The Prime Numbers

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Introduction:

A prime number is a natural number that has Just two divisors: one and itself. From antiquity until our time, scientists are researching mathematical reasoning to understand the prime numbers; eminent scholars had worked on this field before it is abandoned. Mathematicians considered the prime numbers like « building blocs in building natural numbers » and the field of mathematics the most difficult.
Everything is about numbers, everything is about measure, The understanding of the natural numbers and more general the understanding of the numbers depend on the understanding of the prime numbers. This understanding of the prime will gives us greater ease to understand the other sciences. The prime numbers play a very important role for securing information technology hence promotion of the NTIC, Every year, there is a price for persons who will discover the biggest prime "it's the hunt for the big prime"
This first part of this article about the prime numbers has taken a weight off the scientists 's shoulders by highlighting the universe of the prime numbers and has bring the problem of the prime numbers to an end. The mathematical formulas set out in this article allow us to determine all the biggest prime numbers compared to the capacity of our machines.

Summary:
By a little thinking whose origins are first physics before getting to the mathematics, we have written an article about the prime numbers.

In This article:
1). We have established a diagram called **genesis of the "supposed" prime numbers diagram** (this diagram gives us all the natural numbers except the even numbers and integers divisible by three different of two and three).
2) We have represented the chain of the prime numbers (the chain of the prime numbers is **a broken line exhibiting discontinuities**).
3) We have explained **the origin of the twin prime**.
4) We have explained **the difference between a prime number and a "non" prime number** (Note: a "non" prime number is a number which is not prime).
5) We have determined in the order: the set of the "supposed" prime numbers, the set of the "non" prime numbers, the set of the prime numbers and the subset of the prime numbers (**prime numbers small than a given integer, prime numbers between two whole numbers**).
6) We have done an application of the results.

PLAN:

This article comprises six chapters:

Chapter I: Genesis of the "supposed" prime numbers diagram.

Chapter II: The set of the "supposed" prime numbers.

Chapter III: The "non" prime numbers.

Chapter IV: Infinite and ordered set of the prime numbers.

Chapter V: Subset of infinite and ordered set of the prime numbers.

Chapter VI: Applications

Chapter: Genesis of the "supposed" prime numbers diagram.

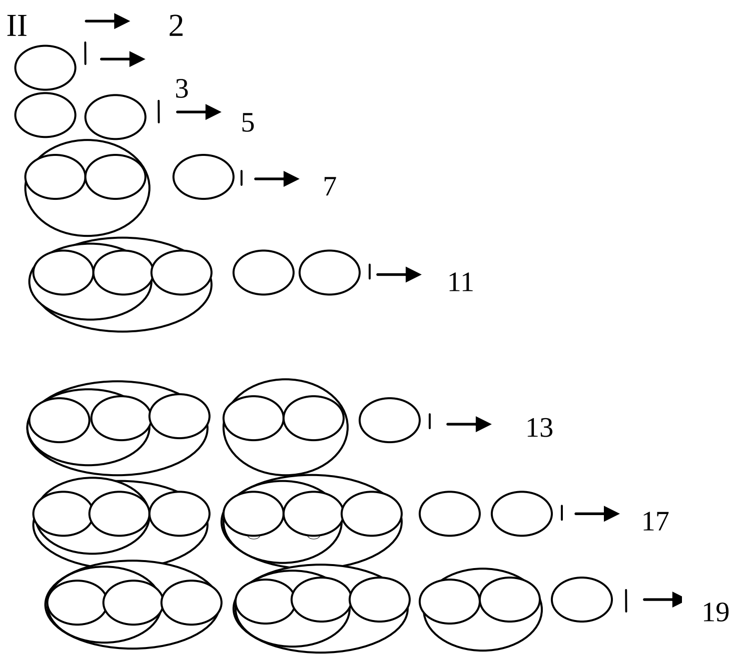
I. Genesis of the "supposed" prime numbers diagram.

1) The diagram

Note:

Each line corresponds to a succession of circles which ends with a bar.

The total number of bars for each line corresponds to a « supposed » prime number.



Note1: <u>The data from the Diagram</u>

1) We could continue to build the diagram until the « supposed » prime number of our choice.
2) Each circle contains at maximum six bars and at minimum two bars.
3) We found that the data from the diagram is all the natural numbers except the even numbers and natural numbers divisible by three.

Note3: <u>The method</u>

1) One will seek to translate the data from the diagram into mathematical formulas; the obtained formulas form the set of the "supposed" prime numbers.
2) The set of the "supposed" prime numbers includes the set of the prime numbers and the set of "non" prime numbers (a "non" prime number is a "supposed" prime which is not prime).
3) In the following time, we will seek to isolate in this previous set the "non " prime numbers, the remaining numbers form the set of the prime numbers.

Note4: <u>the comprehension of the Diagram</u>

1) The logic used to build the diagram is far from the logic used by the mathematicians (One must think like a physician to build the diagram).
2) The hardest to understand in this article is the logic used to build the diagram named "to understand the diagram".
3) **The part named "to understand the diagram" is reserved for the whole book because they are too many theories**.

The whole book is entitled the naturals numbers.

4) Without understanding the logic used to build the diagram you can understand the essence of this article, you only have to read and understand the following pages.

Note5: **Humanity hasn't understand the natural numbers until this moment (naturally, before the onset of this article).**

II. Exploitation of the data from the diagram :

1) Difference between two successive « supposed » prime numbers :

The illustration of these differences amounts to arrange the obtained numbers in increasing order, showing the added value to each number to get the following number.

2 ⟶ +1
3 ⟶ +2
5 ⟶ +2
7 ⟶ +4
11 ⟶ +2
13 ⟶ +4
17 ⟶ +2
19 ⟶ +4
23 ⟶ +2
25 ⟶ +4
29 ⟶ +2
31 ⟶ +4
35 ⟶ +2
37 ⟶ +4

41 → +2
43 → +4
47 → +2
49 → +4
53 → +2
55 → +4
59 → +2
61 → +4

2) Interpretation:

The arrangement of the « supposed » prime, in increasing order makes the diagram ridiculous because you only need, from 5, to add alternatively 2 (to get the following number) and then 4 (to get the number that follow this previous following number) and so on.

Note: The first nine "supposed" prime numbers are all prime numbers,

After 23, we have a mixture of prime numbers and "non" prime numbers based on a logic we don't know yet.

That inspires us mathematical formulas that condition the "supposed" prime numbers,

Hence the need to translate these obtained data into mathematical formulas.

III. Formulas for the numbers from the diagram:

a) The « supposed » prime numbers :

The obtained numbers from the diagram are a mixture of prime numbers and "non" prime numbers, that's why we call them "supposed" prime numbers.

b) Demonstration :

5 → 5
5+2 → 7
5+2+4 → 11
5+2+4+2 → 13
5+2+4+2+4 → 17
5+2+4+2+ 4+2 → 19
- - - - - -

Let $n_1$and n_2 be two parameters such as$n_{1;}n_2 \in N$

n_1: the number of two added to 5 to get a new number.

n_2: the number of four added to 5 to get the same previous new number.

We also see that an obtained number from the diagram is obtained by this following formula:

$5 + 2n_1 + 4n_2$

***Relation between n_1 and n_2:**

There are two possibilities:

$n_1 = n_2$ or $n_1 = n_2+1$ (that means $n_2 = n_1-1$)

For $n_1 = n_2$:

$5 + 2n_1 + 4n_2 = 5 + 2n_1 + 4n_1 = 5 + 6 n_1$ avec $n_1 \geq 0$

For $n_1 = n_2 + 1$ or $n_2 = n_1 - 1$

$5 + 2n_1 + 4n_2 = 5 + 2n_1 + 4 (n_1 - 1) = 5 - 4 + 6n_1 = 6n_1 + 1, n_1 \geq 1$

$5 + 2n_1 + 4n_2 = 5 + 2(n_2 + 1) + 4n_2 = 7 + 6n_2$; $n2 \geq 0$.

