A Sinusoidal Temperature Model for Major Cities in Georgia

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Abstract

One of the periodic natural phenomena in life is temperature patterns. A mathematical model based on periodicity called a sinusoidal temperature model has been formulated to describe and estimate the maximum and minimum temperature characteristics for the major cities in Georgia. The four parameters in the proposed sinusoidal temperature model that are used to predict or estimate temperature patterns are based on a thirty-year monthly means of the maximum and minimum temperature of cities in Georgia obtained from weather.com. The model shows a high level of accuracy in predicting maximum and minimum temperature for major cities in Georgia.

Introduction

Currently the United States and other developed countries invest significant resources monitoring and improving scientific understanding of weather temperature patterns. Given the significant positive and negative impacts of temperature patterns on lives and livestock throughout Georgia, there is a need to formulate a mathematical model that would describe maximum and minimum temperature through a yearly cycle of the cities in Georgia. Weather temperature is a complex phenomena that is periodic in nature and as such we base our model on the periodic functions of sine and cosine. A sinusoidal function is a function which can be expressed in the form:

$$y = D + A * sin(Bx + C) \quad or \quad y = D + A * cos(Bx + C),$$

where the parameters $A, B, C,$ and D are constants (Larson, 2007). We will restrict our model to the sine function only. This paper will take into consideration the work by McCloskey regarding seasonal temperature patterns in Ohio in 1986. Like McCloskey, the basis for the calculation of the parameters for the model is the thirty-year monthly means of maximum and minimum temperature of each of the major cities included in the study. We believe that the thirty-year means will provide us with the necessary sample size to obtain a historical base for comparison of a given year's temperatures, and the model parameters for each city will provide a quantitative way of comparing temperature patterns among the cities in Georgia. A scatter plot of the data for each of the cities clearly revealed a sinusoidal pattern.

Method and Data Set

In this research work we take the following steps to obtain the necessary required parameters for the proposed model: 1) The dataset for a thirty-year period from 1980 to 2010 for the maximum and minimum temperature for each of the major cities in Georgia were obtained from an online website weather.com. 2)

We used excel to compute the thirty-year monthly means for each of the seven cities (as displayed in Figures 1a and 1b), and to draw the scatter plot of the thirty-year monthly means (as shown in Figure 2).

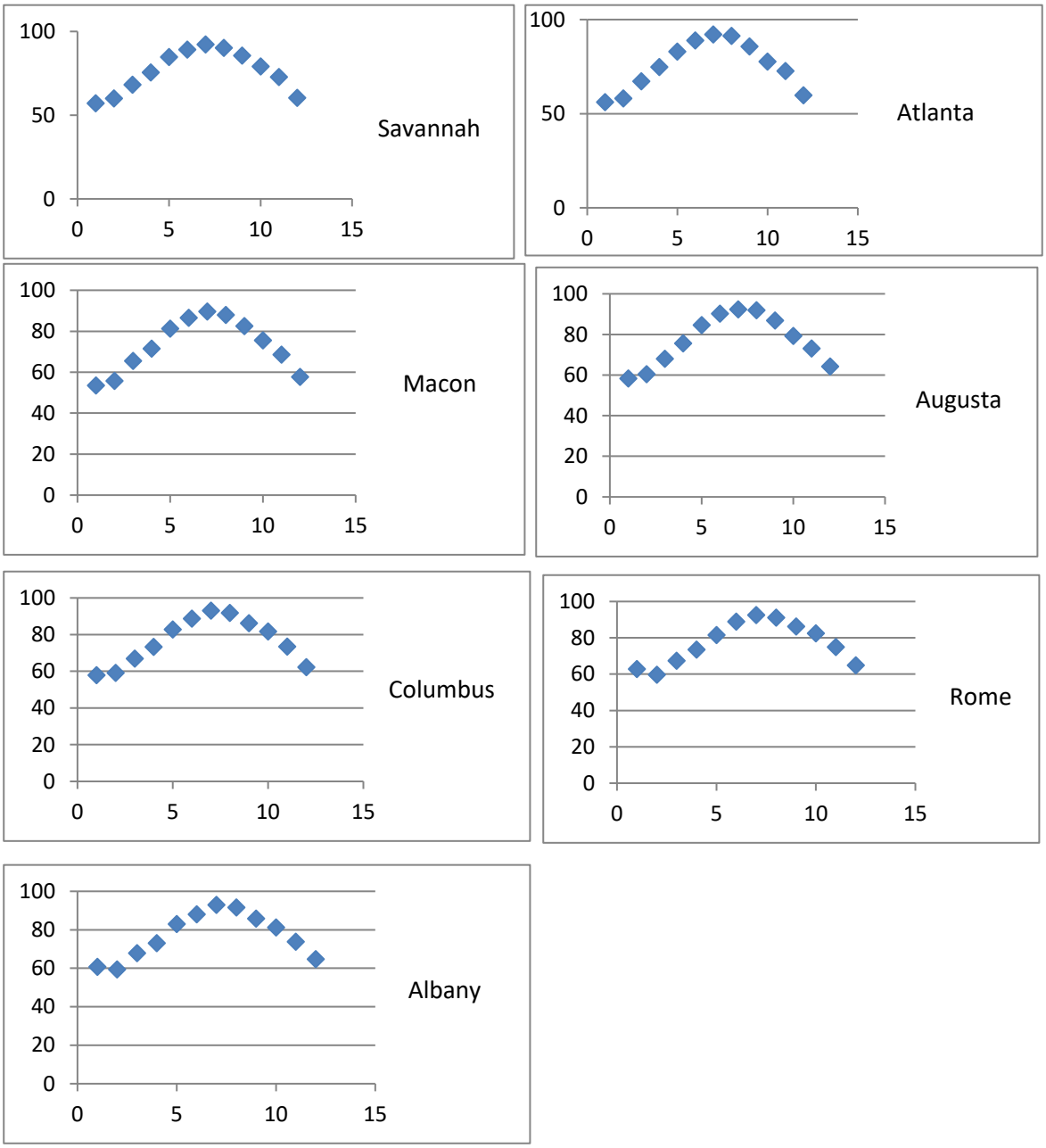
Figure 1a: Table of raw data of thirty-year monthly (maximum) means (from January to December):

