Selection of Real Estate Investment Funds with Application of Fuzzy Logic

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Abstract

The general objective of this work is modeling a system to define which REIT should be acquired. With the identification of relevant variables to the analysis of real estate investment funds, the appropriate values for the variables are delimited using descriptive statistics and then fuzzy logic is applied to select the REIT that should be acquired. With the conclusion of this work, we identified as relevant variables: Book Equity, Manager Characteristics, Market-to-Book, Liquidity, Management Fee and Vacancy. With delimitation of the values to the variables and the use of fuzzy logic to select the REIT, it was identified a positive correlation between the Buying Decision and Yield, proving the reliability of the model within the buying decision of the REIT, according to the investor's risk profile.

Keywords: Real Estate Investment Trust; Fundos de Investimento Imobiliários; Investment Selection; Fuzzy Logic;

1. Introduction

Brazilian REITs (Real Estate Investment Trust), known as FII - Fundos de Investimento Imobiliários , give the investor the possibility of having access to large size properties, such as shopping malls, logistics warehouses, hospitals, hotels, office buildings, among others, in different locations in Brazil, without having to worry about dealing directly with issues of maintenance, acquisition or disposal of the property. The cost of quotas makes FII more affordable than an entire property, and the tax incentives granted in Brazil also turn them more interesting.

In August 2019, 183 FII there are registered in B3 (B3, 2019a), which distinguish by management type and the assets that comprise it, among other characteristics. FII attracts many Individual investors, given that investment in real estate is culturally well-regarded by Brazilians, and the distributed income is exempt from income tax, depending on certain conditions (CVM, 2015).

Choosing financial products for investment requires knowing their characteristics, which factors influence their performance, analyzing the economic environment, and identifying the investor's risk profile to obtain a satisfactory return.

With technology, the adoption of models for decision-making has grown in recent years, along with the amount of information available, making it almost impossible to assemble models with all the information. Therefore, relevant inputs have to be separated from the irrelevant ones to model the situation and analyze them, according to LACHTERMACHER (2016). A thought corroborated by DAMODARAN (2012), who says that, amid such a large amount of data, the selection of information is as important as the evaluation model chosen.

The main objective of this paper is to analyze the application of fuzzy logic to an FII selection model, with its result classified according to investment profiles. Initially, identifying the relevant variables to the analysis of real estate investment funds through documentary research. And then, the appropriate values for the variables are delimited using descriptive statistics.

2. Literature Review
2.1 Fuzzy Logic
In classical logic approach, the sets are clearly defined. Its propositions must be true or false, 0 or 1, therefore there is no ambiguity in the model (WANG, 1997, ZIMMERMANN, 2001).

In 1965, Zadeh observed that "*more often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership*", presenting ambiguity. These are fuzzy sets, which consist of classes of objects with continuous degrees of membership (ZADEH, 1965). These are classes of objects in which the transition from membership to non-membership of a given set is gradual rather than abrupt (ZADEH, 1973). This characteristic makes fuzzy logic tolerant of imprecision and partial truths.

The fuzzy set A in U is represented as a set of ordered pairs of an element x and its membership value:
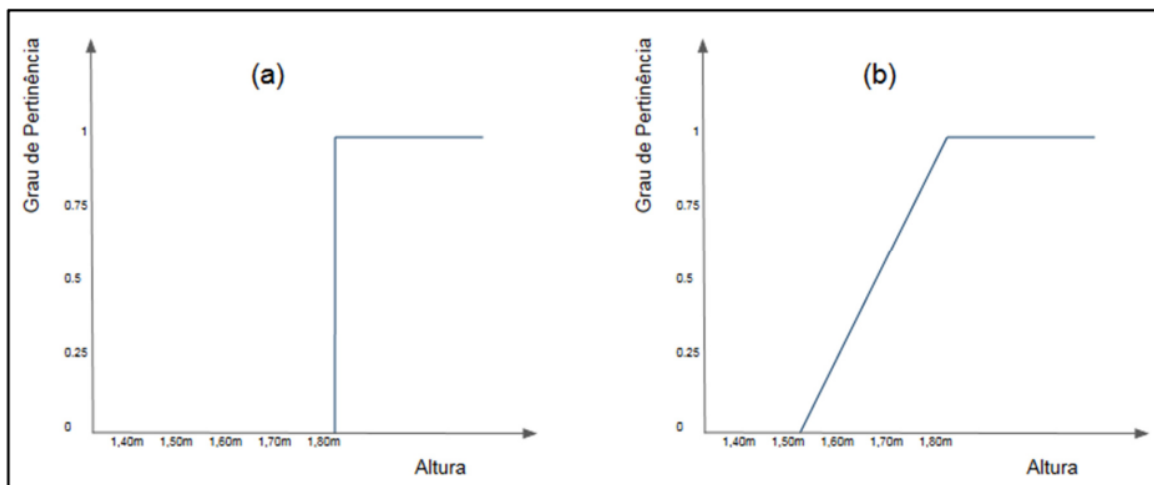
$$A = \{(x, \mu_A(x))\}, x \in U$$