Three mathematical formulas have been obtained:

$6n_1+5$ avec $n_1 \geq 0$; $6n_1+1$ avec $n_1 \geq 1$ ou $6n_2+7$avec $n_2 \geq 0$

Two of these previous three formulas are equivalent, it's:

$6n_1+1$ avec $n_1 \geq 1$ ou $6n_2 + 7$avec $n_2 \geq 0$

When we reduce $n \geq 0$ to $n \geq 1$, we obtain the following two formulas:

$Un = 6n+5$ and $V_n = 6n+7$, $n \in N$

When we reduce $n \geq 1$ to $n \geq 0$, we obtain the following two formulas:

$Un = 6n - 1$ et $V_n = 6n + 1$, $n \in N^*$ Soit $6n \pm 1$, $n \in N$:

For $n_1 = n$; $n_2 = n - 1$

$6n_2 + 5 = 6(n - 1) + 5 = 6n - 1$

In the following pages we will focus on better illustrate the chain of the prime numbers:

$Un = 6n+5$ et $V_n = 6n+7$, $n \in N$

Chapter I: The set of the « supposed » prime numbers.

I. The set of the « supposed » prime numbers:

1. Definition

The « supposed » prime numbers are 2; 3 and numbers which come from the two following formulas:

$U_n = 6n + 5$; $V_n = 6n + 7$, $n \in N$

1. To identify the « supposed » prime numbers except 2 and 3:

a. Parameters of a « supposed » prime number:

$U_n = 6n + 5$; $V_n = 6n + 7$ and $n \in N$

$\frac{U_n - 5}{6} = n$; $\frac{V_n - 7}{6} = n$ and $n \in N$

Consequence:

Let N be a natural number, different of 2 and 3:

If $\frac{N-5}{6} \in N$ or $\frac{N-7}{6} \in N \Rightarrow N$ is a "supposed" prime number.

3. The set of the « supposed » prime numbers:

This set is formed by $6n + 5$ and $6n + 7$, $n \in N$ taking into account 2 and 3.

Let Esp be the **set of the « supposed » prime numbers**

$Es_p = \{2 ; 3 ; 6n + 5 ; 6n + 7, n \varepsilon N\}$ or $Es_p = \{2 ; 3 ; 6n - 1 ; 6n + 1, n \varepsilon N^*\}$

Note:

This set contains all the prime numbers and the « non » prime numbers.

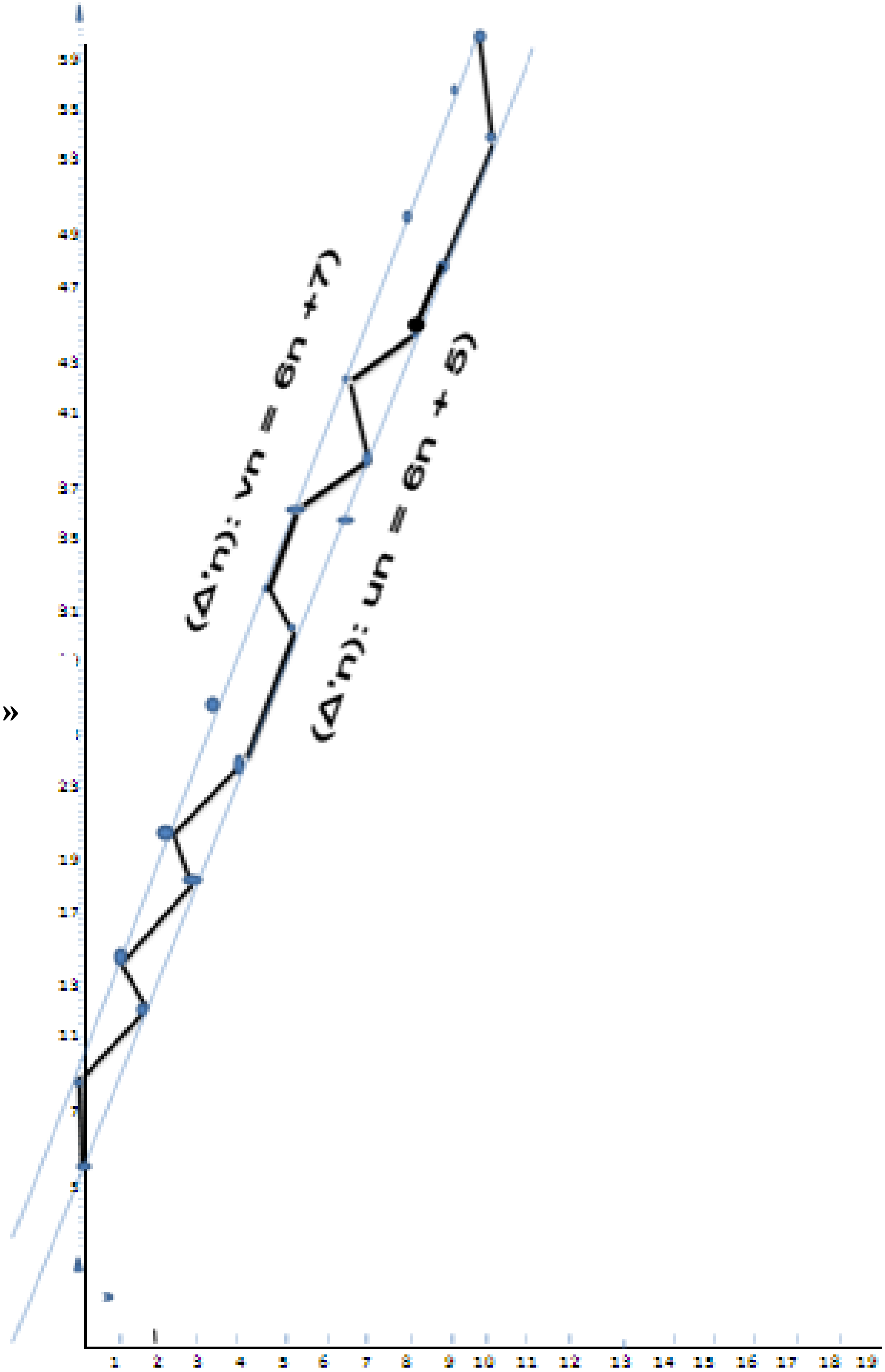
Let Ep be the set of the prime numbers and Enp be the set of the “non” prime numbers.

Esp = EpUEnp

I. The chain of the prime numbers:

1. Graphical Representation in an orthonormal coordinate system of the “supposed” prime numbers:

« Supposed »



n∈ N

2)Analysis:

*$U_n < V_n$ ∀ n∈N

*The couples (U_n ;Vn) are ordered.

*If two « non » prime numbers are obtained by the couple for n defined, we obtain a discontinuity on the chain.

*The first discontinuity is obtained for n = 35

6 x 35 + 5 = 215 divisible by 5

6 x 35 + 7 = 217 divisible by 7

*We deduct that the chain of the prime numbers is **a broken line exhibiting discontinuities.**

Note: The twin prime are two prime numbers which come from the couples ($U_n = 6n + 5$,$V_n = 6n + 7$; n ∈N and n defined).

Twin prime are the same number of Keurs noted k such as k = n + 2, n ∈N**.**

2) Generality:

If (Un = 6n + 5 ,Vn = 6n + 5 n ∈N) are prime, the couple has said twin prime.

If (Un = 6n + 5 , Vn = 6n + 5 n ∈N) are « non » prime, the couple has said twin « non » prime.

If one of (Un = 6n + 5 ,Vn = 6n + 5 n ∈N) is a prime number and the other a "non" prime number, the couple has said twin mixed.