Maximum Temperature							
Savannah	**Atlanta**	**Macon**	**Augusta**	**Columbus**	**Rome**	**Albany**	
57.20968	56.15484	53.53226	58.26129	57.89032	62.97097	60.81935	
60.12258	58.11613	55.76129	60.4871	59.10645	59.72258	59.48387	
68.30323	67.24839	65.55806	68.03871	66.98065	67.53548	67.86452	
75.64839	74.9	71.50323	75.57097	73.33548	73.51935	73.17742	
84.82581	83	81.35484	84.61613	82.87097	81.53548	83.01935	
89.22903	88.93548	86.55484	90.16774	88.77419	88.98065	88.19032	
92.28387	92.1129	89.63871	92.39677	93.04839	92.66452	93.0129	
90.3129	91.31935	88.06774	91.96452	91.84194	91.26452	91.74839	
85.59032	85.72258	82.60968	86.83871	86.29355	86.39677	85.83548	
79.21935	77.69677	75.57742	79.28387	81.75161	82.57419	81.29032	
72.90645	72.71613	68.70323	73.03548	73.51935	74.96774	73.89677	
60.34194	59.76129	57.75806	64.16452	62.18387	64.95161	64.75806	

Figure 1b: Table of raw data of thirty-year monthly (minimum) means (from January to December):

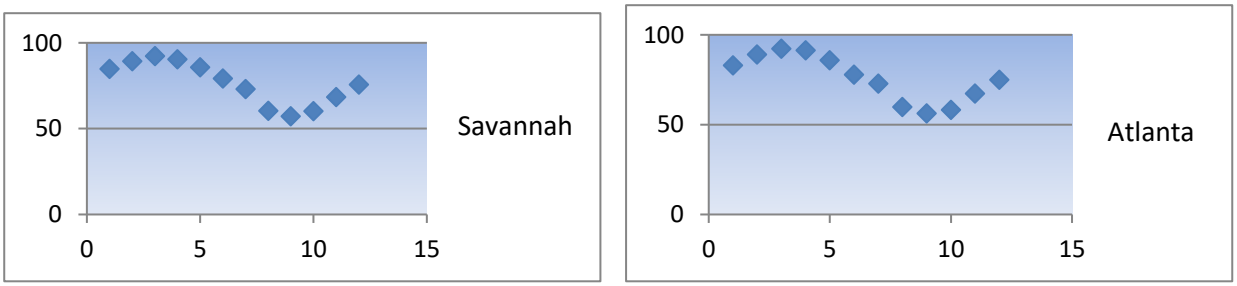
Minimum Temperature							
Savannah	**Atlanta**	**Macon**	**Augusta**	**Columbus**	**Rome**	**Albany**	
38.85806	37.61935	36.97742	39.47692	38.34348	37.60333	43.09667	
40.54194	40.67419	39.00645	43.36538	39.50435	40.95333	41.18	
48.55484	49.87742	48.57419	49.35385	46.7087	51.33	51.23667	
55.77742	53.58065	53.69355	56.56538	54.13043	58.76333	57.20333	
65.76129	64.11935	63.43548	64.46154	63.5	68.89333	66.02667	
72.91613	73.95806	73.49677	74.31154	73.37826	75.74667	76.82	
74.95484	76.13548	76.29355	77.31154	75.97391	78.67333	78.38333	
73.88065	75.79032	74.83871	75.73077	75.96957	78.32333	77.47333	
71.69355	71.36452	72.44516	72.28077	73.77826	73.69333	74.25333	
62.10323	58.53548	59.8129	60.57692	61.36087	62.29333	64.76333	
53.40968	51.32903	50.29032	52.90385	51.52609	53.14333	56.47333	
42.27097	41.02258	39.55484	45.36923	42.19565	43.78	43.56667	