Figure 1 – Membership function for variable Height where horizontal axis represents height and vertical axis is its membership function. Figure (a) represents classical set. Figure (b) represents fuzzy set.

To demonstrate graphically, consider the *Height* represented by a classical set in Figure 1 (a), where *x=1.79m* presents $\mu_A(x) = 0$ is therefore not classified as high, and to *x=1.8m* with $\mu_A(x) = 1$ is considered high, with an abrupt transition. While in Figure 1 (b), which contains a fuzzy set, there is a gradual transition of the degree of relevance of the height until reaching the value 1 at *x=1.8m*, presenting a closer relationship to the way of expressing human reasoning.

Linguistic variables and IF-THEN rules are essential for fuzzy systems. Its composition includes four components: fuzzy rule base, fuzzy inference machine, fuzzifier and defuzzifier. This system is based on knowledge or rules, called fuzzy IF-THEN rules, in which some words are characterized by continuous

membership functions (WANG, 1997). Here is an example of a fuzzy IF - THEN rule:

IF car speed is high THEN apply less force on the accelerator

in which the words "high" and "less" are characterized by membership functions.

To use the fuzzy system in engineering applications, is necessary to add a fuzzifier at the input, which transforms a real-valued variable into a fuzzy set, and a defuzzifier at the output, which transforms a fuzzy set into a real-valued variable, as shown in Figure 2.
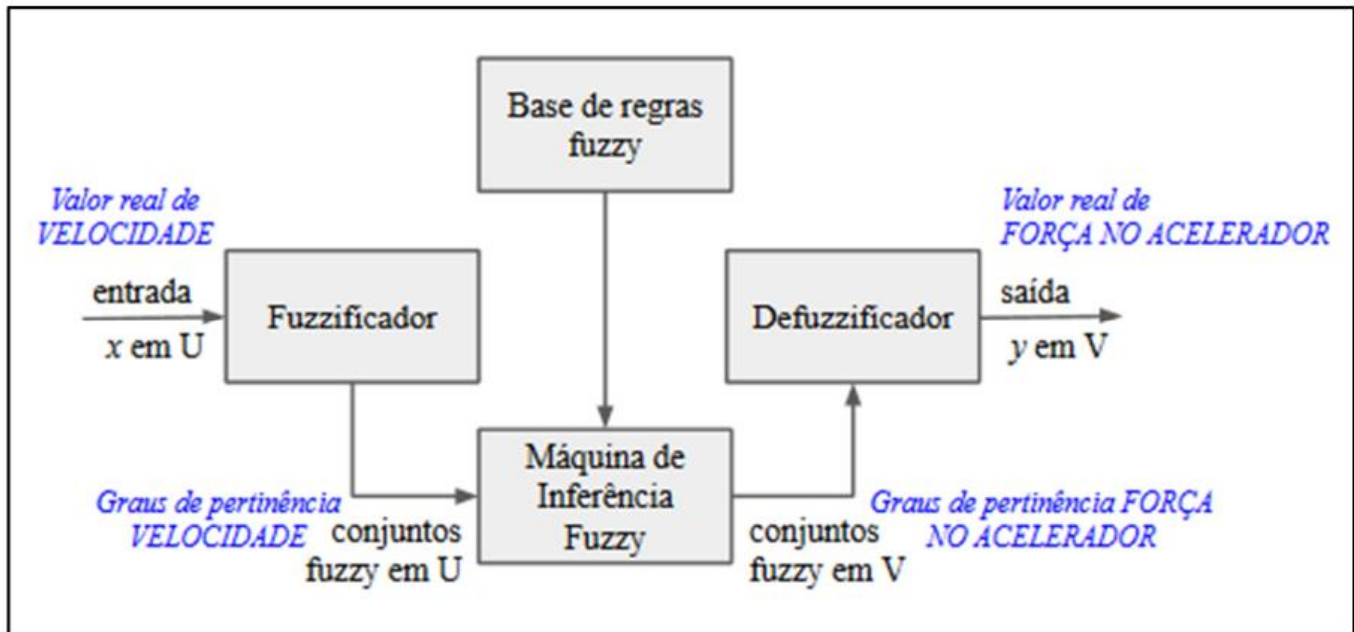