III. Relations

1. Relation between U_n and U_n + 1

$U_n = 6n + 5$; $U_{n+1} = 6(n + 1) + 5 = 6n + 6 + 5 = 6n + 11$

$\Delta Un = U_{n+1} – U_n = 6n + 11 – 6n – 5 = 11 – 5 = 4$

$U_{n+1} = U_n + \Delta Un$ ⇒ $\mathbf{U_{n+1} = U_n + 6}$

2. Relation between V_n and V_{n+1} :

$Vn = 6n + 7$;$\mathbf{V_{n+1}} = 6 (n +1) + 7 = 6n +6 + 7 = 6n + 13$

$\Delta V_n = \mathbf{V_{n+1}} – V_n = 6n + 13 – 6n – 7 = 6$ ⇒$\Delta V_n = 6$

$\mathbf{V_{n+1} = V_n + 6}$

3. Relation between U_n and Vn

$V_n – U_n = 6n + 7 – 6n – 5 = 7 – 5 = 2$

$\mathbf{V_n = U_n + 2}$ And $\mathbf{U_n = V_n - 2}$

4. Sums of the (n + 1) first terms of Un and Vn

$S_{Un} = U_0 + U_1 + . - + U_{n-1} + U_n = 6 \times 0 + 5 + 6 \times 1 + 5 + -- + 6 \times (n - 1) + 5 + 6 \times n + 5$
$= 6 (0 + 1 + 2 +. - - + n) + 5(n + 1)$

$= 6 (1 + 2 + - \quad + n) + 5(n + 1)$ or $1 + 2 + . + n = \frac{n(n+1)}{2}$

$= 6 \times \frac{n(n+1)}{2} + 5(n + 1) = 3n (n + 1) + 5((n + 1) = (n +1) (3n + 5)$

$S_{un} = (n + 1) (3n + 5)$

$S_{vn} = V0 + V1 +. - +V_{n-1} + V_n = 6 \times 0 + 7 + 6 \times 1 + 7 +. - + 6 \times (n - 1) + 7 + 6n + 7$
$= 6 (0 + 1 + 2 + - + n) + 7(n+1)$

$= 6 (1 + 2 + - + n) + 7(n+1) = 6 \times \frac{n(n+1)}{2} + 7 (n + 1)$

$= 3n (n + 1) + 7 (n + 1) = (n + 1) (3n + 7)$

$S_{vn} = (n + 1) (3n + 7)$

5. Sum of sums of the (n+1) first terms of Un andVn
$S_{Un} + S_{vn} = (n +1) (3n + 5) + (n + 1) (3n + 7) = (n +1) [6n + 12] = 6(n + 1) (n + 2)$
$S_{Un ;Vn} = S_{Un} + S_{Vn} = 6 (n + 1) (n + 2)$

Chapter II: The set of the « supposed » prime numbers.

I. The set of the « supposed » prime numbers:

1. definition

The « supposed » prime numbers are 2 ; 3 and numbers which come from the two following formulas :

$$U_n = 6n + 5 \; ; V_n = 6n + 7 \; , n \in N$$

2. To identify the « supposed » prime numbers except 2 and 3:

a. Parameters of a « supposed » prime number:

$$U_n = 6n + 5 \; ; V_n = 6n + 7, n \epsilon N$$

$$\frac{U_n - 5}{6} = n \; ; \; \frac{V_n - 7}{6} = n \; , n \epsilon \; N$$

b. Consequence:

Let N be a natural number, different of 2 and 3:

If $\frac{N-5}{6} \in N$ or $\frac{N-7}{6} \in N \Rightarrow N$ is a "supposed" prime number.

3. The set of the « supposed » prime numbers:

This set is formed by 6n + 5 and 6n + 7, n ε N taking into account 2 and 3.

Let Esp be the **set of the « supposed » prime numbers**

Es_p = {2 ; 3 ; 6n + 5 ; 6n + 7, n ε N} or Es_p = {2 ; 3 ; 6n - 1 ; 6n + 1, n ε N*}

Note: This set contains all the prime numbers and the « non » prime numbers.

Let Ep be the set of the prime numbers and Enp be the set of the "non" prime numbers.

Esp = EpUEnp

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Chapter III : The « non » prime numbers.

I. The « non » prime numbers:

1) Definition

A « non » prime number is a "supposed" prime that has at minimum three divisors.

2)Relation between two « supposed » non-prime numbers.

a) Relation between two « non » prime U:

U_1=6n_1+5, U_2=6n_2+5, n ε N,

Let K_1 and K_2 be two supposed prime.

$K_1U_1=K_2U_2 \Rightarrow K_1(6n_1+5) = K_2(6n_2+5)$

$6K_1n_1+5K_1= 6K_2n_2+5K_2$

$6(K_1n_1-k_2n_2)=5(k_2-k_1) \Rightarrow 5(k_2-k_1) =6n$

Let $n_3=\frac{n}{5}$, $n_3 \in N$ $\Rightarrow k_2-k_1 = 6n_3$

$\Rightarrow K_2= 6n_3+k_1$

Note: To better illustrate the types of « non » numbers, let us consider V = 6n+1.

•If $K_1=1 \Rightarrow K_2= 6n_3+1$ donc $K_2=V \Rightarrow V_1=V_2V_3$

$\Rightarrow [n_1-(6n_3+1)\,n_2] =5n_3 \Rightarrow n_1-6n_3n_2-n_2=5n_3$

$\Rightarrow n_1=5n_3+6n_3n_2+n_2$

$$\mathbf{U_1=U_2V_3 \Leftarrow> n_1=5n_3+6n_3n_2+n_2} \quad (1)$$

•If $K_1=5 \Rightarrow K_2=6n_3+5 \Rightarrow K_2=U \Rightarrow 5U_1 = U_2U_3$

$\Rightarrow 6\,[5n_1-(6n_3+5)\,n_3]=5 \times 6 \times n_3$

$\Rightarrow 5n_1 - 6n_2n_3-5n_2 = 5n_3$

$$\Rightarrow \mathbf{5n_1 = 5n_3+6n_2n_3 + 5n_2 \Rightarrow 5U_1 = U_2U_3} \quad (2)$$

a) Relation between two V:

$K_1V_1=K_2V_2 \Rightarrow K_1\,(6n_1+1)=K_2\,(6n_2+1)$

$\Rightarrow 6K_1n_1+K_1=6K_2n_2+K_2$

$\Rightarrow 6\,(K_1n_1-K_2n_2)=K_2-K_1$

$\Rightarrow K_2=6\,(K_1n_1-K_2n_2)+K_1 \Rightarrow K_2= 6n_3 + K_1$

• If $K_1=1 \Rightarrow K_2 = 6n_3+1 \Rightarrow K_2 =V \Rightarrow V_1=V_2V_3$

$\Rightarrow n_3=[n_1-n_2\,(6n_3+1)]=n_1-6n_2n_3-n_2$

$$\Rightarrow \quad \mathbf{n_3=n_1-6n_2n_3-n_2} \qquad \mathbf{V_1=V_2V_3} \quad (3)$$

• If $K_1=5 \Rightarrow K_2=6n_3+5 \Rightarrow K_2=n_3 \Rightarrow 5V_1=n_3V_2$

$\Rightarrow n_3 = n_1K_1 - n_2K_2 = 5n_1-n_2\,(6n+5) = 5n_1-6n_2n_2-5n_2$

$$\mathbf{5n_1 = n_3 + 6n_2n_3 + 5n_2 \Leftrightarrow 5V_1=U_3V_2} \quad (4)$$

a) Relation between U and V

$K_1U_1=K_2V_2 \Rightarrow K_1\,(6n_1+5) = K_2\,(6n_2+1)$

$K_1\,(6n_1+5) = K_2\,(6n_2+1) \Rightarrow 6K_1n_1 + 5K_1 = 6K_2n_2 + K_2$

$6\,(K_1n_1-K_2n_2) = K_2-5K_1 = 6n \Rightarrow K_2 = 6n_3 + 5K_1$

• Si $K_1 = 1 \Rightarrow K_2 = 6n_3 + 5 \Rightarrow K_2 = U_3 \Rightarrow U_1 = U_3V_2$ $\qquad K_2-5=6n_3$