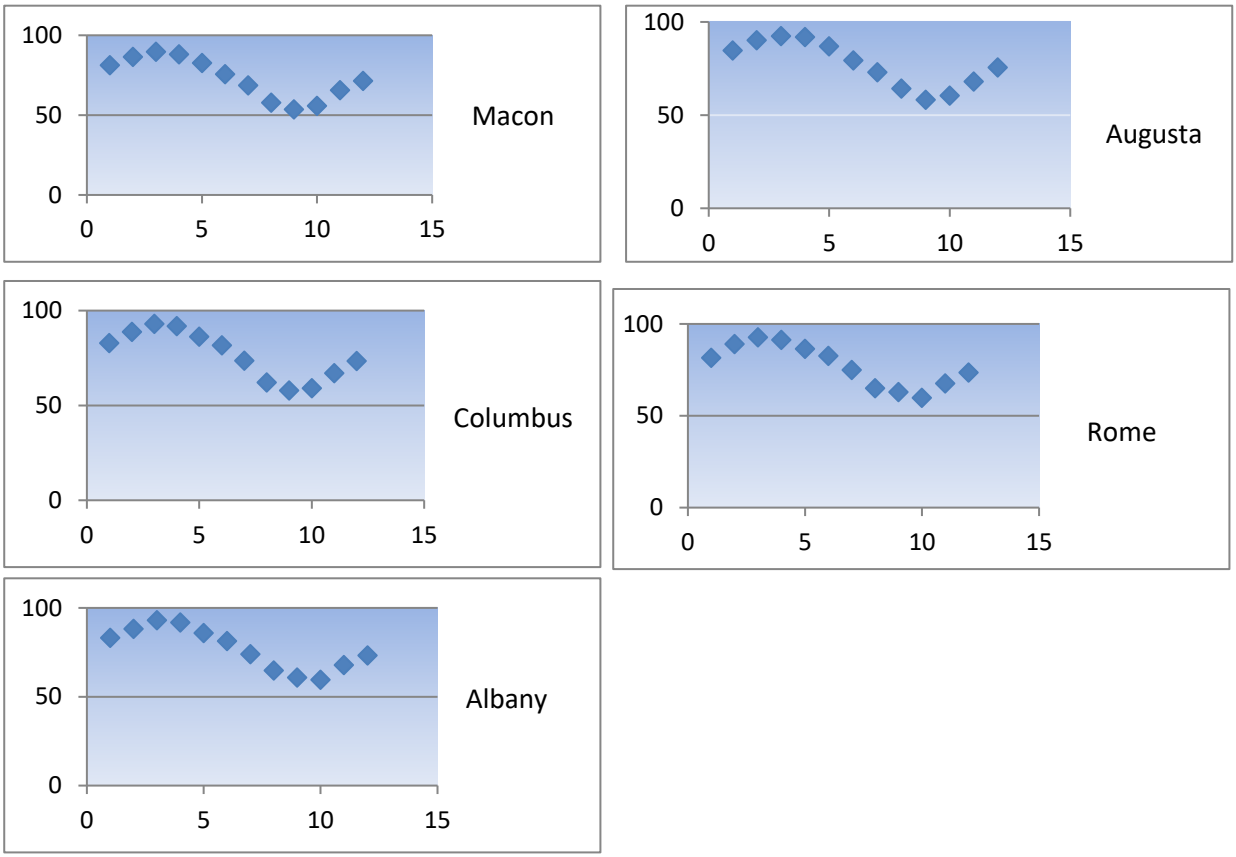
Figure 2: Graphs of raw data of thirty-year monthly means (from January to December)



3) Each of the graphs of the thirty-year monthly means was shifted (horizontal shift) to the right five units (for each city) to match the graph of the periodic function of sine (sinusoidal graph of sine).