Figure 2 – Fuzzy system with fuzzifier and defuzzifier. Author: WANG(2007)

2.2 Brazilian REITs

Brazilian REITs were created by Law No. 8,668/93 and are regulated by CVM Instruction No. 472/08. These assets are traded on the stock exchange, their equity is divided into shares, which correspond to ideal fractions of their equity. The holders of FII shares cannot exercise any real right over the properties and undertakings that are part of the fund's assets.

AMATO *et al*, (2005) concludes in their study that "*some investors consider investing in assets with a high dividend yield comparable to investments in fixed income, as the payment of periodic dividends (in this case, income) can be seen as the payment of interest on a "loan" made on the acquisition of the asset. However, this logic is valid only in the event that the principal, that is, the value of the asset, is preserved over time, being fully refunded at the end of the application period, which, in the case of FII, is not guaranteed.* "

AMATO (2009) highlights the advantages of Brazilian REITs in relation to direct real estate investment: reduction of transaction costs, flexibility in the formation and demobilization of the real estate investment portfolio, agility in transactions, fractioning of the investment, potentialization of diversification, increase in liquidity, increase in of rigor in the inspection of performance and professionalization of the administration. The source of income from FIIs is mainly from the receipt of rents, interest on bonds and capital gains on the sale of assets. The distribution of income from the Brazilian REITs to the shareholders must be done within

one semester, and most funds do so on a monthly basis.

The payment of income, the exemption from income tax, under certain conditions, and the ease of acquiring shares in FII contribute to this asset becoming more attractive for investments.

The gains for the investor in Brazilian REITs occur at the time of the income distribution and acquired share's appreciation. Likewise, losses may occur in the event of a share devaluation or lack of distribution of earnings.

All investments available within the financial market or capital market have risks. The main risks arising from investing in FII shares are (CVM, 2015):

a.   market risks - financial market behavior, impact of relevant macroeconomic factors;

b.   risks related to the real estate market - real estate occupancy rate, property devaluation, regulation of the real estate sector;

c.   risks related to investment in the fund - reduced liquidity, losses and losses of financial investments.

2.3 Profitability of Brazilian REITs

A survey of scientific publications was carried out to analyze variables that influence the return of FII, in order to support the selection of investment funds and identify related searches. Most studies show that the variation in returns is associated with specific risks of the FII, due to its own characteristics, said idiosyncratic risk.

In research carried out in the United States with REITS, LITT et al. (1999) identifies that the size, in terms of book equity, has a significant negative correlation with the return variance, in other words, a fund with lower equity had a bigger variation in profitability.

In Brazil, AMATO (2007) analyzes the impacts of interest rate reduction on the value and liquidity of FII shares and finds that the funds with greater liquidity in their shares were the ones that appreciated the most in the period.

When analyzing variables that influence Brazilian REITs return, YOKOYAMA (2014) identifies the negative correlation between vacancy and FII return.

With a survey to identify the characteristics of the FII exposed in the prospectus that explain its performance, BARRETO (2016) demonstrates that Brazilian REITs with lower management fees had higher returns.