$n_3 = n_1 - n_2\,(6n_3+5) = n_1 + 6n_2n_3 - 5n_2$

$\Rightarrow$ $\boxed{\mathbf{n_1 = n3_3 + 6n_2n_3 + 5n_2 \Leftrightarrow U_1 = U_3V_2}}$ (5)

• If $K_1 = 5 \Rightarrow$ $K_2 = 6n_3 + 25 =$ $6n_3 + 24+1 = 6(n_3 + 4) + 1 = V_4$,$n_4 = n_3 + 4$

$K_2 - 25 = 6n_3$; $K_2 = 6n_3 + 25$

$5n_1 - (6n_3 + 25)\,n_2 = 6n_3 \Rightarrow 5n_1 = 6n_3 + n_2\,(6n_3 + 25) = 6n_3 + n_2\,(6n_3 + 24+1) = 6n_3 + n_2\,(6n_4 + +1) = 6n_3 +6\,n_2n_4 + n_2$

$\Rightarrow$ $\boxed{\mathbf{5n_1} = 6n_3 + 6\,n_2n_4 + n_2 \Leftrightarrow \mathbf{5U_1 = V_4V_2}}$ (6)

$K_1U_1 = K_2V_2 \Leftrightarrow 6(K_2n_2 - K_1n_1) = 5K_1 - K_2 = 6n_3$

$\Leftrightarrow 5K_1 = 6n_3 + K_2$

Si $K_2 = 5 \Rightarrow K_1 = \frac{6n3+5}{5} = \frac{6n3}{5} + 1 \Rightarrow K_1V_3$

$\Rightarrow V_3U_1 = 5\,V_2$

$5K_1 - 5 = 6n_3 \Rightarrow 6\,[5n_2 - (\frac{6n3}{5} + 1)\,n_1] = 6n_3$

$\Rightarrow 5n_2 - \frac{6n3}{5}\ x\ n_1 - \ \ n_1 = n_3$

$\boxed{\mathbf{5n_2 = n_3 + \frac{6n1+n3}{5} + n_1 \Leftrightarrow 5V_2 = U_1V_3}}$ (7)

If $k_2 = 1 \Rightarrow 5K_1 = 6n_3 + 1$

$\Rightarrow V_3 = 5K_1 \Rightarrow K_1 = \frac{V3}{5} \Rightarrow \frac{V3U1}{5} = V_2 \Rightarrow U_1V_3 = 5V_2$

$6[5n_2 - (\frac{6n3}{5} + 1)n_1] = 6n_3 \Rightarrow 5n_2\,\frac{6n3+n1}{5} - n_1 = n_3$

$\boxed{\mathbf{5n_2 = n_3 + \frac{6n1\,n3}{5} + n_1 \Leftrightarrow 5V_2 = U_1V_3}}$ (8)

b) **Note :**

The equalities 7 and 8 are equivalent

As well as the equalities1 and 5 are equivalent.

We obtain these following equalities:

1 : $U_1 = U_2U_3$; **2** : $5U_1 = U_2U_3$; **3** : $V_1 = V_2V_3$; **4** : $5V_1 = V_3V_3$;
5 : $U_1V_3 = 5V_2$; **6** : $5U_1 = V_4V_2$.
We see that 3 and 6 are equivalent because $V_2V_4 = V_2V_3 \Rightarrow V_1 = 5U_1$
If we replace $5U_1$ by V_1 in 2 we obtain:

$$\boxed{\mathbf{V_1 = U_1U\ V_3}}$$

We end up with these following height equalities:
$U_1 = U_2V_2$; $5U_1 = U_2U_3$; $V_1 = V_2V_3$; $5V_1 = U_3V_3$; $U_1U_3 = 5V_2$; $5U_1 = V_4V_2$; $V_1 = U_1U_2$.
Three of them represent the « non » prime numbers, it's:
$U_1 = U_2V_2$; $V_1 = V_2V_3$; $V_1 = U_1U_2$.

c) Consequence:

The « non » prime numbers for U are in the form UV and the « non » prime numbers for V are in the form UU or VV.

II. Formulas for the « non » prime numbers:

1) The parameters of the "non" prime numbers:

Instead of $V_n = 6n + 1$, $n \in N$, we choose $V_n = 6n+7$ for reducing $n \geq 1$ to $n \in N$ $(.n\geq 0\ )$
$U_3 = U_1V_1 \Rightarrow 6n_3 + 5 = (6n_1 + 5)(6n_3 + 7) = 36n_1n_2 + 42n_1 + 30n_2 + 35$
$\Rightarrow 6n_3 = 36n_1n_2 + 42n_1 + 30n_2 + 30$

$$\boxed{\mathbf{U_3 = U_1U_1 \Rightarrow n_3 = 6n_1n_2 + 7n_1 + 5n_2 + 5}}$$

$V_3 = V_1V_2$ ou $V_3 = U_1U_2$
$V_3 = V_1V_2 \Leftrightarrow (6n_3 + 7) = (6n_1 + 7) = 36\ n_1n_2 + 42n_1 + 42n_2 + 49$
$\Leftrightarrow 6n_3 = 36\ n_1n_2 + 42n_1 + 42n_2 + 42$
$\Leftrightarrow n_3 = 6\ n_1n_2 + 7n_1 + 7n_2 + 7$

$$\Rightarrow \boxed{\mathbf{n_3 = 6\ n_1n_2 + 7(n_1 + n_2) + 7 \Leftrightarrow V_3 = V_1V_2}} \quad (2)$$

$6n_3 + 7 = (6n_1 + 5)(6n_2 + 5) = 6\ n_1n_2 + 30n_1 + 30n_2 + 25$
$= 36\ n_1n_2 + 30(n_1 + n_2) + 25$
$\Rightarrow 6n_3 = 36n_1n_2 + 30\ (n_1 + n_2) + 18$

$$\Rightarrow \boxed{\mathbf{n_3 = 6\ n_1n_2 + 5(n_1 + n_2) + 3}} \quad (3)$$

2) Formulas for the « non » prime numbers:

$6n_3 + 5$; $n_3\ \varepsilon$ $\mathbf{\{6\ n_1n_2 + 7n_1 + 5\ n_2 + 5 \text{ and } n_1\ ;\ n_2 \varepsilon\ N^2\}}$
$6n_3 + 7$; $n_3\ \varepsilon$ $\mathbf{\{6\ n_1n_2 + 7(n_1 + n_2) + 7;\ 6n_1n_2 + 5(n_1 + n_2) + 3 \text{ and } n_1\ ;\ n_2\ \varepsilon\ N^2\}}$

Conclusion1:

The above formulas show the famous secret of the « non » prime numbers by making people understand the difference between a prime number and a « non » prime number.

The alternation between prime numbers and "non" prime numbers is not a question of interval or periodicity that depends on the parameters nij of the « non » prime numbers.

nijε **{6ij + 7i + 5j + 5 avec i-j ε N²} u {6ij + 7(i+j) +7 ; 6ij +5(i+j) +3 avec i ; j ε N²}**

Let N be a natural number, N is prime if and only if:

$\frac{N-5}{6}$ **ε N \{6ij + 7i +5j avec i ; j ε N²} or** $\frac{N-7}{6}$ **ε N \{6ij+7(i + j) +7 ; 6ij+7(i+j) +7 avec i ; j ε N²}.**

Chapter IV: Infinite and ordered set of the prime numbers.

I. Infinite and ordered set of the prime numbers :

1) Arrangement of the « supposed » prime numbers :

The couples (6n + 5; 6n + 7, n **ε N**) are ordered and grow following the increasing values of n.

The couple (2; 3) is ordered.

We will need to take into account the couples in order to place the « supposed » prime in order.

2) Infinite and ordered set of the "supposed" prime numbers:

Let Esp be the infinite and ordered set of the "supposed" prime numbers:

Esp = **{**(2 ; 3) ; (6n + 5 ; 6n + 7) avec n**ε N}**

II. Infinite and ordered set of the "supposed" prime numbers:

1) Infinite set of the "non" prime numbers:

a) First form of representation of the set of the « non » prime numbers:

Let Enp be the infinite set of the "non" prime numbers:

Formulas for the « non » prime had been established in the previous chapter.