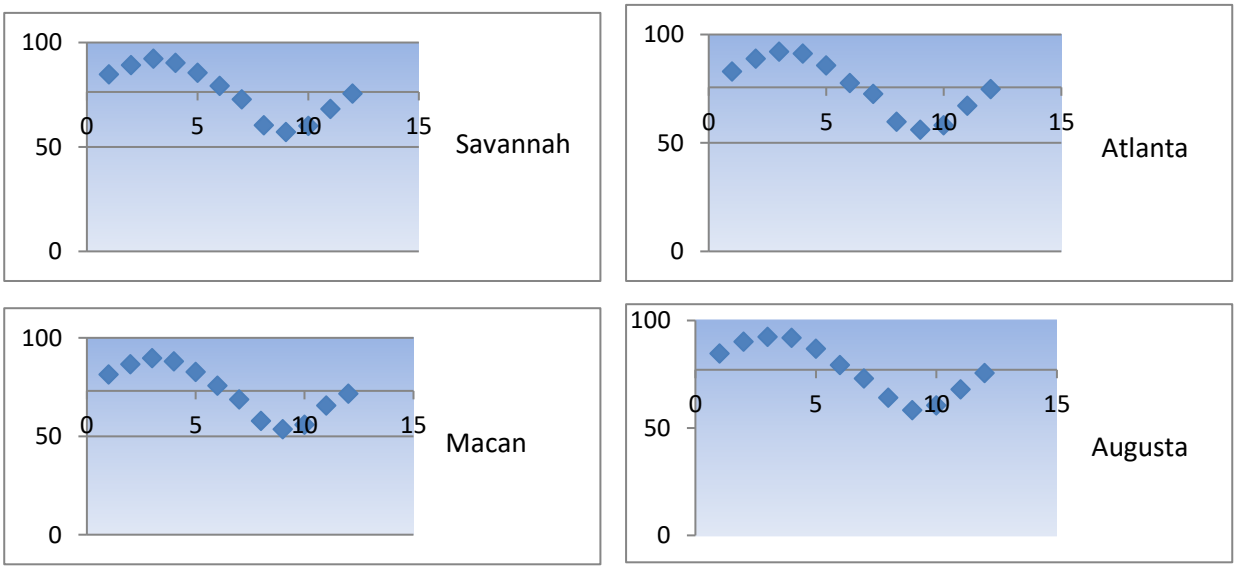
Figure 3: Graphs shifted to the right five units (from May to April)

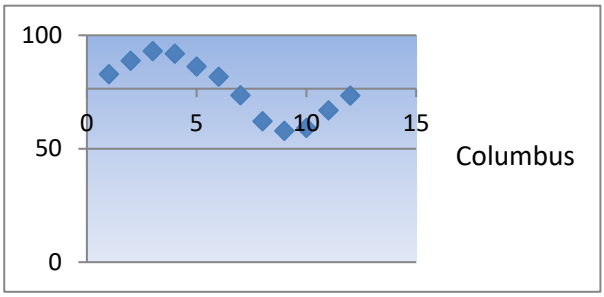


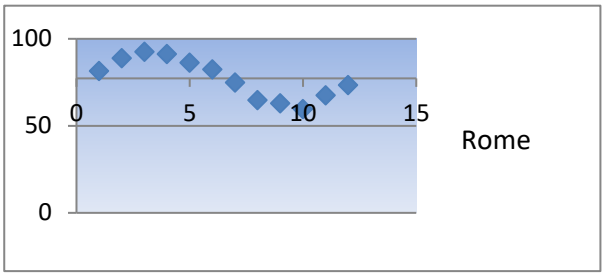


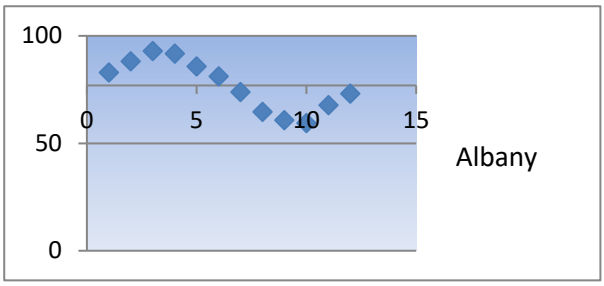
4) Each of the graphs of the thirty-year means was also shifted (vertical shift) upward **D** units to match the periodic functions of sin, where the parameter **D** is the mean yearly temperature for the given city.

