in the figure, the proposed optimal-detection method can achieve satisfactory detection performance even in low SNR conditions. For example, the optimal-detection method can detect primary user signals with 99% probability at SNR of −10dB. However, the detection probability of ED is less than 70% percent.

From the figure, we can also see that for the same SNR, the probability of detection improves as probability of false alarm
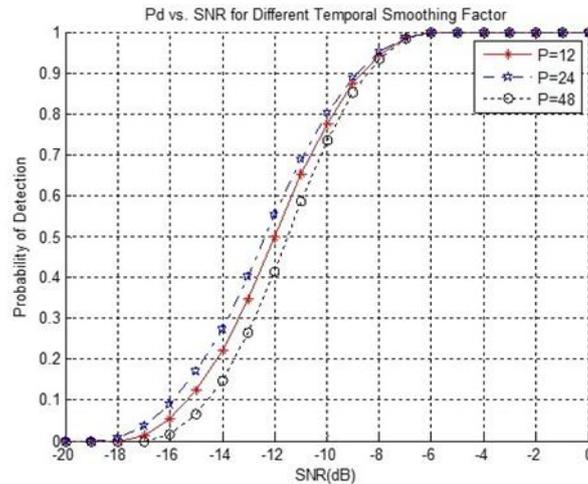

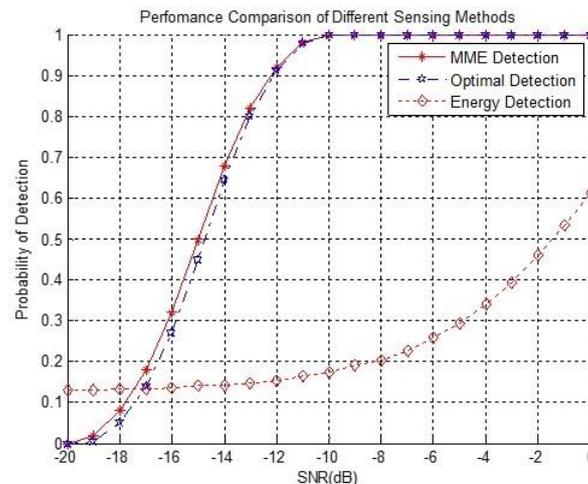Fig. 4. Probability of Detection versus SNR for Different Temporal Smoothing

Fig. 5.Performance Comparison of Different Sensing Methods with P_{FA}= 0.05

Increases. This reflects the trade-off between false alarm and detection probability. The probability of detection versus SNR for different temporal smoothing factors is shown in Figure 4.

The results are taken for P_{FA} = 0.1, and SNRs varying from −20dB to 2dB. It is shown that the detection performance becomes better when P increases from 12 to 24. However, when P turns to 48, the performance detection declines. Therefore, P should be relatively small while using this technique for a given number of samples.

Figure 5 shows the performance comparison of the optimaldetection technique, the MME detection and energy detection. In MME detection, 4 receiving antennas are used for sensing in the radio environment while the optimal detector has a temporal smoothing factor of 16. For all the three methods, a probability factor of P_{FA} = 0.05 is chosen and SNR varied from −20dB to 0dB.

As shown on the figure, the proposed optimal detection technique performs better than the energy detection method.

Also, it can be observed that both MME detection and optimal-detection can detect the primary user signal with 100% probability when the SNR is more than $-10dB$. The performance of the Optimal-detection technique is very close to that of MME detection when the SNR is less than $-10dB$. For example, the detection probabilities of MME detection and optimal-detection are 0.820 and 0.800 at SNR 0f $-13dB$ respectively.

The biggest performance gap between these two methods is only 0.051 with change in SNR. In other words, the proposed optimal-detection method can achieve roughly the same performance as MME detection by using a single antenna. The main reason for this is that the processed data of these two methods have similar structures. The information about the primary user signal is perfectly contained in the data model of both methods, thus they can achieve roughly the same performance.

In summary, all the simulations show that the proposed method works well without using the information of the signal, channel and noise power. The optimal-Detection technique is always better than the energy detection method. Therefore energy detection method is not reliable since it has a low probability of detection and high probability of false alarm when there is noise uncertainty.

IV. Discussions

It is clearly observed that the fundamental problem of spectrum sensing is to discriminate samples that contain only noise from samples that contain a very weak signal embedded in noise. Cognitive radios must be able to detect very weak primary signals. However, fundamental limits arise during detection at low SNR. For example, to set the decision threshold of the energy detector, the noise variance must be known.

If the knowledge of the noise variance is imperfect, clearly the threshold will be erroneous; that is why it is well known that the performance of the energy detector quickly deteriorates if the noise variance is imperfectly known. However, for the optimal detector proposed here, latest random matrix theories have been used to set the decision thresholds and obtain the probability of detection in order to achieve a good detection performance.

Just like the energy detector, the optimal detection method proposed is universal in the sense that it can detect any type of signal, and does not require any knowledge about the signal to be detected. On the other hand, for the same reason, it does not exploit any potentially available knowledge of the signal.

V. Conclusions

A method based on the eigenvalues of the sample covariance matrix of the received signal has been proposed using a single antenna for cognitive radio networks. A temporal smoothing technique is utilized to form a virtual multi-antenna structure.

In order to calculate the maximum and minimum eigenvalues of the covariance matrix obtained by the virtual multi-antenna structure, the proposed method uses power method. Latest random matrix theories have been used to set the decision thresholds and obtain the probability of detection in order to achieve a good detection performance. Simulations using randomly generated signals are presented in order to illustrate the performance of the Optimal-detection method. It has been shown that the performance of Optimaldetection is very close to that of the MME detection with multiple antennas. The method can be used for various signal detection applications without knowledge of signal, channel and noise power. Besides, the proposed optimal-detection method can reduce system overhead and avoid the eigenvalue decomposition processing by utilizing power method. The energy detector is known for its simplicity of implementing and low complexity. However, its weakest point is that it is not effective under the condition that SNR is an unknown, consequently leading to its unguaranteed accuracy.

The eigenvalue-based technique on the other hand, is not as stable as the cyclostationary technique since its threshold varies greatly as it needs to solve the problem of appropriately estimating the size of the covariance matrix. The advantage of the optimal detector, however, is that it does not require knowledge of the primary user signal and performs better than the energy detector.

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