Enp = {$6n_3 + 5$ avec n_3**ε**{$6n_1n_2 + 7n_1 + 5n_2 + 5$ avec n_1, n_2ε N²} ;

$6n_3 + 7$ avec n_3**ε** {$6n_1n_2 + 7(n_1 + n_2) + 7$; $6n_1n_2 + 5(n_1 + n_2) + 3$ avec n_1 ; n_2ε N²}

b) Second form of representation of the set of the « non » prime numbers:

We have demonstrated in the previous chapter that the « non » prime numbers are thefollowing products : UU ; VV or UV.

Let i and j be two parameters such as i ; j ε N²

UiUj: [6i + 5] [6j + 5]; ViVj : [6i + 7] [6j + 7]; UiVj : [6i + 5] [6j + 7]

Enp = {[6i + 5] [6j + 5] ; [6i + 7] [6j + 7] ; [6i + 5] [6j + 7] such as i ; j ε N²}

Note: The members of this set need to be arranged.

2) Infinite and ordered set of the prime numbers :

Let Ep be the infinite and ordered set of prime numbers:

***First form of representation:**

Ep = {(2 ; 3) ; (6n + 5 ; 6n + 7) , n ε N} \ {$6n_3 + 5$, nε {$6n_1n_2 + 7n_1 + 5n_2$, n_1 ; n_2ε N²} ; $6n_3 + 7$, n_3ε {$6n_1n_2 + 7(n_1 + n_2) + 3$, n_1 ; n_2ε N²}}

***Second form of representation:**

Ep= {(2 ; 3) ; (6n + 5 ; 6n + 7) , n ε N} \ {[6i + 5] [6j + 5] ; [6i + 7] [6j + 7] ; [6i + 5] [6j + 7] such as i ; j ε N²}

Note: This second form of the representation is simpler than the previous one.

ChapterV: Subset of the infinite and ordered set of the prime numbers :

I. The ordered set of the prime numbers smaller than a given integer :

1) The set of the « supposed » prime smaller than a given integer :

Let M be an integer ⇒M ε N and $Esp_{<M}$: the set of the prime numbers smaller than M.

$Esp_{<M}$ = {(2 ; 3)< M ; (6n + 5, 6n + 7) < M , n ε N}

***<u>Question</u>**: What is the maximum value of n ?

Let n_{max} be the maximum value of n ?

n_{max} is obtain from 6n + 7.

$6n_{max} + 7 < M \Leftrightarrow n_{max} < \frac{M-7}{6}$

The most logical choice for rn_{max} is $n_{max} = E\left(\frac{M-7}{6}\right)$: that means the integral part of $\frac{M-7}{6}$

$Ep_{<M}$ = {(2 ; 3) < M ; (6n + 5 ; 6n + 7) < M avec n ε {0 ; - ; E $\left(\frac{M-7}{6}\right)$}∩ N}

2) The set of "non" prime numbers smaller than M such as M ε N :

a) The set of "non" prime numbers smaller than M .

Let Enp< M be the set of "non" prime numbers smaller than M

Note: There are two ways to represent this set:

❖ **The first representation:**

Enp < M = {$6n_3 + 5$ < M avec n_3ε {6ij + 7i + 5j + 5 avec i ; jε N²} ;

6n + 7 < M avec ε N {6ij + 7 (i + j) + 7 ; 6ij + 5 (i + j) + 3 avec i ; j ε N²}

❖ **The second representation:**

Enp< M = {[6i + 5] [6j + 5] < M; [6i + 7] [6j + 7] < M; [6i + 5] [6j + 7] < M avec i ; j ε N²}.

Note: The members of this set need to be arranged.

❖ **Calculation Method:**

We will propose a calculation method for the second representation.

<u>* Question:</u>

What are the minimum and the maximum values for i and j ?

- $[6i + 5] [6j + 5] < M$

j_{max} is obtained for r i = 0

$i = 0 \Rightarrow 5 [6j_{max} + 5] < M \Rightarrow j_{max} < \frac{\frac{M}{5} - 5}{6}$

$\Rightarrow j_{max} < \frac{M-25}{30}$

The most logical choice for jmax is the integral part of $\frac{M-25}{30}$.

$i_{max} = E\left(\frac{M-25}{30}\right)$

In the same way $jmax = E\left(\frac{M-25}{30}\right)$

$$\mathbf{I_{max} = j_{max} = E\left(\frac{M-25}{30}\right)}$$

That imply that $[6i + 5] [6j + 5] < M$ and $i ; j \in \{[0 ; E\left(\frac{M-25}{30}\right)] \cap N\}^2$

- $[6i + 7] [6j + 7] < M$

J_{max} is obtained for $i = 0$;

$\Rightarrow 7[6j_{max} + 7] < M \Leftrightarrow j_{max} < \frac{M-49}{42}$

$jmax = E\left(\frac{M-49}{42}\right)$

$$\mathbf{I_{max} = j_{max} = E\left(\frac{M-49}{42}\right)}$$

$\Rightarrow [6i + 7] [6j + 7] < M$ eti ; j ; $\in \{[0 ; E\left(\frac{M-49}{42}\right)] \cap N\}^2$

- $[6i + 5] [6j + 7] < M$

$i=0 \Rightarrow 5 [6j_{max} + 7] < M \Leftrightarrow 6j_{max} + 7 < \frac{M}{5} \Leftrightarrow j_{max} < \frac{M-35}{30} \Rightarrow j_{max} = E\frac{M-35}{30}$

$j=0 \Rightarrow 5 [6i_{max} + 5] < M \Leftrightarrow 6i_{max} + 5 < \frac{M}{7} \Leftrightarrow i_{max} < \frac{M-35}{42} \Rightarrow i_{max} = E\frac{M-35}{42}$

$\Rightarrow$ [6i + 5] [6j + 7] < M avec i ε {[0 ; E ($\frac{M-35}{42}$)] ∩N} et j ε{[0 ; E ($\frac{M-35}{30}$)]

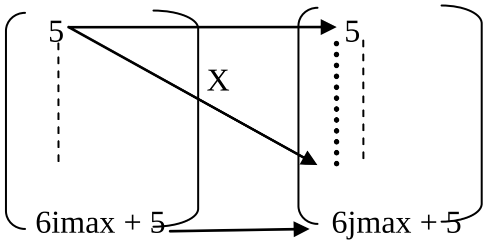
<u>*To calculate:</u>

$[6i + 5]i_{max}$ x $[6j + 5]\ j_{max}$;$[6i + 7]i_{max}$x $[6j + 7]j_{max}$;$[6i + 5]i_{max}$x $[6j + 7]j_{max}$

The method of calculation $[6i + 5]i_{max}$ x $[6j + 5]\ j_{max} < M$:

- **Illustration 1 :** (to calculate [6i + 5] [6j + 5] < M

$[6i + 5]i_{max}$ x $[6j + 5]\ j_{max} < M$ =



< M

For each i, we will distribute the 6i + 5 to all the 6j + 5 until getting a number greater than or equal to M. If the obtained number is greater than M we would eliminate it.

For i = 0 ; it is obvious that all the5(6j + 5) products are lower than M.