In their analysis of multi-asset FII, MORAES and SERRA (2017) state that larger multi-asset funds, measured by market value or equity, tend to offer lower diversifiable risk.

When studying the relative importance of variables that contribute to the profitability of FII, ZANANDREA (2018) identifies that the manager characteristics as a Specialist Manager, who works primarily with real estate investments or responsible for the investment decision of at least five real estate funds traded on the stock exchange, collaborates positively. In the same study, the negative contribution of the management fee was identified, corroborating the finding of BARRETO (2016).

3. Methodology

Supported by the bibliographic review, a survey of related works was made, aiming to obtain the theoretical basis to develop the following stages and identify the relevant variables for the research. From this documental

and bibliographic research, variables that have a positive and negative relationship with the FII were identified, as well as criteria to be observed in the choice of investment, which will be applied in the fuzzy system rule base.

After this step, data were collected to be applied in the fuzzy system. The funds selected were those listed on B3, and their data was retrieved from monthly reports, management reports and quotation history available on the B3 website.

After obtaining the data, the descriptive statistics of each variable were examined, using the R tool, and in this way the values of each corresponding fuzzy set were delimited. And finally, the elaboration of the fuzzy system, with the inclusion of variables, their pertinence functions and rule base to evaluate the results according to the FII data using the Matlab tool.

3.1 Input Variables

Based on the literature, among the variables raised, those applicable to the analysis of the FII were selected, those that were related to their profitability and therefore were highlighted for this research: Book Equity, Manager Characteristics, Market-to-Book, Liquidity, Management Fee and Vacancy. The selected variables are related to the characteristics of the FII (Book Equity, Manager Characteristics and Management Fee) and also to the market (Market-To-Book, Liquidity and Vacancy).

Here are the linguistic values that were applied to the variables and the relationship with the fund's return.:

a. Book Equity – Positive relationship with the performance of the FIIs. Low, medium and high were the values within the linguistic set;

b. Manager Characteristics – It was defined that a specialist manager is the one who manages 5 or more Brazilian REITs. The manager specialized in the real estate market has a positive influence on the FII return. The value of the variable is 1 when the fund manager is considered an expert, and 0 (zero) otherwise;

c. Market-to-Book - significant especially for fixed income funds. Also known as Price-to-Book ratio P/B, its calculation is the traded price of the share (last month closing price) over the Book Equity by the number of shares (both from the month immediately preceding the price of the share). When the MTB value is less than 1, it can be a bargain or greater risk of underperformance. If the MTB is equal to 1, the price can be considered fair. And if the MTB is greater than 1, the market price is above the book equity, which may indicate the fund's quality perceived by the market, but in terms of expected profitability it is above the represented by the book equity;

d. Liquidity - present a positive relationship with the performance of Brazilian REITs. It is also related to the risk of the FII, since a fund with less liquidity represents greater difficulty in getting rid of the quota. The value adopted was the average daily trading volume of securities in the month. The linguistic sets applied to liquidity were low, medium and high;

e. Management Fee - although a higher return is expected with the payment of a higher management fee, the literature portrays the opposite: that lower management fees had a higher return. The value of the variable is a percentage of management fee expense on the fund's equity. There are 3 linguistic values: low, medium and high;

f. Vacancy - has a negative correlation with the return of fund. It is a percentage of the leasable physical area. The variable is applied especially to FIIs backed by pre-existing properties, as there is no way to evaluate the information for funds backed by bonds or under development. Its linguistic values are: low, medium and high.

3.2 Sample and Data

The research covered the period from August/2019 to January/2020. In order to make it possible to obtain the input variables necessary for the application of the model and data analysis, the funds that met the following criteria were included:

- sent monthly reports and management reports to B3, with information on book equity, management fee and vacancy from July to December 2019;
- present the registration data available on the ANBIMA website, referring to the administrator and manager of the fund;
- have been traded on B3, to obtain the amounts of liquidity and trading value of the share from August/2019 to January/2020.

3.3 Membership Functions

The membership sets for the application of fuzzification were obtained by applying descriptive statistics to each of the variables, identifying the minimum, maximum, mean and median values.