Figure 4: Graphs shifted to the right five units, and up between 73-78 units









5) We used algebra to compute the numeric value for the B parameter. This was achieved by using the general formula for computing the period $(P = 12 \text{ months})$ of the sinusoidal function which is , $P = \frac{2\pi}{B}$. The parameter B for the model was found to be $\frac{\pi}{6}$. We also used algebra to compute the parameter C (the phase shift parameter indicating the beginning of the temperature cycle) as follows:

$$T = D + A * \sin(Bx + C)$$
$$= D + A * \sin\left[B\left(x + \frac{C}{B}\right)\right]$$
$$= D + A * \sin\left[\frac{\pi}{6}(x - 5)\right]$$
$$= D + A * \sin\left(\frac{\pi x}{6} - 2.62\right)$$

Calculations on the thirty-year monthly means temperature (T) were used to establish the parameters in the temperature model:

$$T = D + A * \sin\left(\frac{\pi x}{6} - 2.62\right)$$

where x is an integer code 1 to 12 to represent each month throughout the year (1 representing January, 2 for February, and so on). The parameter D is the mean yearly temperature for the given city, and the parameter B is the amplitude for the temperature variation for a given city and can be computed using the formula,

$$A = \frac{|\text{maximum temperature} - \text{minimum temperatur}|}{2} .$$

The following table provides the value of the remaining unknown parameters in the sinusoidal model:

Figure 5: Model parameters for major cities in Georgia:

	Maximum Temperature		**Minimum Temperature**						
City	**Amplitude (A)**	**Vertical Shift (D)**	**Mean**	**S.Dev.**		**Amplitude (A)**	**Vertical Shift (D)**	**Mean**	**S.Dev.**
Savannah	17.54	76.3328	76.3328	12.59		18.05	58.39355	58.39355	13.68
Atlanta	17.98	75.64032	75.64032	13.05		19.26	57.83387	57.83387	14.30
Macon	18.05	73.05161	73.05161	12.94		18.64	57.36828	57.36828	14.79
Augusta	17.07	77.06882	77.06882	12.36		19.14	59.57715	59.57715	13.50
Columbus	17.58	76.4664	76.4664	12.75		18.45	60.91801	60.91801	14.57
Rome	16.47	77.25699	77.25699	11.59		20.53	60.26639	60.26639	14.94
Albany	16.76	76.92473	76.92473	11.90		18.60	60.8736	60.8736	14.09

It is clear from the data in the above table that the major cities in Georgia have (almost) identical temperature patterns. The standard deviation is used as a measure of accuracy of the temperature model. For the maximum temperatures, the standard deviations ranged from 11.59 to 13.05 degrees Fahrenheit, while the minimum temperature ranged from 13.50 to 14.94 degrees Fahrenheit.

Results and Testing the Accuracy of the Model:

The accuracy of the model found in the previous section must be tested. This can be achieved by taking random data (months or dates) from any given year for the purpose of testing them for accuracy. The randomly selected month or date will be substituted for the variable x in the model for a given city. The degree of accuracy is measured by the percentage error as displayed in the table below:

Date	**x-value**	**City**	**Model Estimated High Temperature**	**Actual High Temperature**	**Percentage Error**
7/01/11	7.032258065	Savannah	91.65	91	0.7%
05/05/05	5.161290323	Atlanta	77.12	71.1	8%
10/22/97	10.70967742	Augusta	61.71	57	8%
12/12/88	12.38709677	Rome	63.12	66	4.3%
09/28/08	9.933333333	Albany	85.83	86	0.2%
07/06/11	7.193548387	Savannah	92.32	92	0.3%

It can be seen clearly that the estimated value is very close to the actual value. Furthermore the difference of the estimated and actual value as well as the error never exceeds 10%.

Conclusion

The proposed sinusoidal temperature model provides an excellent fit for the mean monthly temperatures for major cities throughout the state of Georgia. The model is also an important tool for the analysis of temperature variation from many different perspectives. The fact that yearly means and temperature amplitudes vary very, very slightly throughout Georgia is an indication of how identical the weather temperature patterns are throughout the state. The model can also be used to compare the temperature patterns among cities in the study.

References

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