Note1:

A similar logic has also been applied to $[6i + 7]i_{max}$x $[6j + 7]j_{max}$

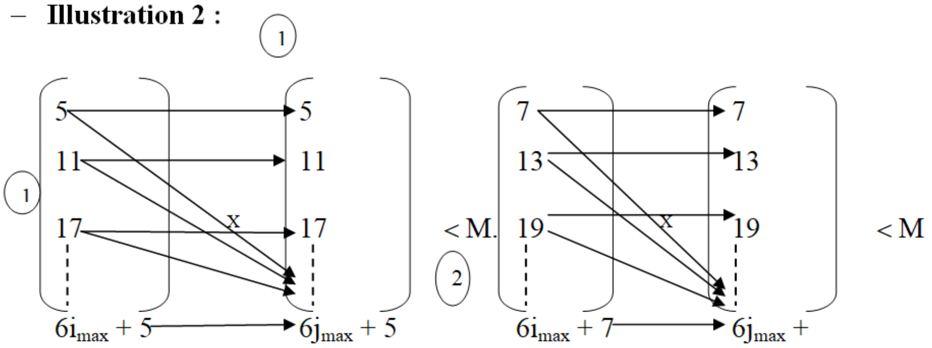
Note2:

For [6i + 5] x [6j + 5] and [6i + 7] x [6j + 7]

For each i calculation starts with the corresponding j.

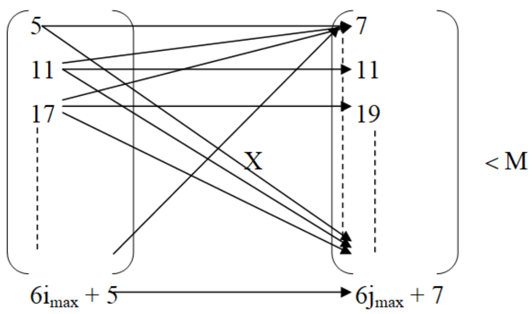
Therefore $(6i + 5)^2$ or $(6i + 7)^2$ is the first obtained number.

- **Illustration 2 :**



Note2: The above illustrations are excluvelly valid for $[6i + 5]i_{max}$x $[6j + 5]j_{max}$ and $[6i + 7]i_{max}$ x $[6j + 7]j_{max}$.

Illustration 3 : For the products $[6i + 5]i_{max}$ $[6j + 7]j_{max}$



***Finally the simplest representation of the set of the « non » prime number is the form bellow*:**

$$\textbf{Enp} < \textbf{M} = \left\{ \begin{array}{l} [6i + 5]i_{max} \times [6j + 5]\, j_{max} < M ,i ; j\ \varepsilon\ \{[o ; E (\frac{M-25}{30})] \cap N\}^2 ; \\ [6i + 7]i_{max} \times [6j + 7]j_{max} < M ,i ; j\ \varepsilon\ \{[o ; E (\frac{M-35}{42})] \cap N\}^2 ; \\ [6i + 5]i_{max} \times [6j + 7]j_{max} < M ,i\ \varepsilon\ \{[o ; E (\frac{M-49}{42})] \cap N\} \text{ et} \end{array} \right\}$$

$$j\ \varepsilon\ \{[o ; E (\frac{M-49}{30})] \cap N$$

3) **The ordered set of the prime numbers smaller than M and M ε N:**

Note: There are two ways to represent this set but this following form is simpler than the other.

$$\textbf{Ep} < \textbf{M} = \left\{ \begin{array}{l} \{ (2 ; 3) < M ; (6n + 5 ; 6n + 7) < M , n\ \varepsilon\ \{[o ; E (\frac{M-7}{6})] \cap N\} \} \\ [6i + 5] [6j + 5] < M ,i ; j\ \varepsilon\ \{[o ; E (\frac{M-35}{42})] \cap N\}^2 ; \\ [6i + 7] [6j + 7] < M ,i ; j\ \varepsilon\ \{[o ; E (\frac{M-49}{42})] \cap N\}^2 ; \end{array} \right\}$$

$$[6i + 5] [6j + 7] < M ,i ;\ \varepsilon\ \{[o ; E (\frac{M-35}{42})] \cap N\} \text{ et}$$

$$j \varepsilon \ \{[o ; E(\frac{M-35}{30})] \cap N\}$$

II. The ordered set of the prime numbers between two whole numbers.

Let M1 and M_2 be two integers such as $M_1 < M_2$

Let M be a prime number between M_1 and $M_2 \Rightarrow M_1 < M < M_2$.

Let Ep< M2 be The ordered set of the prime numbers smaller than.

Let Ep< M1 be the ordered set of the prime numbers smaller than M_1.

Let Ep< M1 < M < M2 be the ordered set of the prime numbers between M_1 and M_2.

Ep < M1 < M < M2 = Ep < M2 \ Ep <

Ep< M1 < M < M2 =

$\{(2 ; 3) < M_2 ; (6n+5 ; 6n+7) < M_2, n \varepsilon \{[o ; n_{max}] \cap N]\}$

$[6i+5][6j+5] < M_2, i ; j \varepsilon \{[o ; E(\frac{M_2-25}{30})] \cap N]\}^2$;

$[6i+7][6j+7] < M_2, i ; j \varepsilon \{[o ; E(\frac{M_2-49}{42})] \cap N]\}^2$;

$[6i+5][6j+7] < M_2, i \ \varepsilon \{[o ; E(\frac{M_2-35}{42})] \cap N]\}$

and $j \varepsilon \{[o ; E(\frac{M_2-35}{30})] \cap N]\}$

\

$\{(2 ; 3) < M_1 ; (6n+5 ; 6n+7) < M_1, n \varepsilon \{[o ; n_{max}] \cap N]\}$

$[6i+5][6j+5] < M_1, i ; j \varepsilon \{[o ; E(\frac{M_1-25}{30})] \cap N]\}^2$;

Chapitre VI : Applications

I. To determine the prime numbers smaller than a given integer and the prime numbers between two given integers.

1) To determine the set of the prime numbers bellow than 1000 :

a) To determine the set of the « supposed » prime numbers bellow than 1000 :

$$Esp_{<1000} = \{(2; 3) < 1000 ; (6n+5 ; 6n+7) < 1000, n\varepsilon \{[o ; E(\frac{1000-7}{6}) \cap N]\}\}$$

$E(\frac{1000-7}{6}) = E(165{,}5) = 165$

The last couple is :(6x165+5; 6x165+7)=(995 ;997)

Esp< 1000 = {(2 ;3) ; (5 ;7) ; (11 ;13) ; (17 ;19) ; (23 ;25) ; (29 ;31) ; (35 ;37) ; (41 ;43) ; (47 ;49) ; (53 ;55) ; (59 ;61) ; (65 ;67) ; (71 ;73) ; (77 ;79) ; (83 ;85) ; (89 ;91) ; (95 ;97) ; (101 ;103) ; (107 ;109) ; (113 ;115) ; (119 ;121) ; (125 ;127) ; (131 ;133) ; (137;139) ; (143 ;145) ; (149;151) ; (155 ;157) ;

(161 ;163) ; (167 ;169) ; (173 ;175) ; (179 ;181) ; (185 ;187) ; (191 ;193) ; (197 ;199) ; (203 ;205) ; (209 ;211) ; (215 ;217) ; (221 ;223) ; (227 ;229) ; (233 ;235) ; (239 ;241) ; (245 ;247) ; (251 ;253) ; (257 ;259) ; (263 ;265) ; (269 ;271) ; (275 ;277) ; (281 ;283) ; (287 ;289) ; (293 ;295) ; (299 ;301) ; (335 ;337) ; (341 ;343) ; (347 ;349) ; (353 ;355) ; (359 ;361) ; (365 ;367) ; (371 ;373) ; (377 ;379) ; (383 ;385) ; (389 ;391) ; (395 ;397) ; (401 ;403) ; (407 ;409) ; (413 ;415) ; (419 ;421) ; (425 ;427) ; (431 ;433) ; (437 ;439) ; (443 ;445) ; (449 ;451) ; (455 ;457) ; (461 ;463) ; (467 ;469) ; (473 ;475) ; (479 ;481) ; (485 ;487) ; (491 ;493) ; (497 ;499) ; (503 ;505) ; (509 ;511) ; (515 ;517) ; (521 ;523) ; (527 ;529) ; (533 ;535) ; (539 ;541) ; (545 ;547) ; (551 ;553) ; (557 ;559) ; (563 ;565) ; (569 ;571) ; (575 ;577) ; (581 ;583) ; (587 ;589) ; (593 ;595) ; (599 ;601) ; (605 ;607) ; (611 ;613) ; (617 ;619) ; (623 ;625) ; (629 ;631) ; (635 ;637) ; (641 ;643) ; (647 ;649) ; (653 ;655) ; (659 ;661) ; (665 ;667) ; (671 ;673) ; (677 ;679) ; (683 ;685) ; (689 ;691) ; (695 ;697) ; (701 ;703) ; (707 ;709) ; (713 ;715) ; (719 ;721) ; (725 ;727) ; (731 ;733) ; (737 ;739) ; (743 ;745) ; (749 ;751) ; (755 ;757) ; (761 ;763) ; (767 ;769) ; (773 ;775) ; (779 ;781) ; (785 ;787) ; (791 ;793) ; (797 ;799) ; (803 ;805) ; (809 ;811) ; (815 ;817) ; (821 ;823) ; (827 ;829) ; (833 ;835) ; (839 ;841) ; (845 ;847) ; (851 ;853) ; (857 ;859) ; (863 ;865) ; (869 ;871) ; (875 ;877) ; (881 ;883) ; (887 ;889) ; (893 ;895) ; (899 ;901) ; (905 ;907) ; (911 ;913) ; (917 ;919) ; (923 ;925) ; (929 ;931) ; (935 ;937) ; (941 ;943) ; (947 ;949) ; (953 ;955) ; (959 ;961) ; (965 ;967) ; (971 ;973) ; (977 ;979) ; (983 ;985) ; (989 ;991) ; (995 ;997). }