The model is applied to each month, so the values of the linguistic sets of each variable are formed according to the market behavior at the time of collection and the information updated by the FII, making the analysis explicit and objective, without losing the characteristics related to the gradual transition between sets.

3.4 Fuzzy Rule Base and Output Variable

Each of the input variables already has its linguistic sets defined, and their respective weights are assigned, varying between 0 and 1, meaning that the lowest value has the worst performance and the highest value has a better result for the Brazilian REITs, as shown in Table 1.

Table 1. Values assigned to linguistic sets of input variables

Variable	Low	Medium	High	Specialized	Not Specialized
Book Equity	0	0,5	1	-	-
Manager Characteristic	-	-	-	1	0
Market-to-Book	0	1	0,5	-	-
Liquidity	0	0,5	1	-	-
Management Fee	1	0,5	0	-	-
Vacancy	1	0,5	0	-	-

To build the fuzzy rule base, a combination of all variables is made, which are independent, each one with all

its linguistic sets, in this way the sum of the weights assigned to the variables indicates what the purchase decision should be, the output, represented in Table 2 and fuzzified in Figure 3.

Table 2. Rules for the output variable Purchase Decision

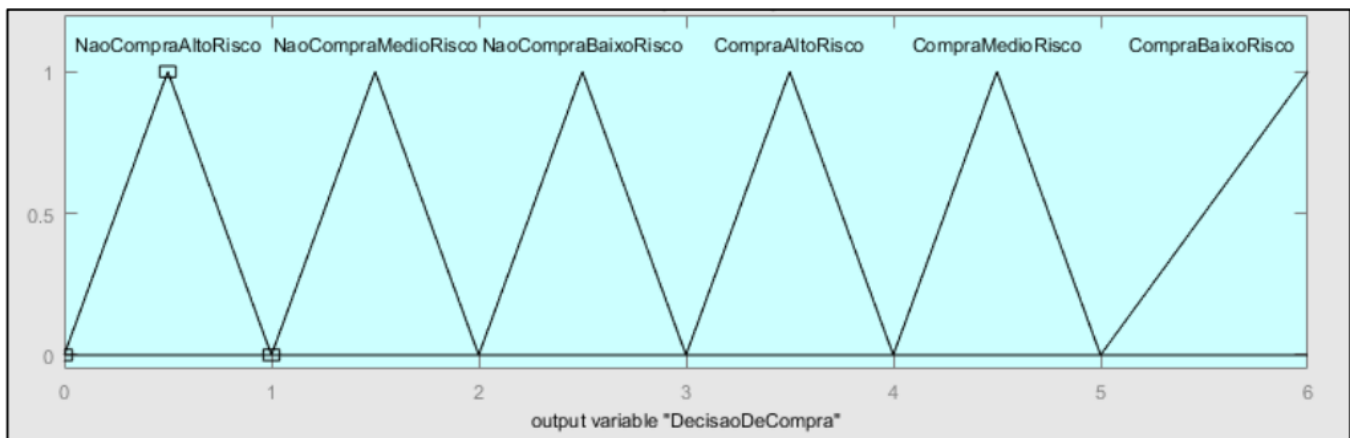|	Rule	Linguistic Value
Purchase Decision	$0 \leq sum < 1$	Don't purchase high risk
	$1 \leq sum < 2$	Don't purchase medium risk
	$2 \leq sum < 3$	Don't purchase low risk
	$3 \leq sum < 4$	High risk purchase
	$4 \leq sum < 5$	Medium risk purchase
	$5 \leq sum \leq 6$	Low risk purchase

Figure 3 – Fuzzified output variable Purchase Decision.

4. Results

Although there were 204 FIIs with at least one variable in August 2019, in order to apply the model, it is necessary for the fund to present all the input variables, which implied the selection of only those that met this criterion, resulting in the permanence of 56 Brazilian REITs. The Table 3 shows the numerical descriptions of the input variables, except for the Manager Characteristics, which was treated in binary mode.