b) To determine the set of the « non » prime numbers bellow than 1000 :

M = 1000.

Enp< 1000 = { [6i + 5] x [6j + 5]< 1000 , i ; j ε{[o ; E($\frac{1000-25}{30}$)]∩ N]}² ;
[6i + 7] x [6j + 7]<1000 , i ; j ε{[o ; E($\frac{1000-49}{42}$)] ∩ N]}² ;
[6i + 5] x [6j + 7] <1000 , i ε {[o ; E($\frac{1000-35}{42}$)]∩ N]} et jε{[o ; E($\frac{1000-35}{30}$)]∩ N]} }

E ($\frac{1000-25}{30}$) = 32, E ($\frac{1000-49}{42}$) = 22 et E ($\frac{1000-35}{42}$) = 22, E ($\frac{1000-35}{30}$) = 32.

- [6i + 5] x [6j + 5]< 1000 , i ; j ε{[o ; 32] ∩ N}.
- [6i + 7] x [6j + 7]<1000 , i ; j ε{[o ; 22] ∩ N}.
- [6i + 5] x [6j + 7] < 1000 , i ; j ε {[o ; 22] ∩ N} and j ε{[o ; 32]∩ N}.

$[6i + 5]_{22} x [6j + 5]_{22} < 1000 =$ x x < 1000

5	5
11	11
17	17
23	23
29	29
35	35
41	41
47	47
53	53
59	59
65	65
71	71
77	77
83	83
85	85
95	95
101	101
107	107
113	113
119	119
125	125
131	131
137	137
143	143
149	149
155	155
161	161

25 ; 55 ; 85 ; 115 ;
145 ; 175 ; 205 ;
235 ; 265 ; 295 ;
325 ; 335 ; 385 ;
415 ; 445 ; 475 ;
505 ; 535 ; 565 ;
595 ; 625 ; 655 ;
685 ; 715 ; 745 ;
775 ; 805 ; 835 ;
865 ; 925 ; 955 ;
985.121 ; 187 ; 253 ;
319 ; 385 ; 451 ;
517 ; 583 ; 649 ;
715 ; 781 ; 847 ;
913 ; 979 ; 289 ;
391 ; 493 ; 595 ;

•$[6i + 7]_{22} x [6j + 7]_{22} < 1000$

7
13
19
25
31
37
43
49
61
67
73
79
85
91
97
103
109
115
121

7
13
19
25
31
37
43
49
61
67
73
79
85
91
97
103
109
115
121

49 ;	91 ;
133 ;	1175 ;
217 ;	259 ;
301 ;	343 ;
385 ;	427 ;
469 ;	511 ;
553 ;	595 ;
637 ;	679 ;
721 ;	763 ;
805 ;	84 ;
889 ;	931 ;
973 ;	169 ;
247 ;	325 ;
403 ;	481 ;
559 ;	637 ;
715 ;	793 ;
871 ;	949 ;
361 ;	475 ;
589 ;	703 ;
817 ;	931 ;
625 ;	775 ;
925 ; 961	

• $[6i + 5]_{22} \times [6j + 7]_{32} < 1000 =$

5
11
17
23
29
35
41
47
53
59
65
71
77
83
89
95
101
107
113
119

x

7
13
19
25
31
37
43
49
61
67
73
79
85
91
97
103
109
115
121
127
133
139
145
151
157
163
169
175

< 10

{
35 ; 65 ; 125 ;
155 ; 185 ; 215 ;
245 ; 275 ; 305 ;
335 ; 365 ; 395 ;
275 ; 305 ; 335 ;
365 ; 395 ; 425 ;
455 ; 485 ; 515 ;
545 ; 575 ; 605 ;
635 ; 665 ; 695 ;
725 ; 755 ; 785 ;
815 ; 845 ; 875 ;
905 ; 935 ; 965 ;
995 ; 77 ; 146 ;
209 ; 275 ;
341 ; 407 ; 473 ;
539 ; 605 ; 671 ;
737 ; 803 ; 869 ;
935 ; 119 ; 221 ;
323 ; 425 ; 527 ;
629 ; 731 ; 833 ;
935 ; 161 ; 299 ;
437 ; 575 ; 713 ;
851 ; 989 ; 203 ;
377 ; 551 ; 725 ;
551 ; 725 ; 899 ;
245 ; 455 ; 665 ;
875 ; 287 ; 533 ;
779 ; 329 ; 611 ;
893 ; 371 ; 689 ;
413 ; 767 ; 455 ;
845 ; 497 ; 923 ;
539 ; 581 ; 623 ;
665 ; 707 ; 749 ;
791 ; 833 ; 875 ;
917 ; 959.

We deduce from the above the « non » prime numbers bellow than 1000:
25 ; 35 ; 49 ; 55 ; 65 ; 77 ; 85 ; 91 ; 95 ; 115 ; 119 ; 121 ; 125 ; 133 ; 143 ; 145 ; 155 ; 161 ; 169 ; 169 ; 175 ; 185 ; 187 ; 203 ; 205 ; 209 ; 215 ; 217 ; 221 ; 235 ; 245 ; 253 ; 259 ; 265 ; 277 ; 287 ; 289 ; 217 ; 221 ; 235 ; 245 ; 247 ; 253 ; 259 ; 265 ; 277 ; 287 ; 289 ; 295 ; 299 ; 301 ; 305 ; 319 ; 323 ; 325 ; 329 ; 335 ; 341 ; 343 ; 355 ; 361 ; 365 ; 371 ; 377 ; 385 ; 391 ; 395 ; 403 ; 407 ; 413 ; 415 ; 425 ; 427 ; 437 ; 445 ; 451 ; 455 ; 469 ; 475 ; 481 ; 485 ; 493 ; 497 ; 505 ; 511 ; 515 ; 517 ; 527 ; 529 ; 533 ; 535 ; 539 ; 545 ; 551 ; 553 ; 559 ; 565 ; 575 ; 581 ; 583 ; 589 ; 595 ; 605 ; 611 ; 623 ; 625 ; 629 ; 635 ; 637 ; 349 ; 665 ; 667 ; 671 ; 679 ; 685 ; 689 ; 695 ; 697 ; 703 ; 707 ; 713 ; 715 ; 721 ; 721 ; 725 ; 731 ; 737 ; 745 ; 749 ; 755 ; 763 ; 767 ; 775 ; 779 ; 781 ; 785 ; 791 ; 713 ; 715 ; 721 ; 725 ; 731 ; 737 ; 745 ; 749 ; 755 ; 763 ; 767 ; 775 ; 779 ; 781 ; 785 ; 791 ; 793 ; 799 ; 803 ; 805 ; 815 ; 817 ; 833 ; 855 ; 841 ; 845 ; 847 ; 851 ; 865 ; 869 ; 871 ; 875 ; 889 ; 893 ; 899 ; 901 ; 905 ; 913 ; 917 ; 923 ; 925 ; 931 ; 935 ; 943 ; 949 ; 955 ; 959 ; 961 ; 965 ; 973 ; 979 ; 985 ; 989 ; 995.

2) To determine the prime numbers bellow than 1000 :

We deduce from the above the prime numbers bellow than 100:
2 ; 3 ; 5 ; 7 ; 11 ; 13 ; 17 ; 19 ; 23 ; 29 ; 31 ; 37 ; 41 ; 43 ; 47 ; 53 ; 59 ; 61 ; 67 ; 71 ;73 ; 79 ; 83 ; 89 ; 97 ; 101 ; 103 ; 107 ; 109 ; 113 ; 127 ; 131 ; 137 ; 139 ; 149 ; 151 ; 157 ; 163 ; 167 ; 173 ; 179 ; 181 ; 191 ; 193 ; 197 ; 199 ; 211 ; 223 ; 227 ; 229 ; 233 ; 239 ; 241 ; 251 ; 257 ; 263 ; 269 ; 271 ; 277 ; 281 ; 283 ; 293 ; 307 ; 311 ; 313 ; 317 ; 331 ; 337 ; 347 ; 349 ; 353 ; 359 ; 367 ; 373 ; 379 ; 383 ; 389 ; 397 ; 401 ; 409 ; 419 ; 421 ; 431 ; 433 ; 439 ; 443 ; 449 ; 453 ; 457 ; 461 ; 463 ; 467 ; 479 ; 487 ; 491 ; 499 ; 503 ; 509 ; 521 ; 523 ; 541 ; 547 ; 557 ; 563 ; 569 ; 571 ; 577 ; 587 ; 593 ; 599 ; 601 ; 607 ; 613 ; 617 ; 619 ; 631 ; 641 ; 647 ; 653 ; 659 ; 661 ; 673 ; 677 ; 683 ; 691 ; 701 ; 709 ; 719 ; 727 ; 733 ; 739 ; 743 ; 751 ; 757 ; 761 ; 769 ; 773 ; 787 ; 797 ; 809 ; 811 ; 821 ; 823 ; 827 ; 829 ; 839 ; 853 ; 857 ; 859 ; 863 ; 877 ; 881 ; 883 ; 887 ; 907 ; 911 ; 919 ; 929 ; 937 ; 941 ; 947 ; 953 ; 967 ; 971 ; 977 ; 983 ; 991 ; 997.

.

Note :

Verification : « table et répartition des nombres premiers inférieurs à 10 000 (via-Google) ».

3) **To determine the prime numbers bellow than 100:**

a) To determine the "supposed" prime numbers bellow than 100:

$$Esp_{<100} = \left\{ (2\ ;3) < 100\ ;(6n+5\ ;6n+7) < 100 \text{ avec } n\varepsilon \left\{[o\ ;E\left(\frac{100-7}{6}\right) \cap N]\right\}\right.$$

$E\left(\frac{100-7}{6}\right) = E\left(\frac{93}{6}\right) = 15$; the couple (95; 97) is the last couple.

$$\Rightarrow Esp_{<100} = \left\{ \begin{array}{l} (2\ ;3)\ ;(5\ ;7)\ ;(11\ ;13)\ ;(17\ ;19)\ ;(23\ ;25)\ ;(29\ ;31)\ ;(35\ ;37)\ ;(41\ ;43) \\ (47\ ;49)\ ;(53\ ;55)\ ;(59\ ;61)\ ;(71\ ;73)\ ;(77\ ;79)\ ;(83\ ;85)\ ;(89\ ;91)\ ; \\ (95\ ;97). \end{array} \right\}$$

b) To determine the "non" prime numbers bellow than 100:

M = 100

$$Enp_{<100} = \left\{ \begin{array}{l} [6i + 5] \times [6j + 5] < 100 , i ; j \ \varepsilon \{[o ; E(\frac{100-25}{30}) \cap N]\}^2 ; \\ [6i + 7] \times [6j + 7] < 100 , i ; j \ \varepsilon \{[o ; E(\frac{100-49}{42}) \cap N]\}^2 ; \\ [6i + 5] \times [6j + 7] < 100 , i ; j \ \varepsilon \{[o ; E(\frac{100-35}{42}) \cap N]\} \text{ and } \\ \qquad j\varepsilon\{[o ; E(\frac{100-35}{30}) \cap N]\} \end{array} \right\}$$

•$[6i + 5]_2 \times [6j + 5]_2 < 100$, i ; j ε{o ; 1 ; 2}².
•$[6i + 7]_1 \times [6j + 7]_1 < 100$, i ; j ε{o ; 1}².
•$[6i + 5]_1 \times [6j + 7]_2 < 100$, i ; j ε {o ; 1 ; 2} and j ε{o ; 1}.

25;
5 5 55;
• $[6i + 5]_2 \times [6j + 5]_2 < 100$ = [11] x [11] < 100 = {85; 85;}
17 17 55; 187;
121; 289
18

c) To determine the prime numbers bellow than 100:

We deduce from the above the prime numbers bellow than 100:

2 ; 3 ; 5 ; 7 ; 11 ; 13 ; 17 ; 19 ; 23 ; 29 ; 31 ; 37 ; 41 ; 43 ; 47 ; 53 ; 59 ; 61 ; 67 ; 71 ; 73 ; 79 ; 83 ; 89 ; 97.

4) To determine the prime numbers between 100 and 1000 :

$$Ep_{100 < M < 1000} = Ep_{<1000} \setminus Ep_{<100}$$

We deduce from the above the prime numbers between 100 and 1000:

101 ; 103 ; 107 ; 109 ; 113 ; 127 ; 131 ; 137 ; 139 ; 149 ; 151 ; 157 ; 163 ; 167 ; 173 ; 179 ; 181 ; 191 ; 193 ; 197 ; 199 ; 211 ; 223 ; 227 ; 229 ; 233 ; 239 ; 241 ; 251 ; 257 ; 263 ; 269 ; 271 ; 277 ; 281 ; 283 ; 293 ; 307 ; 311 ; 313 ; 317 ; 331 ; 337 ; 347 ; 349 ; 353 ; 359 ; 367 ; 373 ; 379 ; 383 ; 389 ; 397 ; 401 ; 409 ; 419 ; 421 ; 431 ; 433 ; 439 ; 443 ; 449 ; 453 ; 457 ; 461 ; 463 ; 467 ; 479 ; 487 ; 491 ; 499 ; 503 ; 509 ; 521 ; 523 ; 541 ; 547 ; 557 ; 563 ; 569 ; 571 ; 577 ; 587 ; 593 ; 599 ; 601 ; 607 ; 613 ; 617 ; 619 ; 631 ; 641 ; 647 ; 653 ; 659 ; 661 ; 673 ; 677 ; 683 ; 691 ; 701 ; 709 ; 719 ; 727 ; 733 ; 739 ; 743 ; 751 ; 757 ; 761 ; 769 ; 773 ; 787 ; 797 ; 809 ; 811 ; 821 ; 823 ; 827 ; 829 ; 839 ; 853 ; 857 ; 859 ; 863 ; 877 ; 881 ; 883 ; 887 ; 907 ; 911 ; 919 ; 929 ; 937 ; 941 ; 947 ; 953 ; 967 ; 971 ; 977 ; 983 ; 991 ; 997.

Conclusion:

The results of this work (the integral article…) permit us to have simple method for:
Determining the prime numbers smaller than and given integer.
Determining the prime numbers between two integers..
Understanding the even prime
Building the chain of the prime numbers…

Note: There are two other articles and the four articles form a book entitled entitle the natural numbers.