Table 3. August 2019 input variables

	Book Equity (R$)	P/B	Liquidity (R$)	Management Fee(%)	Vacancy (%)
Minimum	25942373,81	0,48	21102,58	0,0063	0
Mean	469353069,51	1,08	1274889,05	0,0526	11,72
Median	220725824,33	1,09	348125,81	0,0347	3,20
Maximum	3559171465,75	1,48	8052429,16	0,6419	100,00

With the great amplitude presented between the minimum and maximum values of the Book Equity variable, a logarithmic transformation was applied, show in Table 4.

Table 4. August 2019 input variables with natural logarithm of Book Equity

	Ln(Book Equity) (R$)	P/B	Liquidity (R$)	Management Fee(%)	Vacancy (%)
Minimum	20,34	0,48	21102,58	0,0063	0
Mean	23,56	1,08	1274889,05	0,0526	11,72
Median	23,64	1,09	348125,81	0,0347	3,20
Maximum	25,33	1,48	8052429,16	0,6419	100,00

Each of the input variables for the month of August/2019 was fuzzified with the values identified from Table 4. Linguistic variables are represented by triangular numbers, with the following composition:
- Low Value Set = (0, minimum, median);
- Average Value Set = (minimum, median, maximum);
- High Value Set = (median, maximum, maximum).

The Figure 4 shows that the model was applied with the Mamdani applied inference type, widely used and appropriate for rules based on human knowledge (MATLAB, 2020). The defuzzification method adopted was the centroid, often used because it presents good results (MARQUES, 2017; MATLAB, 2020).
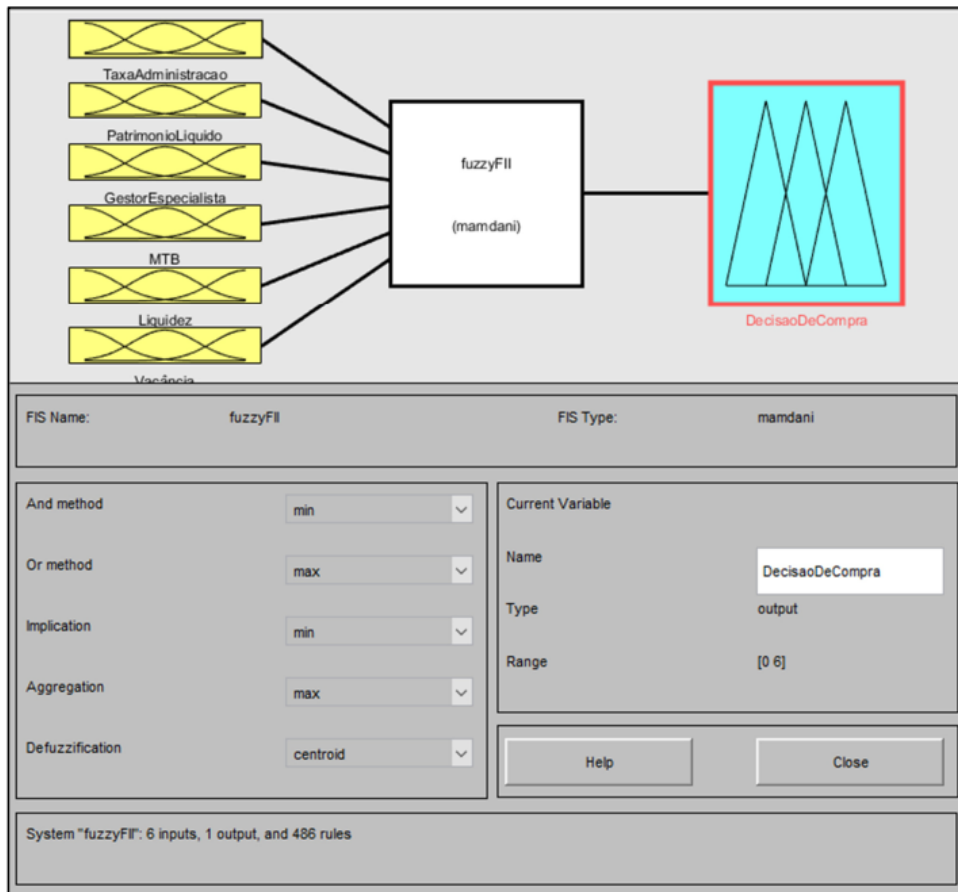
Figure 4 – Inference Model.

The membership functions of the input and output variables are included in the model, along with the rules that make up the inference engine, producing 486 possible combinations defining the rule base, shown in Figure 5.

Figure 5 – Inference Model.

After applying the model in August 2019, the following fund amounts were obtained for each purchase decision:

- Low risk purchase – 2;
- Medium risk purchase – 24;
- High risk purchase – 22;
- Don't purchase low risk – 6;
- Don't purchase medium risk – 1;
- Don't purchase high risk – 1.

In applying the model, the HGBS11 fund showed a degree of relevance (GP) of 5.24 and the KNRI11 fund resulted in 5.15. Both of them fall into the Low Risk Purchase set, as shown in Table 5, whose rules are generally defined in Table 2.

Table 5. Low risk purchase FII for August 2019

Input						Output
Ln (Book Equity) (R$)	**Manager Charact.**	**P/B**	**Liquidity (R$)**	**Management Fee (%)**	**Vacancy. (%)**	**GP**
25,33 High	1 Yes	1,18 Medium and High	6532617,66 Medium and High	0,0594 Medium and High	4,87 Medium and High	5,24
25,33 High	1 Yes	1,05 Low and Medium	6071280,76 Medium and High	0,113 Medium and High	3,01 Low and Medium	5,15

Applying the model for all months, we have the Table 6 with 15 funds classified as Low Risk Purchase.

Table 6. Low risk purchase FII for August 2019 – January 2020

Month	Year	FII	Input						Output
			Ln (Book Equity) (R$)	**Manager Charact.**	**P/B**	**Liquidity (R$)**	**Management Fee (%)**	**Vacancy. (%)**	**GP**
8	2019	HGBS	25,33 High	1 Yes	1,18 Medium and High	6532617,66 Medium and High	0,0594 Medium and High	4,87 Medium and High	5,24
8	2019	KNRI	25,33 High	1 Yes	1,05 Low and	6071280,76 Medium and	0,113 Medium and	3,01 Low and	5,15

					Medium	High	High	Medium	
9	2019	KNRI	25,33 High	1 Yes	1,06 Low and Medium	7354659,58 Medium and High	0,1083 Medium and High	2,78 Low and Medium	5,33
9	2019	RBED	23,14 Low and Medium	1 Yes	1,05 Low and Medium	524787,54 Medium and High	0,0139 Low and Medium	0 Low	5,13
9	2019	HGLG	25,33 High	1 Yes	1,2 Medium and High	5509506,33 Medium and High	0,0601 Medium and High	6,8 Medium and High	5,12
9	2019	ABCP	25,23 Medium and High	1 Yes	1,26 Medium and High	445565,09 Medium and High	0,0083 Low and Medium	0,9 Low and Medium	5,11
9	2019	HGBS	25,33 High	1 Yes	1,17 Medium and High	5183693,56 Medium and High	0,0582 Medium and High	4,88 Medium and High	5,09
10	2019	FVPQ	24,69 Medium and High	1 Yes	1,02 Low and Medium	1191067,49 Medium and High	0,0119 Low and Medium	2,5 Low and Medium	5,14
10	2019	ABCP	25,23 Medium and High	1 Yes	1,27 Medium and High	361565,78 Low and Medium	0,0083 Low and Medium	1 Low and Medium	5,06
10	2019	KNRI	25,33 High	1 Yes	1,13 Medium and High	8252470,1 Medium and High	0,1251 Medium and High	5,05 Medium and High	5,05
10	2019	HGLG	25,33 High	1 Yes	1,27 Medium and High	6111776,23 Medium and High	0,0681 Medium and High	6,8 Medium and High	5
12	2019	ABCP	25,33 High	1 Yes	1,29 Medium and High	287268,79 Low and Medium	0,0083 Low	1 Low and Medium	5,2
12	2019	HGRU	25,33 High	1 Yes	1,29 Medium and High	2624038,97 Medium and High	0,0705 Medium and High	0 Low	5,16

| 1 | 2020 | HGRU | 25,33 High | 1 Yes | 1,21 Medium and High | 14471817,62 Medium and High | 0,0763 Medium and High | 0 Low | 5,26 |
| 1 | 2020 | ABCP | 25,33 High | 1 Yes | 1,25 Medium and High | 3421072,12 Medium and High | 0,0083 Low and Medium | 1,3 Low and Medium | 5,23 |

All months showed indication of purchase of Low Risk Purchase, except the month of November/2019. Highlight for the following points in the Brazilian REITs from Table 6:

- All have a specialist manager, which, as it is a binary variable, has a great impact on the result;
- In 11 of 15 funds, the Book Equity presented the maximum value, classified as High, resulting in better performance for the fund;
- In the 4 funds in which the Equity variable is offset by the good performance of the MTB, Management Fee and Vacancy variables.

The variables related to the characteristics of the FII (Book Equity, Manager's Characteristics and Management Fee) showed values that had greater influence on the indication of Low Risk Purchase compared to the variables related to market behavior (MarketTo-Book, Liquidity and Vacancy).

Thereafter, we proceed to analyze the result of the model using the income of the FII. For this, we collected the income of each FII of the sample universe, for a period of 12 months after the indicated purchase decision. It was expected that the Brazilian REITs indicated as Low Risk Purchase would present a higher frequency of income payments for the lowest suggested risks and lower payment frequency for the highest risks.

The examination of the income of the FII in the 12 months following the fuzzy inference performed was translated by the percentage of the accumulated income, presented in Table 7.

Table 7 - Percentage of FII's Accumulated Income in the 12-month period after purchase decision

	Minimum	Maximum	% Amplitude
Low Risk Purchase	2,63	7,39	4,76
Medium Risk Purchase	0,31	26,92	26,61
High Risk Purchase	0	131,47	131,47
Don't Purchase Low Risk	0,82	96,75	95,93
Don't Purchase Medium Risko	0	5,95	5,95
Don't Purchase High Risk	0	0	0

Current legislation, art. 10, p.u., of Law 8.668/93, determines that the FII must distribute to its shareholders

at least 95% of the profits earned every six months, so there are 2 semesters for examination. It should also be noted that the income is an amount paid separately to the shareholder in his account, and is not incorporated into the price of the share.

Income cannot be expressed as a negative number. The worst result that can be obtained is the absence of income. For this aspect, the accumulated income per set of Purchase Decisions was examined.

The sets that show performance in all FIIs were Low Risk Purchase, Medium Risk Purchase and Don't Purchase Low Risk. The others contain in their items at least one FII that had no income distribution. Altogether we have 5 FII without income distribution, and some appear in more than a month: ALMI11, ATSA11, BBVJ11, HTMX11 and XTED11.

The Low-Risk Purchase set had a lower range of income percentage, and also all of its FII made payments, testifying to the expected results for this purchase indication.

In the positive buy indication set, the amplitude increases as the risk of the buy indication increases. In the set with the refusal to buy, the opposite occurs, but it is the group in which there is more lack of payments.

5. Conclusions

This paper carried out the modeling of a fuzzy system to define which Brazilian REITs should be acquired, classifying the purchase indication according to the risk profile, and a satisfactory result was identified.

Six input variables were used for the model, 3 related to FII characteristics (Book Equity, Manager's Characteristics and Management Fee) and 3 related to market behavior (MarketTo-Book, Liquidity and Vacancy).

With the result presented by the model, it was identified that the variables related to the characteristics of the FII presented values that had greater influence on the indication of Low Risk Purchase in comparison with the variables related to the behavior of the market.

To validate the result, the profitability of the funds was analyzed for the 12 months following the application of the model.

Profitability presented a positive correlation with the Purchase Decision of the positive purchase indication sets, demonstrating the model's reliability in the purchase decision of FII according to the risk tolerance profile.

The set with a negative purchase indication had the largest amount of funds that did not pay income, which is the worst expected result in relation to income payment.

This article presents an indicator to help in the screening of the FII that will be analyzed, given the amount of funds available and information to be studied at the time of selection for purchase. It does not diminish the importance or remove the responsibility of the interested party to know the fund's prospectus, fund manager and management reports.

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